INTERNATIONAL CONFERENCE ON ALGEBRAIC GRAPH THEORY, GRAPH THEORY AND TOPOLOGY

9th & 10th January 2025

CONFERENCE PROCEEDINGS (ICAGT-2025)

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Organised by

RESEARCH DEPARTMENT OF MATHEMATICS

NESAMONY MEMORIAL CHRISTIAN COLLEGE

Marthandam - 629 165

Kanniyakumari District, Tamil Nadu

Affiliated to Manonmaniam Sundaranar University
Re-accredited with 'A' Grade by NAAC
42nd position in National Rankings - 2024 for colleges by NIRF

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Editors

Dr. S. Asha

Dr. D. Nidha

Dear Delegates, Researchers and Readers,

I am delighted to share this message through the Proceedings of the International Conference on Algebraic Graph Theory, Graph Theory and Topology. This conference exemplifies the power of mathematics in bridging geographical gaps, fostering collaboration and advancing knowledge.

Algebraic Graph Theory and Topology have profound implications in pure mathematics, data science, artificial intelligence and network analysis. Our hybrid conference featured esteemed mathematicians worldwide, showcasing the diversity and depth of contributions from leading researchers and emerging scholars.

I extend my sincere gratitude to the authors, reviewers and editors, for their tireless efforts in making this publication possible. I also applaud the organizing committee's vision and hard work in making this conference a resounding success.

I hope these proceedings inspire the scholarly community to further develop these fascinating fields of mathematics.

Warm regards and best wishes,

Prof. Dr. A. J. S. Pravin
Correspondent/Secretary
Nesamony Memorial Christian College
Marthandam

Dear Delegates and Participants,

I am extremely excited that the Research Department of Mathematics is hosting an International Conference on Algebraic Graph Theory, Graph Theory and Topology on January 9 and 10, 2025.

I extend my warmest greetings to the contributors and readers of the International Conference Proceedings. This collection showcases outstanding research, creative thinking and teamwork from global scholars, researchers and students. The conference's topics - Algebraic Graph Theory, Graph Theory and Topology - illustrate the connections between pure mathematics and various fields including science and technology.

The proceedings serve as a valuable resource for further research and a testament to the concepts explored during the conference. I express my sincere gratitude to the organizers for compiling this collection and commend the authors to advance in mathematical knowledge.

May the proceedings inspire readers and scholars to push the boundaries of knowledge, foster meaningful collaborations and promote mathematics' continued growth as a science that shapes our understanding of the world.

Congratulations to all contributors. I wish you great success in your future endeavours.

Warm regards,

Dr. R. Sheela Christy
Principal *i/c*
Nesamony Memorial Christian College
Marthandam

Dear colleagues and participants,

It is with immense pleasure and pride that I welcome you to the proceedings of the "International Conference on Algebraic Graph Theory, Graph Theory and Topology". This event marks a significant step in advancing research and fostering collaboration in the realms of mathematics and its interdisciplinary applications.

Algebraic Graph theory, Graph theory and Topology are pivotal areas of study, offering powerful tools and insights that span various scientific domains. By bringing together scholars, researchers, and practitioners from around the globe, this conference aims to spark innovative ideas and promote meaningful discussions.

I extend my heartfelt gratitude to the organizing committee, contributors, and participants for their dedication and commitment to making this event a success. Your collective efforts serve as a testament to the vibrant intellectual community that we are privileged to be part of.

As you engage with the content of this publication, we invite you to reflect on the broader implications of these works and their potential to create meaningful impact in our fields and beyond. May this document serve as a valuable resource for advancing your endeavours and fostering further collaborations.

Best wishes for a fruitful and engaging experience!

Sincerely,
Dr. A. Pramila Inpa Rose
Head, Department of Mathematics
Nesamony Memorial Christian College
Marthandam

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SOME NEW GRAPH PARAMETERS

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Abstract

In this talk we shall have an insight into the following four new graph parameters.

(i) Strongly Regular graphs
(ii) Triameter of a graph
(iii) Proper diameter of a graph
(iv) Neighbourhood Polynomial of a graph

A graph G is said to be strongly regular with parameters $(n; k; \lambda; \mu)$ if it is a k-regular n-vertex graph in which any two adjacent vertices have λ common neighbours and any two non-adjacent vertices have μ common neighbours.

The triameter of G is defined as $max\{d(u,v) + d(v,w) + d(u,w) : u, v, w \in V(G)\}$ and is denoted by $tr(G)$. It is obvious that $3 \le tr(G) \le 2n-2$. We determine several upper bounds for this parameter and prove that they are best possible. We also determine the relationship between this parameter and several other parameters.

A proper edge-coloring of a graph is a coloring in which adjacent edges receive distinct colors. A path is properly colored if consecutive edges have distinct colors, and an edge-colored graph is properly connected if there exists a properly colored path between every pair of vertices. In such a graph, we introduce the notion of the graphs proper diameter which is a function of both the graph and the coloring and define it to be the maximum length of a shortest properly colored path between any two vertices in the graph.

For $0 \le i \le n-2$, the $i-$common neighbor set of G is defined as $N(G,i) = \{(u,v) : u,v \in V(G), u \neq v \text{ and } |N(u) \cap N(v)| = i\}$. The common neighbor polynomial of G denoted by $N[G;x]$ is defined as $N[G;x] = \sum_{i=0}^{n-2} |N(G,i)|\, x_i$. Note that $N[G,x]$ is a polynomial of degree at most n-2.
Also isomorphic graphs have same common neighbor polynomials.

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ON (a, d) −HYPEREDGE ANTIMAGIC LABELING OF CERTAIN CLASSES OF HYPERGRAPHS: A NEW NOTION

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Abstract

By a hypergraph G, we mean a generalization of a graph G in which an edge can join any number of vertices. In an ordinary graph, an edge connects exactly two vertices, but in hypergraph, an edge or *hyperedge* may connect more than two vertices. Let $G = (V, E)$ be a hypergraph, thus V contains a finite set of vertices, and E contains a hyperedge of subset of V. Some vertices are said to be adjacent if they are elements of a hyperedge. A vertex v is said to be incident to an hyperedge e If $v \in e$. Similarly, a hyperedge e is said to be incident to vertex v if $v \in e$. Furthermore, a bijection f from $V(G)$ into $\{1,2,3,...,|V|\}$ is called and (a, d) −hyperedge antimagic labeling of hypergraph G if the hyperedge weights $W(e) = \sum_{v \in e} f(v)$Form an arithmetic progression starting from a and having common difference d. In this paper, we initiate to study hyperedge antimagic labeling of certain classes of hypergraphs, including analyze the properties of the antimagicness of any hypergraph.

Keywords : Hypergraphs, Hyperedge Antimagic Labeling, Hyperedge weights.

THE K-UNIFORM HYPER GRAPH OF COMMUTATIVE RINGS

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Abstract

The idea of k −zero-divisor hypergraph of a commutative ring R was introduced by Ch. Eslanchi and A. M. Rahimi [1] in 2007. Actually they extended the concept of zero-divisor of a commutative ring R to that of k-*zero-divisor* and investigating the interplay between the ring-theoretic properties of R and the hypergraph-theoretic properties of its associated k-uniform hypergraph. They defined, for $k \geq 2$, a non zero non unit element a_1 as a k-*zero-divisor* in R if there exist $k-1$ distinct elements $a_2, \ldots, a_k$ different from a_1 such that $a_1, \ldots, a_k = 0$, and no product of elements of any proper subset of $\{a\_1, \ldots, a\_k\}$ is zero and denote $Z(R,k)$ as the set of all k-zero-divisors in R. The k-*zero-divisor hypergraph* of R, denoted by $H_k(R)$, is a hypergraph with vertex set $Z(R,k)$, and for distinct elements $x_1, x_2, \ldots, x_k$ in $Z(R,k)$, the set $\{x_1, x_2, \ldots, x_k\}$ is an edge of $H_k(R)$ if and only if $\prod_{i=1}^{k} x_i = 0$ and the product of any $(k-1)$ elements of $\{x_1, x_2, \ldots, x_k\}$ is non zero. In this talk, we discuss some properties of k −zero-divisor hypergraph of R and we will generalize this notion by replacing elements whose product is zero with elements whose product lies in some ideal I of R.

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OMEGA INVARIANT AND ALGEBRAIC GRAPHS

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Abstract

The omega invariant was introduced in 2018 to determine several algebraic, combinatoric and topological properties of all realizations of a given degree sequence or of any given graph. It is directly related to the Euler characteristic and the cyclomatic number of the graph. It helps one to find many algebraic, geometric, graph theoretical, number theoretical, topological and combinatorial properties of all the realizations of the given degree sequence including cyclicity, connectivity, numbers of components, multiple edges, loops, cycles, chords, pendant and support vertices, etc. Since 2019, several applications of this invariant have been found. In this work, we shall recall the omega invariant together with some fundamental combinatoric properties and also apply it to study the constructive properties of idempotent total graphs as algebraic graphs.

Keywords: graph characteristic, degree sequence, omega invariant, idempotent total graph.

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Contents

CONNECTED GEODETIC DOMINATION NUMBER OF A FUZZY GRAPH

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Abstract

Let S be a subset of $V(G)$ and let $G:(V,\sigma,\mu)$ be a fuzzy graph. A connected geodetic dominating set of a fuzzy graph $G:(V,\sigma,\mu)$ is a geodetic dominating set S such that the sub graph induced by S, $\langle S\rangle$, is connected. The minimum cardinality among all the connected geodetic dominating set of G is called the connected geodetic domination number of G and is denoted by $\gamma_{fcg}(G)$. In this paper the concept of connected geodetic domination number of fuzzy graph is introduced and also proves some important results related to connected geodetic domination number of fuzzy graph.

Keywords: geodesic set, dominating set, geodetic dominating set, connected geodetic dominating set, connected geodetic domination number.

2020 Mathematics Subject Classification (AMS): 05C72, 05C69, 05C12.

1. Introduction

Zadeh in 1965[12] developed a mathematical phenomenon for describing the uncertainties prevailing in day-to-day life situations by introducing the concept of fuzzy sets. The theory of fuzzy graphs was later on developed by Rosenfeld in the year 1975[7]. A fuzzy graph is a triplet $G:(V,\sigma,\mu)$ where V is a vertex set, σ is a fuzzy subset on V and μ is a fuzzy relation on σ such that $\mu(x,y) \leq \sigma(x)\wedge\sigma(y) \forall x,y \in V$. We assume that V is finite and nonempty, μ is reflexive and symmetric. In all the examples σ is chosen suitably. Also we denote the underlying crisp graph by $G^*:(\sigma^*,\mu^*)$ where $\sigma^* = \{x \in V: \sigma(x) > 0\}$ and $\mu^* = \{(x,y) \in V \times V: \mu(x,y) > 0\}$. Here we take $\sigma^* = V$. For basic fuzzy graph theoretic terminology we refer to Nagoorgani and Chandrasekaran VT [4]. A fuzzy graph $G:(V,\sigma,\mu)$ is a complete fuzzy graph if $\mu(x,y) \leq \sigma(x)\wedge\sigma(y)$ for every $x,y \in \sigma^*$.

