Chapter- 13

Some Properties of Fuzzy Chordal Graphs

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Abstract
This paper explores properties of fuzzy chordalgraphs as initially defined by John N. Moder- son and Premchand S. Nair. We examine these graphs in comparison to chordal graphs within crisp graph theory and fuzzy chordal graphs, providing illustrative counter examples where appropriate.

Keywords: Fuzzy graph,fuzzy chordal graph,eccentricity

1 Introduction

Graphs are a widely used tool for modeling relationships, where vertices rep- resent objects and edges denote connections between them. When ambiguity exists in defining these objects or their interconnections, a fuzzy graph model becomes useful. The concept of fuzzy subsets, introduced by L.A. Zadeh [**1**]in 1965, provided a foundation for managing uncertainty. Later, Rosen- feld [**5**]formalized the fuzzy graph model in 1975, allowing Yeh and Bang[**6**] to explore various connectivity concepts in fuzzy graphs, such as vertex and edge connectivity, and to apply these models in data clustering. Influential fig- ures like Bhutani, Moderson, Vijayakumar, Sunitha, and Sunil Mathew further advanced fuzzy graph connectivity concepts following Zadeh and Rosenfeld's pioneering work.

In crisp graph theory, chordal graphs hold significance due to their treelike structure composed of complete graphs, which enables some NP-hard problemsto be solved in polynomial time. A chordal graph is characterized as a simple graph where every cycle longer than three has an additional edge, or "chord," connecting two non-consecutive vertices within the cycle.[2]

2 Preliminaries

2.1 Definition

[9, 8] Let $G(V, E)$ be a graph with the pair of mappings $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that the condition $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $(x, y) \in E \subseteq V \times V$, then $G(V, E, \mu, \sigma)$ is called a **fuzzy graph**.

2.2 Definition

[7, 8, 9] A **Path** P of length n is a sequence of distinct nodes x_0, $x_1, x_2, \ldots, x_n$ such that $\mu(x_{i-1}, x_i) > 0$, for $i = 1, 2, \ldots, n$ and the degree of membership ofthe weakest arc is defined as its strength. If $x_0 = x_n$ and n 3 then P is called a cycle

2.3 Definition

[7, 8, 9, 4] The strength of connectedness between two nodes u and v is defined as the maximum of strengths of all paths between u and v and is denoted by CONN-G(u, v).

2.4 Definition

[9] A chord (u, v) in a cycle of length greater than 3 of a fuzzy graph $G(V, E, \mu, \sigma)$is said to be a **fuzzy strong chord** if $\mu(u, v)$ $S(u, v)$, where $S(u, v)$ is the strength of connectedness between the vertices u and v.A graph has fuzzy strong chord then it is fuzzy chordal graph