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Domination in fuzzy graph is one of the widest fields which have witness a tremendous growth recently. The term domination in crisp graph was first introduced by Ore[6]. The concept of domination in fuzzy graph was introduced by A. Somasundaram and S. Somasundaram [9]. Let $G: (V, \sigma, \mu)$ be a fuzzy graph. Let x and y be any two vertices of G. We say that x dominates y if (x, y) is a strong arc. A subset D of V is called a dominating set of G if for every $y \notin D$, there exists $x \in D$ such that x dominates y. A dominating set S is a connected dominating set if it induces a connected sub graph in G.

If there is no shorter strong path from x to y, then a strong path P from x to y is said to be geodesic, and the length of a $x - y$ geodesic is the geodesic distance from x to y, indicated by (x, y). Let S represent the collection of vertices in a fuzzy connected graph G. The set of all vertices in S as well as the vertices that lie on the geodesic between S's vertices is known as the geodesic closure (S) of S. Any set of G with a minimum number of vertices is referred to as a geodesic basis for G, and S is said to be a geodesic set of G if $(S) = V(G)$. The number of vertices on a geodesic basis determines its order. A fuzzy graph's geodesic number, indicated by the symbol $gn(G)$, is the order of a geodesic basis of G. In this paper, the connected geodetic domination number of fuzzy graph is introduced and its limiting bounds are identified.

2. Connected geodetic domination number of a fuzzy graph

In this section, we introduce the concept of connected geodetic domination in fuzzy graphs and its bounds are discussed.

Definition 2.1. A connected geodetic domination set of a fuzzy graph $G: (V, \sigma, \mu)$ is a geodetic dominating set S such that the sub graph induced by S, $\langle S \rangle$, is connected. The minimum cardinality among all the connected geodetic dominating set of G is called the connected geodetic domination number of G and is denoted by $\gamma_{fcg}(G)$.

Example 2.2. For the fuzzy graph given in Fig.1. the arcs (v_1, v_2) and (v_5, v_6) are δ- arcs and all the other arcs are strong arcs. Here $S_1 = \{v_1, v_2, v_5, v_6\}$ is a minimum geodetic dominating set and the geodetic domination number is $\gamma_{fg}(G) = 2.1$. Also, $S_2 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is a minimum connected geodetic dominating set and the connected geodetic domination number is $\gamma_{fcg}(G) = 3$. Thus the geodetic domination number and connected geodetic domination number are different.

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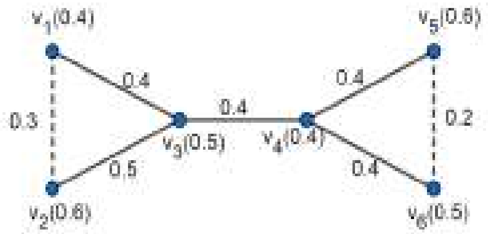

Fig. 1. Illustration of connected geodetic domination number of a connected fuzzy graph.

Theorem 2.3. For the complete fuzzy graph $G = K_n: (V, \sigma, \mu)$, $(n \geq 2)$, $\gamma_{fcg}(G) = p$, where $p = \sum_{u \in V} \sigma(u)$.

Proof. Since G is a complete fuzzy graph, all arcs are strong arcs and each vertex is adjacent to all other vertices. No vertex will lie on the geodesic path of any pair of vertices $(x, y) \in \mu^*$. Therefore the complete vertex set is the only connected geodetic dominating set and hence $\gamma_{fcg}(G) = p$.

Proposition 2.4. Any connected geodetic dominating set of a fuzzy graph $G: (V, \sigma, \mu)$ is a geodetic dominating set of G.

Remark 2.5. The converse of Proposition 2.4 need not be true.

Example 2.6. Consider the fuzzy graph in Fig. 2. In this graph $S = \{a, c\}$ is a geodetic dominating set, but not a connected geodetic dominating set, since the induced sub graph $\langle S \rangle$ is not connected.

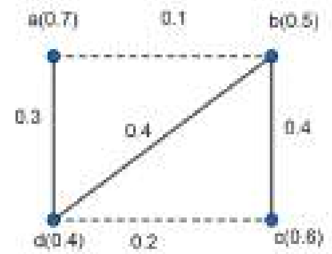

Fig.2. Example of geodetic dominating set but not a connected geodetic dominating set.

Proposition 2.7. For any connected fuzzy graph $G: (V, \sigma, \mu)$, $\gamma_{fg}(G) \leq \gamma_{fcg}(G)$.

Proposition 2.8. If $G: (V, \sigma, \mu)$ is a non-trivial fuzzy path P_n then $\gamma_{fcg}(G) = p$, where $p = \sum_{u \in V} \sigma(u)$.

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Definition 2.9. A minimal connected geodetic dominating set S in a fuzzy graph $G:(V,\sigma,\mu)$ is a connected geodetic dominating set which contains no connected geodetic dominating set as a proper set.

Remark 2.10. For any minimal connected dominating set S of a fuzzy graph $G:(V,\sigma,\mu)$, if S is also a geodetic set of G, then S is minimal connected geodetic dominating set in a fuzzy graph G.

Example 2.11. Consider the fuzzy graph in Fig.3. In this graph, the minimal connected dominating set is $S=\{a,b\}$, which is also a geodetic set of G. Hence $S=\{a,b\}$ is a minimal connected geodetic dominating set of G.

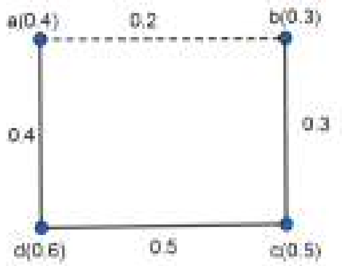

Fig. 3. Minimal connected geodetic dominating set.

Remark 2.12. [1] For a connected fuzzy graph $G:(V,\sigma,\mu)$ on n vertices , $0 \le \gamma_{fg}(G) \le p$, where $p=\sum_{u\in V}\sigma(u)$.

Remark 2.13. For a connected fuzzy graph $G:(V,\sigma,\mu)$ on n vertices, $0 \le \gamma_{fcg}(G) \le p$, where $p=\sum_{u\in V}\sigma(u)$.

Theorem 2.14. For connected fuzzy graph $G:(V,\sigma,\mu)$ having maximum degree $\Delta = \vee \{d(v)/v \in V\}$, $\delta = \wedge \{d(v)/v\in V\}$ order $p=\sum_{u\in V}\sigma(u)$, and size $q=\sum_{u\neq v}\mu(u,v)$,then $\frac{p}{1+\Delta} \le \gamma_{fcg}(G) \le 2\left(q-\frac{p}{2}\right)+2$.

Proof. The vertex set V of the fuzzy graph $G:(V,\sigma,\mu)$ is a connected geodetic dominating set of G with order p. But it may not be the minimum one. Therefore, $\gamma_{fcg}(G) \le p$ and for all connected fuzzy graph G we have $q \ge p-1$. Thus, $\gamma_{fcg}(G) \le p = 2(p-1)-p+2 \le 2\left(q-\frac{p}{2}\right)+2$. For the time being, we assume that $\gamma_{fcg}(G)=\delta$, where $\delta \le p$. Consider a connected geodetic dominating set $S=\{u_1,u_2,\dots,u_\delta\}$ with $\gamma_{fcg}(G)=\delta$. For

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$a \in \{1, 2, \ldots, \delta\}$, we can write $\sum_{a=1}^{\delta} 1 + \deg(u_a) \geq p$ and $\deg(u_a) \leq \Delta$ for each a,

$$p \leq \sum_{a=1}^{\delta}(1 + \deg(u_a)) \leq \sum_{a=1}^{\delta}(1 + \Delta) \leq \delta(1 + \Delta).$$

It follows that $\frac{p}{1+\Delta} \leq \gamma_{fcg}(G) \leq 2\left(q - \frac{p}{2}\right) + 2$.

Remark 2.15. For any connected fuzzy graph $G: (V, \sigma, \mu)$ on n vertices, $0 \leq \gamma_{fg}(G) \leq \gamma_{fcg}(G) \leq p$, where $p = \sum_{u \in V} \sigma(u)$.

Proof. By Remark 2.12, it is clear that $\gamma_{fg}(G) \geq 0$. Now by Definition 2.1, every connected geodetic dominating set is also a geodetic dominating set of G and so $\gamma_{fg}(G) \leq \gamma_{fcg}(G)$. Also note that $V(G)$induces a connected geodetic dominating set of G and it is obvious that $\gamma_{fcg}(G) \leq p$. Thus $0 \leq \gamma_{fg}(G) \leq \gamma_{fcg}(G) \leq p$.

Corollary 2.16. Let $G: (V, \sigma, \mu)$ be any connected fuzzy graph on n vertices. If $\gamma_{fg}(G) = p$, then $\gamma_{fcg}(G) = p$, where $p = \sum_{u \in V} \sigma(u)$.

Definition 2.17. [8] A vertex v in a fuzzy graph $G: (V, \sigma, \mu)$ is called extreme vertex, if the fuzzy sub graph induces by its neighbors is a complete fuzzy graph.

Proposition 2.18. Each extreme vertex of a fuzzy graph $G: (V, \sigma, \mu)$ belongs to every geodetic dominating set of G.

Proof. Let S be a geodetic dominating set of G and x be an extreme vertex of G. Let $\{x_1, x_2, \ldots, x_n\}$ be the neighbors of x and (x, x_i) $(1 \leq i, j \leq n)$ be the edges incident on x. Since x is an extreme vertex, x_i and $x_j$are adjacent for $i \neq j$ $(1 \leq i, j \leq n)$. Then any geodetic dominating set which contains x, is either (x_i, x) $(1 \leq i \leq n)$ or $y_1, y_2, \ldots, y_m, x_i, x$ where each y_i $(1 \leq i \leq n)$ is different from x_i. Thus each extreme vertex of a fuzzy graph $G: (V, \sigma, \mu)$ belongs to every geodetic dominating set of G.

Proposition 2.19. Each extreme vertex of a fuzzy graph $G: (V, \sigma, \mu)$ belongs to every connected geodetic dominating set of G.

Proof. Since every connected geodetic dominating set is also a geodetic dominating set, the result follows from Proposition 2.19.

Proposition 2.20. Let $G: (V, \sigma, \mu)$ be a connected fuzzy graph such that the underlying crisp graph G^* contains at least one cut-vertex and let S be a connected geodetic dominating set of G. If x is a cut-vertex of G^*, then every component of $G^* - \{x\}$ contains an element of S.

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Proof. Let x be a cut-vertex of G^* and let S be a connected geodetic dominating set of G. Suppose that there exists a component say ${G_1}^*$, of $G^* - \{x\}$ such that ${G_1}^*$ contains no vertex of S. Let $y \in V({G_1}^*)$. Since S is a connected geodetic dominating set of G, there exists a pair of vertices a and b in S such that y lies on some $a - b$ geodesic path $P: a = y_0, y_1, \dots, y, \dots, y_n = b$ in G. Since x is a cut-vertex of G^*, the $a - y$ geodesic sub path of P and the $y - b$ geodesic sub path of P both contain x. Then it follows that P is not a path, contrary to assumption.

Proposition 2.21. Let $G: (V, \sigma, \mu)$ be a connected fuzzy graph such that G^* contains at least one cut-vertex. Then every cut-vertex of G^* belongs to every connected geodetic dominating set of G.

Proof. Let x be a cut-vertex of G^* and let ${G_1}^*, {G_2}^*, \dots, {G_a}^*$ $(a \geq 2)$ be the components of $G - \{x\}$. Let S be any connected geodetic dominating set of G. Then by Proposition 2.20, S contains at least one element from each component ${G_i}^*$ $(l \leq i \leq a)$. Since $\langle S \rangle$ is connected, it follows that $x \in S$.

Theorem 2.22. For a complete bipartite fuzzy graph $G = K_{r,s} = (V_1 \cup V_2, \sigma, \mu)$ with partite sets V_1 and V_2 having number of vertices r and s respectively,

(i) $\gamma_{fcg}(G) = p$ where $p = \sum_{u \in V} \sigma(u)$, if $r = 1, s \geq 1$.

(ii) $\gamma_{fcg}(G) = \min\{\sum_{a \in V_1} \sigma(a) + min_{b \epsilon V_2} \sigma(b), \sum_{b \in V_2} \sigma(b) + min_{a \epsilon V_1} \sigma(a)\}$, if $r = s = 2$.

(iii) $\gamma_{fcg}(G) = \sum_{a \in V_1} \sigma(a) + min_{b \epsilon V_2} \sigma(b)$ if $r = 2, s \geq 3$.

(iv) $\gamma_{fcg}(G) = min_{a_1, a_2 \in V_1} [\sigma(a_1) + \sigma(a_2)] + min_{b_1, b_2 \in V_2} [\sigma(b_1) + \sigma(b_2)]$, if $r, s \geq 3$.

Proof.

(i) If the set V_1 having single vertex then the underlying crisp graph $K_{1,s}$ $(s \geq 1)$ then any connected geodetic dominating set must contains every vertices in G. Therefore $\gamma_{fcg}(G) = \sum_{u \in V_1 \cup V_2} \sigma(u) = p$.

(ii) If the sets V_1 and V_2 each having two vertices. In $K_{2,2}$, all the arcs are strong. Also each vertex in V_1 is adjacent with all the vertices in V_2. Therefore, the minimal connected geodetic dominating sets consists of three vertices such that two vertices from one partite set and the other is from the other partite set. Hence $\gamma_{fcg}(G) = \min\{\sum_{a \in V_1} \sigma(a) + min_{b \epsilon V_2} \sigma(b), \sum_{b \in V_2} \sigma(b) + min_{a \epsilon V_1} \sigma(a)\}$.

(iii) Suppose V_1 consists of two vertices and V_2 consists of more than two vertices. So in

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this complete bipartite fuzzy graph any two vertices of a partite set is geodetic dominate with all the vertices of the other partite set. But it is not connected. Hence the minimum connected geodetic dominating sets consists of three vertices from V_1 and the other one is from V_2. Hence $\gamma_{fcg}(G) = \sum_{a \in V_1} \sigma(a) + min_{b \epsilon V_2} \sigma(b)$.

(iv) Let $V_1 = \{a_1, a_2, \dots, a_m\}$ and $V_1 = \{b_1, b_2, \dots, b_n\}$ be the partitions of the complete bipartite graph G. In G every vertex in V_1 is linked with every vertex in V_2. Moreover all the arcs are strong and also any vertex will never lies on the shortest path between any other pair other pair vertices. Also two vertices from each partite set of G say $S = \{a_i, a_{i+1}, b_j, b_{j+1}\}$. Each path a_i, a_{i+1} contains all the vertices of V_2 as an internal vertices and the path b_j, b_{j+1} contains all the vertices of V_1 as an internal vertices. Clearly the set S is connected geodetic dominating set. Therefore the minimal connected geodetic dominating sets are the sets consists of four vertices such that two vertices in V_1 and the remaining two vertices in V_2. Thus $\gamma_{fcg}(G) = min_{a_1, a_2 \in V_1}[\sigma(a_1) + \sigma(a_2)] + min_{b_1, b_2 \in V_2}[\sigma(b_1) + \sigma(b_2)]$.

Example 2.23. Consider the complete bipartite fuzzy graph $G = K_{r,s} : (V_1 \cup V_2, \sigma, \mu)$ is shown in Fig.4. with partition sets are $V_1 = \{a_1, a_2, a_3, a_4\}$, $V_2 = \{b_1, b_2, b_3, b_4, b_5\}$ with $\sigma(a_1) = 0.4, \sigma(a_2) = 0.2, \sigma(a_3) = 0.6, \sigma(a_4) = 0.5, \sigma(a_1) = 0.4 \sigma(b_1) = 0.2, \sigma(b_2) = 0.1, \sigma(b_3) = 0.8, \sigma(b_4) = 0.4, \sigma(b_5) = 0.6$ and edge membership values are shown in Fig. 4. Here $r, s \geq 3$, so $\gamma_{fcg}(G) = min_{a_1, a_2 \in V_1}[\sigma(a_1) + \sigma(a_2)] + min_{b_1, b_2 \in V_2}[\sigma(b_1) + \sigma(b_2)]$=0.6+0.3=0.9. Thus the minimum connected geodetic dominating set is $S = \{a_1, a_2, b_1, b_2\}$ and the connected geodetic domination number is 0.9.

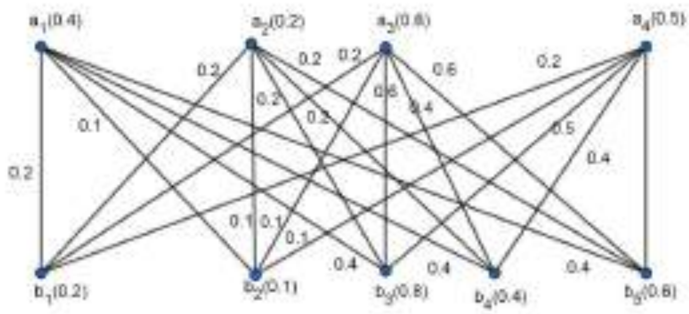

Fig. 4. Connected geodetic domination number of a complete bipartite fuzzy graph.

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Proposition 2.24. If $G:(V,\sigma,\mu)$ is a connected fuzzy graph on $n \geq 3$ vertices containing no δ- arcs such that x is a cut-vertex of G^* of degree $n-1$, then $\gamma_{fcg}(G) = p$ where $p = \sum_{u \in V} \sigma(u)$.

Proof. Let S be any connected geodetic dominating set of G and x be a cut-vertex of G^* of degree $n-1$. Then by Proposition 2.21, $x \in S$.

Claim: $S = V(G)$ is a minimum connected geodetic dominating set of G.

Otherwise, there exists a set $W \subset V(G)$ such that W is a connected geodetic dominating set of G. By Proposition 2.21, $x \in W$. Since $W \subset V(G)$, there exists a vertex $a \in V$ such that $a \notin W$. Since W is a connected geodetic dominating set of G, the vertex a lies on a geodesic joining a pair of vertices u and v of W. Let the geodesic be $P: u, \ldots, x, a, \ldots, b$. Then we have $a \neq u, v$.

Case (i): Suppose $u = x$, then the arc (x, v) is the only geodesic joining x and v, since x is adjacent to every vertex of G.

Case (ii): Suppose $u \neq x$, then $u - x - v$ is the only geodesic joining u and v. Thus in any case P is not an $u - v$ geodesic, which is a contradiction. So $S = V(G)$ is the only connected geodesic domination number of G. Hence $\gamma_{fcg}(G) = p$.

Remark 2.25. The converse of Proposition 2.24 is not true. For the fuzzy graph G given in Fig. 5, $S = \{a, b, c, d, e\}$ is a minimum connected geodetic dominating set of G and then $\gamma_{fcg}(G) = 1.5 = p$. But no vertex of degree G^* of degree $n-1$.

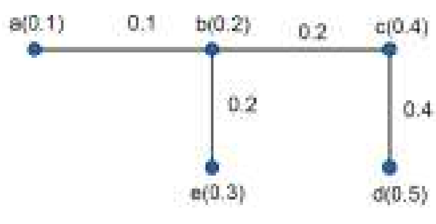

Fig. 5

Theorem 2.26. For any pair r, n of integers with $3 \leq r \leq n$, there exists a connected fuzzy graph $G:(V,\sigma,\mu)$ on n vertices such that $\gamma_{fcg}(G) = \sum_{i=1}^{r} \sigma(a_i)$.

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Proof. We construct a connected fuzzy graph $G:(V,\sigma,\mu)$ on n vertices having connected geodetic domination number $\sum_{i=1}^{r}\sigma(a_i)$ as follows:

Let $P_r: a_1, a_2, a_3, \dots, a_r$ be a path on r vertices with $\sigma(a_i) = 0.i\ (0 \le i \le r)$ such that $\mu(a_i, a_{i+1}) = \sigma(a_i) \wedge \sigma(a_{i+1})\ (1 \le i \le r-1)$. Add new vertices $b_1, b_2, \dots, b_{n-r}$, each having membership value $\sigma(b_j) = \wedge\{\sigma(a_i)\}\ (1 \le i \le r)$ and join each $b_j\ (1 \le j \le n-r)$ with a_1 and a_3 taking $\mu(a_i, b_j) = \sigma(a_i) \wedge \sigma(b_j)$, $i = 1,3$ and $1 \le j \le n-r$, thereby obtaining a fuzzy graph G as shown in Fig. 6.

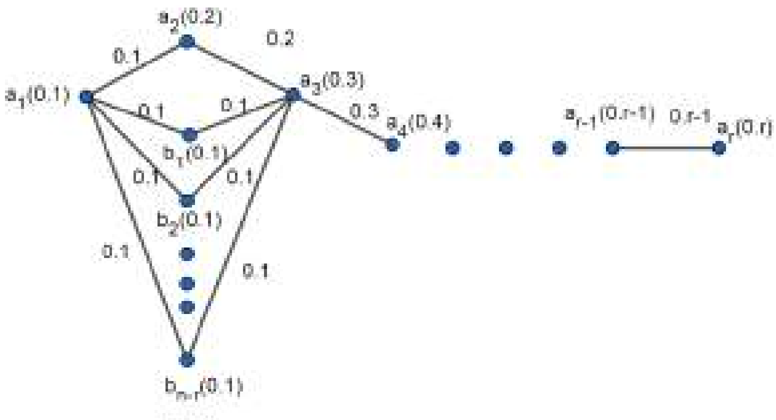

Fig. 6.