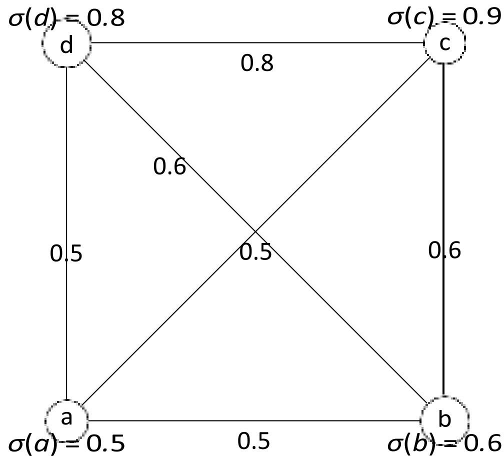
3.Properties of Fuzzy Chordal Graphs

Theorem

[3, 9, 10] Every complete fuzzy graph is fuzzy chordal

Example

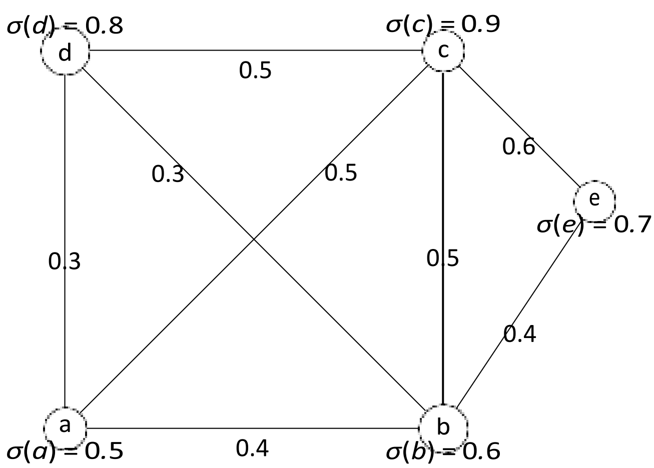
Let G be a complete fuzzy graph (see figure) with four vertices a, b, c, d and(a, c) and (b, d) are fs-chord. so$G$ is fuzzy chordal



Remark

The converse of above need not true.There exist fuzzy chordal graph which isnot complete fuzzy graph

Example



Here this graph fuzzy chordal but not complete fuzzy graph.its membershipvalue not exactly minimum of membership values of vertices. .

Theorem [3]

If G is chordal and connected, then

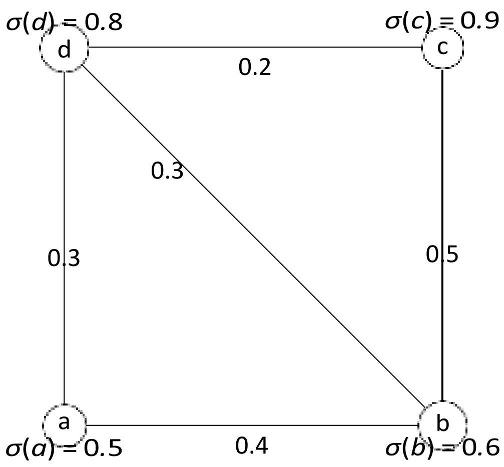
$$\frac{1}{2}d(G) \leq r(G) \leq \left\lceil \frac{1}{2}d(G) \right\rceil + 1.$$

Proposition

If G is fuzzychordal and connected.Then

$$\frac{1}{2}d(G) \leq r(G) \leq \left\lceil \frac{1}{2}d(G) \right\rceil + 1.$$

Example



$d_f(a, b) = 0.4 d_f(c, b) = 0.4$
$d_f(d, b) = 0.4$
$d_f(d, c) = 0.2$
$d_f(a, d) = 0.3$
$d_f(a, c) = 0.4$

eccentricity
$e(a) = 0.4$
$e(b) = 0.4$
$e(c) = 0.4$
$e(d) = 0.4$

$d(G) = 0.4$
$r(G) = 0.4$
so

$$\frac{1}{2}d(G) + 1 = 1.2$$

The inequality satisfied

4. Conclusion

In conclusion, this paper has presented an in-depth exploration of fuzzy chordalgraphs, distinguishing them from crisp chordal graphs by examining their unique properties and connectivity structures. Through theoretical analysis and illustrative examples, we demonstrated that while every complete fuzzy graph is fuzzy chordal, the converse does not necessarily hold, underscoring the nuances in the relationship between completeness and chordality in fuzzy graph theory. The study of fuzzy strong chords, connectivity strengths, and the specific conditions required for a graph to be fuzzy chordal highlight the adaptability of these graphs in capturing relationships marked by uncertainty. Applications of fuzzy chordal graphs are particularly relevant in fields where ambiguous connections arise, such as data clustering, network analysis, and fields requiring flexible yet structured graph representations. These findings contribute to a deeper understanding of fuzzy graph theory and lay the ground-work for future research to further explore complex fuzzy graph structures and potential practical implementations. Future studies could investigate ex- tensions of fuzzy chordal graph properties and their real-world applications, potentially enhancing methodologies for managing uncertainty in complex sys-tems.

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