Then G is a connected fuzzy graph on n vertices and $S = \{a_3, a_4, \dots, a_r\}$ is the set of all cut-vertices of the underlying crisp graph G^* and all the extreme vertices of G. It follows from Proposition 2. 19 and 2.21 that $\gamma_{fcg}(G) \ge \sum_{i=3}^{r}\sigma(a_i)$. Clearly, S is not a geodetic dominating set of G, since $(S) \ne V(G)$ and thus not a connected geodetic dominating set of G. So, $\gamma_{fcg}(G) > \sum_{i=3}^{r}\sigma(a_i)$.

Note that neither $S \cup \{b_j\}\ (1 \le j \le n-r)$ nor $S \cup \{a_2\}$ is a geodetic dominating set of G. Thus, $R = S \cup \{a_1\}$ is a geodetic dominating set of G but $\langle R \rangle$ is not connected. However, $R \cup \{a_2\}$ is a connected geodetic dominating set of G of minimum cardinality. Hence the connected geodetic domination number is $\gamma_{fcg}(G) = \sum_{i=1}^{r}\sigma(a_i)$.

3. Conclusion

In this paper, the concept of connected geodetic domination number of a fuzzy graph is introduced and its limiting bounds are identified. It is proved that all extreme vertices of a connected fuzzy graph G and all cut-vertices of its underlying crisp graph G^* belong to its

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connected geodetic dominating set. Also the connected geodetic domination number of complete fuzzy graph and complete bipartite fuzzy graphs are obtained. We extend this concept to other distance related parameters in fuzzy graph.

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EXPLORING REGULAR AND TOTAL REGULAR CUBIC FUZZY GRAPHS

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Abstract

This paper introduces and investigates the concept of regular and total regular cubic fuzzy graphs. This research involves a combination of theoretical and analytical approaches to define and analyze the degree of a vertex and total degree of a vertex in the context of cubic fuzzy graphs. The main findings of this paper include the specification of regular and total regular cubic fuzzy graphs are defined, along with illustrative examples. Additionally, some characterization of results on a cycle with some specific membership values has been analyzed. This research provides a new perspective on cubic fuzzy graphs, extending the existing literature on fuzzy graphs and opening up new avenues for future research in this area.

Keywords: Degree of a vertex in Fuzzy Graphs, Total degree of a vertex in Fuzzy Graphs, Regular Fuzzy Graphs, Total Regular Fuzzy Graphs, Cubic Fuzzy Graphs.

2020 Mathematics Subject Classification (AMS): 05C72, 03E72.

1. Introduction

Fuzzy set theory, introduced by Lotfi A. Zadeh in 1965 [18]. Zadeh further extended this concept to fuzzy relations in 1971 [19], where the relationship between elements is represented by a membership function. The fundamental characteristic of fuzzy sets is the membership function, which assigns a degree of membership to each element in the set. The integration of fuzzy set theory and graph theory gave rise to a new class of graphs known as fuzzy graphs.

Fuzzy graph theory, introduced by Kaufmann in 1973 [10]. Azriel Rosenfeld developed fuzzy graph operations such as union, intersection, and complement in 1975 [17]. Although this field is relatively young, it has rapidly expanded and found numerous applications across various disciplines. Recently, researchers have continued to contribute to the field with notable

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contributions including the introduction of interval-valued fuzzy graphs (IVFGs) by Akram and Dudek in 2011 [1], the exploration of Totally Regular Fuzzy Graphs by Edward Samuel and C. Kayalvizhi in 2016 [3] and the presentation of A New Approach to Regular Fuzzy Graphs by Kailash Kumar Kakkad and Sanjay Sharma in 2017 [9]. Additionally, Huda Mutab Al Mutab conducted a study on fuzzy graphs in 2019 [5], further advancing the field.

Cubic Fuzzy Sets (CFS) are a mathematical framework that combines fuzzy sets and intuitionistic fuzzy sets to provide a more comprehensive and flexible approach to modeling uncertainty and imprecision. Introduced by Jun et al. [8], Cubic Fuzzy Set integrate fuzzy sets (FS) and intuitionistic fuzzy sets (IVFS). Rashid et al. [15] extended this idea to Cubic Fuzzy Graphs, introducing various types of graphs and their applications. Kishore Kumar et al. [11] investigated the concept of regularity in Cubic Fuzzy Graphs., while Muhiuddin et al. [12] provided a modified definition of Cubic Fuzzy Graphs, along with notions such as strong edges, paths, path strength, bridges and cut vertices. Furthermore, Rashmanlon et al. [6, 16] further elaborated on various aspects of Cubic Fuzzy Graphs.

Cubic Fuzzy Graphs represent a novel extension of fuzzy graph theory, combining the concepts of fuzzy sets and graph theory. However, the study of regular and total regular cubic fuzzy graph remains a relatively unexplored area. Nagoor Gani and Radha introduced the concept Total Degree and Total Regular Fuzzy Graphs in 2008 [13]. The existing literature on fuzzy graphs lacks a comprehensive study on regular and total regular fuzzy graphs, which motivates our research. This paper aims to explore the degree and total degree of a vertex in cubic fuzzy graph. We conduct a comparative study of regular and total regular cubic fuzzy graph through various examples. Additionally, we characterize cycles with specific membership function providing a comprehensive study on this topic. This research will contribute to the development of fuzzy graph theory and its application.

2. Basic Definitions

Definition 2.1. A **graph** $G: (V, E)$ consists of a finite set denoted by V as $V(G)$ and a collection E as $E\ (G)$ are unordered pairs (u, v) of distinct elements from V. Each element of V is called a vertex or a point and each element of E is called an edge or a line.

Definition 2.2. A **fuzzy graph** $C^*: (\sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma: V \rightarrow [0, 1]$ is a fuzzy subset of a non-empty set V and $\mu: V \times V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that, $\forall\ u, v$ in V, the relation $\mu\ (u, v) \leq \sigma\ (u) \wedge \sigma\ (v)$ is satisfied.

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Definition 2.3. An **interval-valued fuzzy set** A on V is defined as $A = \{[\alpha(u), \beta(u)]/u \in V\}$, where α and β are fuzzy sets of V, such that $\alpha(u) \leq \beta(u), \forall u \in V$.

Definition 2.4. Let U be a non-empty set. A **cubic set** is a structure of the form $C = \{u, A(u), B(u)\}$, where $A(u)$ is an interval valued fuzzy set in U and $B(u)$ is a fuzzy set in U.

Definition 2.5. A **cubic fuzzy set** in V is described as $X = \{([\alpha(u), \beta(u)], \gamma(u))/u \in V\}$, where $[\alpha(u), \beta(u)]$ is named the IVF- membership value and $\gamma(u)$ is named the F-membership value of u, such that $\alpha, \beta, \gamma: V \rightarrow [0, 1]$. X is named an internal CFS if $\gamma(u) \in [\alpha(u), \beta(u)]$ and an external CFS whenever $\gamma(u) \notin [\alpha(u), \beta(u)], \forall u \in V$.

Definition 2.6. A **cubic fuzzy graphs** on $G: (V, E)$ is a pair of functions C^*: (A,B), where $A = ([\alpha_1, \beta_1], \gamma_1)$ such that $[\alpha_1, \beta_1]: V \rightarrow [0,1]$ and $\gamma_1 : V \rightarrow [0,1]$ is a Cubic Fuzzy Set on the vertex set V and B = $([\alpha_2, \beta_2], \gamma_2)$ such that $[\alpha_2, \beta_2] : E \rightarrow [0,1]$ and $\gamma_2 : E \rightarrow [0,1]$ is a Cubic Fuzzy Set on the edge set E, satisfying the following conditions:

$$\alpha_2(u, v) \leq min\ \{\alpha_1(u), \alpha_1(v)\}, \forall (u, v) \in E$$

$$\beta_2(u, v) \leq min\ \{\beta_1(u), \beta_1(v)\}, \forall (u, v) \in E$$

$$\gamma_2(u, v) \leq min\ \{\gamma_1(u), \gamma_1(v)\}, \forall (u, v) \in E$$

Definition 2.7. Let G^*: (σ, μ) be a fuzzy graph on $G: (V, E)$. **The degree of a vertex** u in G is denoted by $d(u)$ and is defined as $d(u) = \sum \mu(u, v), \forall (u, v) \in E$ and $d(u) = 0, \forall (u, v) \notin E$.

Definition 2.8. Let G^*: (σ, μ) be a fuzzy graph on $G: (V, E)$. **The total degree of a vertex** u is denoted by $td(u)$ and is defined as $td(u) = \sum \mu(u, v) + \sigma(u), \forall (u, v) \in E$. It can also defined as $td(u) = d(u) + \sigma(u)$.

Definition 2.9. Let G^*: (σ, μ) be a fuzzy graph on $G: (V, E)$. If $d(v) = k, \forall v \in V$, i.e. if each vertex has the same degree k, then G is, then G^* is said to be a regular fuzzy graph of degree k or **a k-regular fuzzy graph**.

Definition 2.10. If each vertex of G^* has the same total degree k, then G^* is said to be a totally regular fuzzy graph of total degree k, or a **k- totally regular fuzzy graph**.

Definition 2.11. A **cycle** of length n in a graph G, denoted by C_n is a sequence $(u_0, u_1, u_2, ..., u_{n-1}, u_0)$ of vertices of G, such that, for $1 \leq i \leq n - 2$, the vertices u_i and u_{i+1} are adjacent; u_{n-1} and u_0 are also adjacent and $u_0, u_1, u_2, ...u_{n-1}$ are distinct.

A cycle C_n of length n is called an **even cycle** or **odd cycle** according as n is even or odd.

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3. Degree and total degree of a vertex in a cubic fuzzy graph

Definition 3.1. Let C^*: (A, B) be a Cubic Fuzzy Graph on G: (V, E). The degree of a vertex u in C^* is an interval valued fuzzy membership number such that $d[\alpha_1, \beta_1](u) = \sum [\alpha_2, \beta_2](u, v)$, $\forall (u, v) \in$ E and also $d[\alpha_1, \beta_1](u) = 0$, $\forall (u, v) \notin E$. The degree of a vertex u in C∗ is a fuzzy membership number such that $d\ [\gamma_1]\ (u) = \sum\ [\gamma_2]\ (u, v)$, $\forall\ (u, v) \in$ E and also $d[\gamma_1](u) = 0$, $\forall$ (u, v) $\notin$ E. Therefore, **the degree of a vertex in a Cubic Fuzzy Graph** is defined as $d(u) = (d\ [\alpha_1, \beta_1]\ (u), d[\gamma_1]\ (u))$.

Example 3.2. Consider a Cubic Fuzzy Graph C^*: (A, B) on G: (V, E).

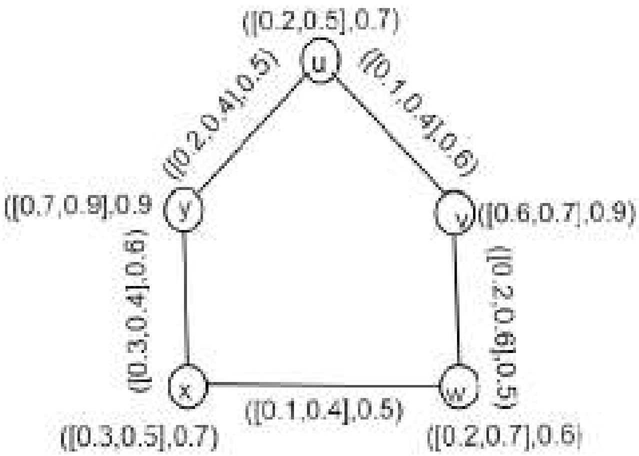

Figure. 1

$d(u)$ = ([0.2 + 0.1, 0.4 + 0.4], 0.5 + 0.6) = ([0.3, 0.8], 1.1). Similarly, $d(v)$ = ([0.3, 1.0], 1.1), $d(w)$ = ([0.3, 1.0], 1.0), $d(x)$ = ([0.4, 0.8], 1.1) and $d(y)$ = ([0.5, 0.8], 1.1).

Definition 3.3. Let C^*: (A, B) be a Cubic Fuzzy Graph on $G: (V, E)$. **The total degree of a vertex u∈V in a Cubic Fuzzy Graph** is defined $td(u) = (td[\alpha_1, \beta_1](u), td[\gamma_1](u))$, where $td[\alpha_1, \beta_1](u) = \sum [\alpha_2, \beta_2]\ (u, v) + [\alpha_1, \beta_1](u), \forall (u, v) \in E$ and $td[\gamma_1](u) = \sum [\gamma_2](u, v) + [\gamma_1](u), \forall (u, v) \in E$. It can also be defined as $td(u) = d(u) + A(u)$, where $A(u) = ([\alpha_1, \beta_1](u), [\gamma_1](u))$.

Example 3.4. Consider a Cubic Fuzzy Graph C^*: (A, B) on $G: (V, E)$. In Figure 1, $td(u)$ = ([0.3, 0.8], 1.1) + ([0.2, 0.5], 0.7) = ([0.5, 1.3], 1.8). Similarly, $d(v)$ = ([0.9, 1.7], 2.0), $d(w)$ = ([0.5, 1.7], 1.6), $d(x)$ = ([0.7, 1.3], 1.8) and $d(y)$ = ([1.2, 1.7], 2.0).

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4. Regular and total regular cubic fuzzy graphs

Definition 4.1. Let C^*: (A, B) be a Cubic Fuzzy Graph on $G: (V, E)$. If $d(u)$ = ([k_1, k_2], k_3), $\forall u \in V$, i.e., if each vertex has the same degree ([k_1, k_2], k_3) then C^* is said to be a ([k_1, k_2], k_3) - Regular Cubic Fuzzy Graph.

Example 4.2. Consider a Cubic Fuzzy Graph C^*: (A, B) on $G: (V, E)$.

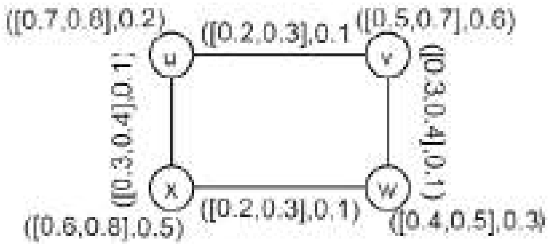

Figure. 2

$d(u)$ = ([0.5, 0.7], 0.2), $\forall u \in V$. This graph is a ([0.5, 0.7], 0.2) - Regular Cubic Fuzzy Graph.

Definition 4.3. Let C^*: (A, B) be a Cubic Fuzzy Graph on $G: (V, E)$. If each vertex of C^* has the same total degree ([k_1,k_2],k_3), then C^* is said to be a Total ([k_1,k_2],k_3) - Regular Cubic Fuzzy Graph.

Example 4.4. Consider a Cubic Fuzzy Graph C^*: (A, B) on $G: (V, E)$.

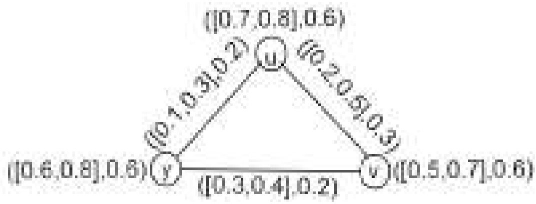

Figure. 3

$td(u)$ = ([1.0, 1.6], 1.1), $\forall u \in V$. If each vertex has the same total degree ([1.0, 1.6], 1.1), then this graph is a Total ([1.0, 1.6], 1.1) - Regular Cubic Fuzzy Graph. However, it is observed that $d(u) \neq d(w)$. Hence, C^* is not a ([k_1,k_2],k_3) - Regular Cubic Fuzzy Graph.

Remark 4.5. From Example 4.4, it is clear that a Total ([k_1,k_2],k_3) - Regular Cubic Fuzzy Graph is not necessarily a ([k_1,k_2],k_3)) - Regular Cubic Fuzzy Graph.

Example 4.6. Consider a Cubic Fuzzy Graph C^*: (A, B) on G: (V, E). Figure 2 shows that $d(u)$ = ([0.5, 0.7], 0.2), $\forall u \in V$. If each vertex has the same degree ([0.5, 0.7], 0.2), then this

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graph C^* is a ([0.5, 0.7], 0.2) - Regular Cubic Fuzzy Graph. However, it is observed that $td(u) \neq td(w)$. Hence, C^* is not a Total ($[k_1,k_2],k_3$) - Regular Cubic Fuzzy Graph.

Remark 4.7. From Example 4.6, it is clear that a ($[k_1,k_2],k_3$) - Regular Cubic Fuzzy Graph is not necessarily a Total ($[k_1,k_2],k_3$) - Regular Cubic Fuzzy Graph.

Example 4.8. Consider a Cubic Fuzzy Graph C^*: (A, B) on G: (V, E).

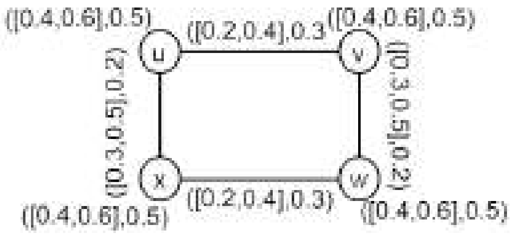

Figure. 4

$d(u) =$ ([0.5, 0.9], 0.5), $\forall u \in V$ and $td(u) =$ ([0.9, 1.5], 1.0), $\forall u \in V$. If each vertex has the same degree ([0.5, 0.9], 0.5), then this graph is a ([0.5, 0.9], 0.5) - Regular Cubic Fuzzy Graph. Additionally, if each vertex has the same total degree ([0.9, 1.5], 1.0), then C^* is a Total ([0.9, 1.5], 1.0) - Regular Cubic Fuzzy Graph.

Remark 4.9. From Example 4.8, it is clear that a ($[k_1,k_2],k_3$) - Regular Cubic Fuzzy Graph is also a Total ($[k_1,k_2],k_3$) - Regular Cubic Fuzzy Graph.

Theorem 4.10. Let C^*: (A, B) be a Cubic Fuzzy Graph on $G: (V, E)$. Then A is a constant function if and only if the following conditions are equivalent.

(i) C^* is a Regular Cubic Fuzzy Graph.

(ii) C^* is a Total Regular Cubic Fuzzy Graph.

Proof. Consider $A(u) = ([c_1,c_2],c_3)$, $\forall u \in V$. Assume that C^* is a ($[k_1,k_2],k_3$) - Regular Cubic Fuzzy Graph. Then $d(u) = ([k_1,k_2],k_3)$, $\forall u \in V$.
So, $td(u) = d(u) + A(u) \Rightarrow td(u) = ([k_1,k_2],k_3) + ([c_1,c_2],c_3) \Rightarrow td(u) = ([k_1+c_1, k_2+c_2], k_3+c_3)$, $\forall u \in V$. Hence C^* is a Total Regular Cubic Fuzzy Graph. Thus (i) ⇒ (ii) is proved. Now, suppose C^* is a Total ($[k_1+c_1, k_2+c_2], k_3+c_3$) - Regular Cubic Fuzzy Graph.
Then $td(u) = ([k_1+c_1, k_2+c_2], k_3+c_3)$, $\forall u \in V$. This implies $d(u) + A(u) = ([k_1+c_1, k_2+c_2], k_3+c_3)$, $\forall u \in V$. Therefore, $d(u) + ([c_1,c_2],c_3) = ([k_1,k_2],k_3) + ([c_1,c_2],c_3)$, $\forall u \in V$. Hence $d(u) = ([k_1,k_2],k_3)$, $\forall u \in V$.

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Thus C^* is a Regular Cubic Fuzzy Graph. Therefore (ii) ⇒ (i) is proved. Conversely, assume that (i) and (ii) are equivalent. Suppose $A(u)$ is not a constant function. Then $A(u) \neq A(w)$ for at least one pair $u, w \in V$, i.e., $td(u) \neq td(w)$. Let C^* be a Regular Cubic Fuzzy Graph. Then, $d(u) = d(w)$ = ([k_1,k_2],k_3).

So, $td(u) = d(u) + A(u), td(w) = d(w) + A(w) \Rightarrow td(u)$ = ([k_1,k_2],k_3) + $A(u), td(w)$ = ([k_1,k_2],k_3) + $A(w)$. Since $A(u) \neq A(w)$, i.e., $td(u) \neq td(w)$ = ([k_1,k_2],k_3) + $A(u) \neq$ ([k_1,k_2],k_3) + $A(w)$.⇒ $td(u) \neq td(w)$.

So, C^* is not Total Regular Cubic Fuzzy Graph. This contradicts our assumption. Now, Let C^* be a Total Regular Cubic Fuzzy Graph. Then, $td(u) = td(w)$.⇒ $d(u) + A(u) = d(w) + A(w) \Rightarrow d(u) \neq d(w)$. So C^* is not a Regular Cubic Fuzzy Graph. This is a contradiction. Thus, it can be concluded that A is a constant function.

5. Characterization of a cycle with some specific membership values

Theorem 5.1. Let C^*: (A, B) be a Cubic Fuzzy Graph on $G: (V, E)$ which is an odd cycle. If B is a constant function, then C^* is a Regular Cubic Fuzzy Graph.

Proof. If B is a constant function, then $B\ (u\ v)$ = ([c_1,c_2],c_3), ∀u ∈ V. Then, d (u) = ([c_1,c_2],c_3) + ([c_1,c_2],c_3) = ([2c_1, 2c_2], 2c_3). Hence C^* is a Regular Cubic Fuzzy Graph.

Conversely, suppose that C^* is a ([k_1,k_2],k_3) - Regular Cubic Fuzzy Graph. Let e_1,e_2,e_3,...,e_{2n}, e_{2n+1} be the edges of an odd cycle of C^*. Let $\alpha_2(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 = k_1 & \text{if } i \text{ is even} \end{cases}$'

$\beta_2(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 = k_3 & \text{if } i \text{ is even} \end{cases}$ and $\gamma_2(e_i) = \begin{cases} k_5 & \text{if } i \text{ is odd} \\ k_6 = k_5 & \text{if } i \text{ is even} \end{cases}$

Then, $d(v_1)$ = ([$\alpha_2(e_1)$, $\beta_2(e_1)$], $\gamma_2(e_1)$) + ([$\alpha_2(e_{2n+1})$, $\beta_2(e_{2n+1})$], $\gamma_2(e_{2n+1})$)

= ([k_1,k_3],k_5) + ([k_1,k_3],k_5) = ([2k_1, 2k_3], 2k_5)

$d(v_2)$ = ([$\alpha_2(e_2)$, $\beta_2(e_2)$], $\gamma_2(e_2)$) + ([$\alpha_2(e_1)$, $\beta_2(e_1)$], $\gamma_2(e_1)$)

= ([k_1,k_3],k_5) + ([k_1,k_3],k_5) = ([2k_1, 2k_3], 2k_5)

For i = 3,4,5,…,2n

Proceeding similarly, we get $d(v_{2n})$ = ([2k_1, 2k_3], 2k_5) and $d(v_i)$ = ([2k_1, 2k_3], 2k_5). Hence, C^* is a Regular Cubic Fuzzy Graph. But it is obtained as B is not a constant function.

Remark 5.2. If a Cubic Fuzzy Graph C^* on G which is an odd cycle and A is not a constant function, then C^* is not a Total Regular Cubic Fuzzy Graph.

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Example 5.3. Consider a Cubic Fuzzy Graph C^*: (A, B) on $G: (V, E)$ which is an odd cycle. In Figure 3, $td(u)$ = ([1.0, 1.6], 1.1), $\forall\ u \in V$. Then, this graph is a Total ([1.0, 1.6], 1.1) - Regular Cubic Fuzzy Graph. But it is obtained as B is not a constant function.

Theorem 5.4. Let C^*: (A, B) be a Cubic Fuzzy Graph on $G: (V, E)$ which is an even cycle. If B is a constant function or the alternate edges have the same IVF membership number and fuzzy membership number, then C^* is a Regular Cubic Fuzzy Graph.

Proof. If B is a constant function or the alternate edges have the same IVF membership number and fuzzy membership number, then C^* is a Regular Cubic Fuzzy Graph.
Conversely, suppose that C^* is a ([k_1,k_2],k_3) - Regular Cubic Fuzzy Graph. Let e_1,e_2,e_3,...,e_{2n} be the edges of an even cycle of C^*. Let $\alpha_2(e_i) = \begin{cases} k_1 \text{ if } i \text{ is odd} \\ k_2 \text{ if } i \text{ is even} \end{cases}$, $\beta_2(e_i) = \begin{cases} k_3 \text{ if } i \text{ is odd} \\ k_4 \text{ if } i \text{ is even} \end{cases}$ and

$\gamma_2(e_i) = \begin{cases} k_5 \text{ if } i \text{ is odd} \\ k_6 \text{ if } i \text{ is even} \end{cases}$

If $d(v_1) = d(v_i)$, then B is a constant function. If $d(v_1) \neq d(v_i)$, then the alternate edges have the same IVF membership number and fuzzy membership number.

Remark 5.5. If a Cubic Fuzzy Graph C^* on G which is an even cycle and alternate edges have the same IVF membership number and fuzzy membership number, then since A is not a constant function C^* is not a Total Regular Cubic Fuzzy Graph.

Example 5.6. Consider a Cubic Fuzzy Graph C^*: (A, B) on $G: (V, E)$ which is an even cycle. In Figure 4, $td(u)$ = ([0.9, 1.5], 1.0), $\forall\ u \in V$. Then C^* is a Total ([0.9, 1.5], 1.0) - Regular Cubic Fuzzy Graph. However, this is only possible when B is not a constant function or when alternate edges have the same IVF membership number and fuzzy membership number.

6. Conclusion

This study has explored the concepts of regular and total regular cubic fuzzy graphs, providing definitions, examples and characteristics of these graphs and have investigated their relationships and differences. The study of regular and total regular cubic fuzzy graphs has provided new insights into the structure and behaviour of complex systems and has opened up new avenues for research in fuzzy graph theory.

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FUZZY DOUBT Z-IDEAL OF Z-ALGEBRA

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Abstract

The Concept of this effort is to present the definition of fuzzy doubt z- ideals in Z - algebras and several properties related to fuzzy doubt z-ideals are discussed. The Cartesian product and homomorphic of fuzzy doubt z-ideal is also discussed And at the same time we specify some common theorem belonging to them with examples.

Key words: Z-algebra, Fuzzy set, Fuzzy z-ideal, Fuzzy Doubt z-Subalgebra, Fuzzy Doubt z-Ideal, Intersection.

2020 Mathematical Subject Classification (AMS): 03E72

1. Introduction

Fuzzy mathematics is the branch of mathematics including fuzzy set theory and fuzzy logic that deals with partial inclusion of elements in a set on a spectrum as opposed to simple binary "yes' or 'no" (0 or 1) inclusion. Fuzzy mathematics has its orgin on fuzzy set introduced by Lofti Asker Zadeh [1]. Fuzzy set theory has been developed in many directions by many scholars and has evolved a great deal of interest among mathematicians working in various fields of mathematics. As a advancement of these research works we get, the idea of intuitionistic fuzzy sets propounded by T. Atanassov in 2012 [10], that is a generalisation of the notion of fuzzy set. Imai and Iseki [2] introduced two classes of abstract algebras BCK-algebras and BCI-algebras. It is known that the class of BCK-algebra is a proper subclass of the class of BCI-algebra. In 2017, M. Chandramouleeswaran [6], introduced the concept of Z-algebras. Then in 2020, S. Sowmiya [7] gave another concept of fuzzy ideals of z-algebras. Following the same route, S. Sowmiya [8] established the definition of the intuitionistic fuzzy

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sub-algebra and intuitionistic fuzzy ideal in z –algebras. In the last two decades interest of many mathematicians has shifted to the development of fuzzy algebra in view of generalisation of the well-known rules of algebraic structures. Many mathematicians have been involved in extending the concepts and outcomes of various algebra.

2. Preliminaries

We first list some basic concepts which are needed for our work.

Definition 2.1. [6] A **Z-algebra** (A, *, 0) is a nonempty set X with a constant 0 and a binary operation * satisfying the following conditions:

- a * 0 = 0
- 0 * a = a
- a * a = a
- a * b = b * a when a ≠ 0 and b ≠ 0 for every a, b ϵ A.

Throughout this paper A means a Z-algebra without any specification. We also include some basic results that are necessary for this paper.

Definition 2.2. [6] A subset I of a Z-algebra A is called an **ideal** of A if it satisfies

- 0 ∈ I,
- a ∗ b ∈ I and b ∈ I imply a ∈ I, for all a, b ∈ A.

Definition 2.3. [6] Let (A,*, 0) and (A',*,0) be two Z-Algebras. A mapping h:A→A' be Z-homomorphism of Z-Algebras if h : (A,*,0) → (A',*,0) is said to be a **Z-homomorphism** of Z-algebras if h (x*y) = h(x) * h(y) for all x, y ϵ X.

Definition 2.4. [7] Let h be a Z-homomorphism of Z-algebra (A, ,*,0) → (A',*,0), then h is called

- A Z-monomorphism of Z-algebras if h is 1-1.
- An Z-epimorphism of Z-algebras if h is onto.
- An Z-endomorphism of Z-algebras if h is mapping (A, ,*,0) into itself.

Definition 2.5. [7] A fuzzy set δ of a Z-algebra A is called a **fuzzy ideal** of A if it satisfies

- δ(0) ≥ δ(a)
- δ (a) ≥ min {δ (a ∗ b) , δ (b)} , for all a,b ∈ A.

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Definition 2.6. [9] Let μ be a fuzzy set in a Z-algebra A. then μ is called a **fuzzy subalgebra of A**, if it satisfies

- μ (xy) ≥ μ(x). μ(y), for all x,y ϵ X.

Definition 2.7. [9] Let μ be a fuzzy set in a BCI-algebra X. then μ is called a **fuzzy ideal of A**, if it satisfies

- μ(0) ≥ μ(x)
- μ (xy) ≥ μ(x). μ(y), for all x,y ϵ X.

3. Fuzzy Doubt z-Ideal

Definition 3.1. Let δ_A be a fuzzy set in Z-Algebra A, Then δ_A is called a **fuzzy doubt z-subalgebra** of A, if it satisfies

- $\delta_A(\omega) \wedge \delta_A(\kappa) \leq \delta_A(\omega\kappa)$

Definition 3.2. Let δ_A be a fuzzy set in Z-Algebra A,Then δ_A is called a **fuzzy doubt z-ideal** of δ, if it satisfies

- $\delta_A(0) \leq \delta_A(\omega\kappa)$
- $\delta_A(\omega\kappa) \wedge \delta_A(\kappa) \leq \delta_A(\omega)$

Throughout this concept FDzI means Fuzzy Doubt z-Ideal

Theorem 3.3. Let g: A → A' be a homomorphism of A. If A_2 is a FDzI of A', then the pre image $g^{-1}(A_2)$ of A_2 under g is a FDzI of A.

Proof. for any $\omega_1, \kappa_1 \in$ A we have

$$\delta_{g^{-1}(A_2)}(0) = \delta_{A_2}(g(0))$$

$$\leq \delta_{A_2}(g(\omega_1\kappa_1))$$

$$= \delta_{g^{-1}(A_2)}(\omega_1\kappa_1)$$

$$\delta_{g^{-1}(A_2)}(\omega_1) = \delta_{A_2}(g(\omega_1))$$

$$\geq \delta_{A_2}(g(\omega_1\kappa_1) \wedge \delta_{A_2}(g(\kappa_1)).$$

$$= \delta_{A_2}(g(\omega_1\kappa_1)) \wedge \delta_{A_2}(g(\kappa_1)).$$

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$= \delta_{g^{-1}(A_2)}(\omega_1\kappa_1) \wedge \delta_{g^{-1}(A_2)}(\kappa_1)$

Hence, $g^{-1}(A_2)$ is a FDzI of A.

Theorem 3.4. Let A_1 and A_2 be a FDzI of A and A' respectively, then the cross product $A_1 \times A_2$ of A_1 *and* A_2 defined by $\delta_{A_1\times A_2}(\omega_1,\omega_2) = \delta_{A_1}(\omega_1).\ \delta_{A_2}(\omega_2)$ for all $(\omega_1,\omega_2) \in$ A $\times A'$ is a FDzI of A$\times A'$.

Proof. For all $(\omega_1,\omega_2) \in$ A$\times A'$, we have

$\delta_{A_1\times A_2}(0, 0) = \delta_{A_1}(0).\ \delta_{A_2}(0)$

$\leq \delta_{A_1}(\omega_1) \wedge \delta_{A_2}(\omega_2)$

$= \delta_{A_1\times A_2}(\omega_1,\omega_2)$

Now, for all (ω_1,ω_2), $(\kappa_1,\kappa_2) \in$ A$\times A'$, we have

$\delta_{A_1\times A_2}(\omega_1,\omega_2) = \delta_{A_1}(\omega_1) \wedge \delta_{A_2}(\omega_2)$,

$\geq (\delta_{A_1}(\omega_1\kappa_1).\ \delta_{A_1}(\kappa_1)) \wedge ((\delta_{A_2}(\omega_2\kappa_2).\ \delta_{A_2}(\kappa_2))$

$= (\delta_{A_1}(\omega_1\kappa_1).\ (\delta_{A_2}(\omega_2\kappa_2))) \wedge (\delta_{A_1}(\kappa_1).\delta_{A_2}(\kappa_2))$

$= (\delta_{A_1\times A_2}(\omega_1\kappa_1,\omega_2\kappa_2) \wedge \delta_{A_1\times A_2}(\kappa_1,\kappa_2))$

$= (\delta_{A_1\times A_2}((\omega_1,\omega_2)(\kappa_1, \kappa_2)) \wedge \delta_{A_1\times A_2}(\kappa_1, \kappa_2))$

Thus $A_1 \times A_2$ is a FDzI of A $\times A'$.

Theorem 3.5. Let A_1 and A_2 be a FDzI of A and A' respectively, then the cross product $A_1 \times A_2$ is a FDzI of A$\times A'$, then A_1 *or* A_2 must be a fuzzy doubt ideal.

Proof. Let $A_1 \times A_2$ is FDDzI of A$\times A'$.

We assume that A_1 *or* A_2 satisfies $\delta_{A_1}(0) \leq \delta_{A_1}(\omega_1)$ or $\delta_{A_2}(0) \leq \delta_{A_2}(\omega_2)$.

Suppose $\delta_{A_1}(0) > \delta_{A_1}(\omega_1)$ and $\delta_{A_2}(0) > \delta_{A_2}(\omega_2)$ for some $(\omega_1,\omega_2) \in$ A$\times A'$,

Then, we have

$\delta_{A_1\times A_2}(0, 0) = \delta_{A_1}(0) \wedge \delta_{A_2}(0)$

$> \delta_{A_1}(\omega_1) \wedge \delta_{A_2}(\omega_2)$

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$= \delta_{A_1 \times A_2}(\omega_1,\omega_2)$ which is a contradiction.

Therefore $\delta_{A_1}(0) \leq \delta_{A_1}(\omega_1)$ or $\delta_{A_2}(0) \leq \delta_{A_2}(\omega_2)$.

Suppose that the condition

$\delta_{A_1}(\omega_1) \leq \delta_{A_1}(\omega_1,\kappa_1) \wedge \delta_{A_1}(\kappa_1)$ or $\delta_{A_2}(\omega_2) \leq \delta_{A_2}(\omega_2,\kappa_2) \wedge \delta_{A_2}(\kappa_2)$ is not true,

Then (ω_1,ω_2), $(\kappa_1,\kappa_2) \in$ A$\times A'$,

We have

$\delta_{A_1 \times A_2}(\omega_1,\omega_2) = \delta_{A_1}(\omega_1) \wedge \delta_{A_2}(\omega_2)$,

$< (\delta_{A_1}(\omega_1\kappa_1) \wedge \delta_{A_1}(\kappa_1)) \wedge ((\delta_{A_2}(\omega_2\kappa_2) \wedge (\kappa_2))$

$= (\delta_{A_1}(\omega_1\kappa_1) \wedge \ (\delta_{A_2}(\omega_2\kappa_2))) \wedge (\delta_{A_1}(\kappa_1).\delta_{A_2}(\kappa_2))$

$= (\delta_{A_1 \times A_2}(\omega_1\kappa_1,\omega_2\kappa_2) \wedge \delta_{A_1 \times A_2}(\kappa_1,\kappa_2))$

$= (\delta_{A_1 \times A_2}((\omega_1,\omega_2)(\kappa_1,\kappa_2)) \wedge \delta_{A_1 \times A_2}(\kappa_1,\kappa_2))$ which is impossible.

Hence $\delta_{A_1}(\omega_1) \geq \delta_{A_1}(\omega_1,\kappa_1) \wedge \delta_{A_1}(\kappa_1)$ or $\delta_{A_2}(\omega_2) \leq \delta_{A_2}(\omega_2,\kappa_2) \wedge \delta_{A_2}(\kappa_2)$ is true.

Thus A_1 or A_2 is a FDzI of A $\times$ A'.

Theorem 3.6. Let M be a nonempty subset of A and δ_M be a fuzzy set in A defined by $\delta_M(\omega) = \alpha$ if $\omega \in$ M and $\delta_M(\omega) = \beta$ otherwise $\alpha, \beta \in [0,1]$ with $\alpha > \beta$. Then δ_M is a FDzI of A. if M is a ideal of A.

Proposition 3.7. Every FDzI of A is a FDzS of A.

Remark 3.8. The converse of proposition 7 may not be true in the following example.

Example 3.9. Suppose A = {0, *β, ω, κ*} the operation is given by the table

*	0	β	ω	κ
0	0	β	ω	κ
β	0	β	κ	β
ω	0	κ	ω	β
κ	0	β	β	κ

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Then (A, ∗, 0) is a Z-algebra. We define δ: A→ [0,1] by δ (0)=0.6, δ (β)=0.9, δ (ω)=0.7 and δ (κ)=0.9. By simple calculations show that δ is FDzI as well as FDzS.

Preposition 3.10. If δ_{A_1} and δ_{A_2} are FDzI of A, then so is $\delta_{A_1} \cap \delta_{A_2}$.

Proof. Let ω, κ ϵ A.

Then, $(\delta_{A_1} \cap \delta_{A_2})$ (0) = ∩ $((\delta_{A_1}(0), \delta_{A_2}(0))$

≥ ∩ $((\delta_{A_1}(\omega\kappa), \delta_{A_2}(\omega\kappa))$

= $(\delta_{A_1} \cap \delta_{A_2})$ (ωκ)

Also, $(\delta_{A_1} \cap \delta_{A_2})$ (ω) = ∩ $(\delta_{A_1}(\omega), \delta_{A_2}(\omega))$

≥ ∩ $(\delta_{A_1}(\omega\kappa) \wedge \delta_{A_1}(\kappa)), (\delta_{A_2}(\omega\kappa) \wedge \delta_{A_1}(\kappa))$

= $(\delta_{A_1} \cap \delta_{A_2})(\omega\kappa) \wedge (\delta_{A_1} \cap \delta_{A_2})(\kappa)$

Hence $\delta_{A_1} \cap \delta_{A_2}$ is a FDzI of A.

Theorem 3.11. Let A_1 be a fuzzy subset of A, assume that δ_{A_1} be a fuzzy subset of A × A defined by $\delta_{A_1}(\omega, \kappa) = A_1(\omega). A_1(\kappa)$ for all ω, κ ϵ A. Then A_1 is FDzI of A if and only if δ_{A_1} is a FDzI of A × A.

Proof. Suppose A_1 is a FDzI of A. For all ω, κ ϵ A.

$\delta_{A_1}(0, 0) = A_1(0) \wedge A_1(0)$

$\leq A_1(\omega\kappa) \wedge A_1(\omega\kappa)$

$= \delta_{A_1}(\omega\kappa, \omega\kappa)$

For any $\omega_1, \kappa_1, \omega_2$ *and* κ_2 ϵ A.

Also, We have

$\delta_{A_1}((\omega_1, \omega_2)\ (\kappa_1, \kappa_2)) = \delta_{A_1}(\omega_1\kappa_1, \omega_2\kappa_2) \wedge \delta_{A_1}(\kappa_1, \kappa_2)$

$= (A_1(\omega_1\kappa_1) \wedge A_1(\omega_2\kappa_2)) \wedge (A_1(\kappa_1) \wedge A_1(\kappa_2))$

$= (A_1(\omega_1\kappa_1) \wedge A_1(\kappa_2)) \wedge (A_1(\omega_2\kappa_2) \wedge A_1(\kappa_2))$

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$\leq A_1\ (\omega_1) \wedge A_1(\omega_2)$

$= \delta_{A_1}$(ω, ω)

Therefore, δ_{A_1} is a FDzI of A ✕ A.

Conversely, suppose δ_{A_1} is a FDzI of A ✕ A.

Obviously, A_1 (ω) ≥ δ_{A_1} (ωκ). A_1(κ) is a FDzI.

Theorem 3.12. Let A_1 be a fuzzy subset of A, assume that δ_{A_1}be a fuzzy subset of A✕ A defined by A_1 (ω) = δ_{A_1} (0, ω). for all ω ϵ A. If δ_{A_1} is a FDzI of A ✕ A Then A_1 is FDzI of A.

Proof. for all ω ϵ A.

We have, A_1(0) = δ_{A_1} (0, 0)

≤ δ_{A_1} (0, ω)

= A_1 (ω)

For all ω, κ ϵ A,

A_1(ωκ) A_1(κ) = δ_{A_1} (0, ωκ) δ_{A_1} (0, κ)

= δ_{A_1} (00, ωκ) δ_{A_1} (0, κ)

= δ_{A_1} ((0, ω) (0, κ)) δ_{A_1} (0, κ)

≤ δ_{A_1} (0, ω)

= A_1 (ω)

Thus, A_1 is a FDzI.

Proposition 3.13. If $\delta_{A_1}$and its complement ${\delta_{A_1}}^c$ are a FDzI, then $\delta_{A_1}$is constant.

Proof. we know that, δ_{A_1} (0) ≤ $\delta_{A_1}(\omega)$ …………. (1)

Then, ${\delta_{A_1}}^c$(0) ≤ ${\delta_{A_1}}^c$(ω)

1-δ_{A_1} (0) ≤ 1-$\delta_{A_1}(\omega)$

δ_{A_1} (0) ≥ $\delta_{A_1}(\omega)$…………………… (2)

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From (1) and (2) δ_{A_1} is a constant.

4. Conclusion

Through this work, we present the definitions of the FDzI and study some relationship among these types. The goal of our future effort is to study some concepts such as p-ideals, h-ideals. To develop the theory of Z-algebras, the fuzzy ideal plays an important role. Also, we have developed several theorem of FDzI in z-algebras. Using above notion we can conclude that the research along this path can be continued for further developments of intuitionistic fuzzy doubt z-ideals in Z-algebras and their applications in various algebra.

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A CONTEMPORARY APPROACH ON α- LEVEL SET OF A PENTAPARTITIONED NEUTROSOPHIC BINARY SUBGROUPS

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Abstract

Pentapartitioned Neutrosophic Binary Set is a new concept endowed with five degrees of membership functions over two universes. It is an important tool to deal certain problems that require two universes rather than a single one. In this paper, the concept of α-level set of a pentapartitioned neutrosophic binary subgroups are studied and also its some interesting theorems are analyzed.

Keywords: level set, pentapartitioned neutrosophic binary set, pentapartitioned neutrosophic binary subgroup.

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1. Introduction

The concept of the neutrosophic set was presented by Smarandache in 1998. As a continuation of neutrosophic set, Pentapartitioned neutrosophic set was established by Surpati Pranamik and Rama Malik. It's a five-valued logic set where each x in X has a membership that represents a truth, a contradiction, ignorance, unknown, and falsehood. In 2024, A. Anit Yoha and M. Jaslin Melbha established a new set called Pentapartitioned Neutrosophic Binary Set and applied it in a group structure. Awolola introduced the Concept of $\boldsymbol{\alpha}$ - Level Sets of Neutrosophic Set in 2020. This paper concentrates on α-level set of a pentapartitioned neutrosophic binary subgroups and its theoretical implementations

2. Preliminaries

Definition 2.1. A neutrosophic set (NS) $\tilde{A}$ over X is defined as follows:

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$\tilde{A} = \{< u, \mu_{\tilde{A}}(u), \nu_{\tilde{A}}(u), \gamma_{\tilde{A}}(u) >: u \in X\}$, Where $\mu_{\tilde{A}}(u), \nu_{\tilde{A}}(u), \gamma_{\tilde{A}}(u)$ are the truth, indeterminant, and falsity membership values of each $u \in X$.

So, $0 \leq \mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u) + \gamma_{\tilde{A}} \leq 3$.

Definition 2.2. Let U and V be two universes of discourse. The *Pentapartitioned neutrosophic binary set* $(\tilde{A}_1, \tilde{A}_2) \subseteq (U, V)$ is given by

$$(\tilde{A}_1, \tilde{A}_2) = \left\{ \begin{matrix} < u, \mu_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \vartheta_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), \gamma_{\tilde{A}_1}(u) >, \\ < v, \mu_{\tilde{A}_2}(v), \sigma_{\tilde{A}_2}(v), \vartheta_{\tilde{A}_2}(v), \phi_{\tilde{A}_2}(v), \gamma_{\tilde{A}_2}(v) >: u \in U, v \in V \end{matrix} \right\}$$

Where $\mu_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \vartheta_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), \gamma_{\tilde{A}_1}(u): U \to [0,1]$ are the degrees of the membership of truth, contradiction, ignorance, unknown, and falsity membership values of $u \in U$ and $\mu_{\tilde{A}_2}(v), \sigma_{\tilde{A}_2}(v), \vartheta_{\tilde{A}_2}(v), \phi_{\tilde{A}_2}(v), \gamma_{\tilde{A}_2}(v): V \to [0,1]$ are the degrees of the membership of truth, contradiction, ignorance, unknown, and falsity membership values of $v \in V$ such that $0 \leq \mu_{\tilde{A}_1}(u) + \sigma_{\tilde{A}_1}(u) + \vartheta_{\tilde{A}_1}(u) + \phi_{\tilde{A}_1}(u) + \gamma_{\tilde{A}_1}(u) \leq 5$ and $0 \leq \mu_{\tilde{A}_2}(v) + \sigma_{\tilde{A}_2}(v) + \vartheta_{\tilde{A}_2}(v) + \phi_{\tilde{A}_2}(v) + \gamma_{\tilde{A}_2}(v) \leq 5$.

Definition 2.3. Suppose $(\tilde{A}_1, \tilde{A}_2)$ represents a Pentapartitioned Neutrosophic Binary Set (PNBS) over two universes U and V. A Pentapartitioned Neutrosophic Binary Subgroup (PNBSG) is a structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = (G_{(\tilde{A}_1,\tilde{A}_2)}, *)$ where $G_{(\tilde{A}_1,\tilde{A}_2)} = (G = \{U \cup V\}, *)$ forms a group under a binary operation $*$ which satisfies, the following $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)}$ inequality:

(i) $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m, n) \succcurlyeq \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n)$

(ii) $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}) \succcurlyeq \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m)$; for every $m, n \in G$

That is, for every $m, n \in G$,

(i) $\mu_{(\tilde{A}_1,\tilde{A}_2)}(m, n) \geq \mu_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \mu_{(\tilde{A}_1,\tilde{A}_2)}(n)$,
$\sigma_{(\tilde{A}_1,\tilde{A}_2)}(m, n) \geq \sigma_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \sigma_{(\tilde{A}_1,\tilde{A}_2)}(n)$,
$\vartheta_{(\tilde{A}_1,\tilde{A}_2)}(m, n) \leq \vartheta_{(\tilde{A}_1,\tilde{A}_2)}(m) \vee \vartheta_{(\tilde{A}_1,\tilde{A}_2)}(n)$,
$\phi_{(\tilde{A}_1,\tilde{A}_2)}(m, n) \leq \phi_{(\tilde{A}_1,\tilde{A}_2)}(m) \vee \phi_{(\tilde{A}_1,\tilde{A}_2)}(n)$,
$\gamma_{(\tilde{A}_1,\tilde{A}_2)}(m, n) \leq \gamma_{(\tilde{A}_1,\tilde{A}_2)}(m) \vee \gamma_{(\tilde{A}_1,\tilde{A}_2)}(n)$ and

(ii) $\mu_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}) \geq \mu_{(\tilde{A}_1,\tilde{A}_2)}(m)$,$\sigma_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}) \geq \sigma_{(\tilde{A}_1,\tilde{A}_2)}(m)$,
$\vartheta_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}) \leq \vartheta_{(\tilde{A}_1,\tilde{A}_2)}(m)$, $\phi_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}) \leq \phi_{(\tilde{A}_1,\tilde{A}_2)}(m)$;
$\gamma_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}) \leq \gamma_{(\tilde{A}_1,\tilde{A}_2)}(m)$.

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Definition 2.4. Let $\tilde{A}$ be any neutrosophic set in a non-empty set X. Then for any $\alpha \in [0,1]$, the α- lower level and the α- upper level sets of $\tilde{A}$ denoted by $L(\tilde{A},\alpha)$ and $U(\tilde{A},\alpha)$ are respectively defined as follows:

$$L(\tilde{A},\alpha) = \{u \in X: \mu_{\tilde{A}}(u) \geq \alpha, \nu_{\tilde{A}}(u) \geq \alpha, \gamma_{\tilde{A}}(u) \leq \alpha\} \text{ and}$$
$$U(\tilde{A},\alpha) = \{u \in X: \mu_{\tilde{A}}(u) \leq \alpha, \nu_{\tilde{A}}(u) \leq \alpha, \gamma_{\tilde{A}}(u) \geq \alpha\}$$

Proposition 2.5. If $(\tilde{A}_1,\tilde{A}_2)$ represents a PNBSG with structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = (G_{(\tilde{A}_1,\tilde{A}_2)},*)$ iff $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m * n^{-1}) \succcurlyeq \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n)$, for every $m,n \in G_{(\tilde{A}_1,\tilde{A}_2)}$.

Remark 2.6. Every subgroup of an abelian group is abelian.

3. Main Results

Definition 3.1. If $(\tilde{A}_1,\tilde{A}_2)$ represents a PNBSG with structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = (G_{(\tilde{A}_1,\tilde{A}_2)},*)$ over U and V then the α- level set of $(\tilde{A}_1,\tilde{A}_2)$ denoted by $(\tilde{A}_1,\tilde{A}_2)_\alpha$ and is defined as follows: for any $\alpha \in (0,1]$,

$$(\tilde{A}_1,\tilde{A}_2)_\alpha = \{m \in G_{(\tilde{A}_1,\tilde{A}_2)}: \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \succcurlyeq \alpha\}$$

Theorem 3.2. If $(\tilde{A}_1,\tilde{A}_2)$ represents a PNBS over U and V. Then $(\tilde{A}_1,\tilde{A}_2)$ is PNBSG of a group $G_{(\tilde{A}_1,\tilde{A}_2)} = (U \cup V,*)$ iff $(\tilde{A}_1,\tilde{A}_2)_\alpha$ is a subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ for all $\alpha \in (0,1]$, where $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(e) \succcurlyeq \alpha$ and e appears as the identity in $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Proof. Assume $(\tilde{A}_1,\tilde{A}_2)$ is PNBSG of a group $G_{(\tilde{A}_1,\tilde{A}_2)} = (U \cup V,*)$, where $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(e) \succcurlyeq \alpha$ and e appears as the identity in $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Clearly, $(\tilde{A}_1,\tilde{A}_2)_\alpha \neq \emptyset$ as $\in (\tilde{A}_1,\tilde{A}_2)_\alpha$. Let $m,n \in (\tilde{A}_1,\tilde{A}_2)_\alpha$ be any two elements.

Then $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \succcurlyeq \alpha$ and $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n) \succcurlyeq \alpha$

$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m * n^{-1}) \succcurlyeq \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n) \succcurlyeq \alpha$ [As $(\tilde{A}_1,\tilde{A}_2)$ is a PNBSG of $G_{(\tilde{A}_1,\tilde{A}_2)}$]

$\Rightarrow m * n^{-1} \in (\tilde{A}_1,\tilde{A}_2)_\alpha$

$\Rightarrow (\tilde{A}_1,\tilde{A}_2)_\alpha$ is a subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Conversely, Let $(\tilde{A}_1,\tilde{A}_2)$ represents a PNBS over U and V such that $(\tilde{A}_1,\tilde{A}_2)_\alpha$ is a subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ for all $\alpha \in (0,1]$.

Let $m,n \in G_{(\tilde{A}_1,\tilde{A}_2)}$ and let $\alpha = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n)$

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Then $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \succcurlyeq \alpha$ and $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n) \succcurlyeq \alpha$

That is $m,n \in \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$

$$\Rightarrow\ m * n^{-1} \in \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha \text{ [since } \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha \text{ is a subgroup of } G_{(\tilde{A}_1,\tilde{A}_2)}]$$

$$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m * n^{-1}) \succcurlyeq \alpha = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n)$$

$$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m * n^{-1}) \succcurlyeq \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \wedge \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n)$$

Therefore, $\left(\tilde{A}_1,\tilde{A}_2\right)$ is PNBSG of a group $G_{(\tilde{A}_1,\tilde{A}_2)}$. [By Proposition 3.5]

Definition 3.3. If $\left(\tilde{A}_1,\tilde{A}_2\right)$ represents a PNBSG with structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = \left(G_{(\tilde{A}_1,\tilde{A}_2)},*\right)$ over U and V, then it is said to be pentapartitioned neutrosophic binary normal subgroup (PNBNSG) in $G_{(\tilde{A}_1,\tilde{A}_2)}$ if $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m * n) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n * m)$ for every $m,n \in G_{(\tilde{A}_1,\tilde{A}_2)}$.

Remark 3.4. If $\left(\tilde{A}_1,\tilde{A}_2\right)$ represents a PNBSG with structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = \left(G_{(\tilde{A}_1,\tilde{A}_2)},*\right)$ over U and V, then it is said to be normal in $G_{(\tilde{A}_1,\tilde{A}_2)}$ iff $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(g^{-1}mg) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m)$ for every $m \in \left(\tilde{A}_1,\tilde{A}_2\right), g \in G_{(\tilde{A}_1,\tilde{A}_2)}$.

Theorem 3.5. If $\left(\tilde{A}_1,\tilde{A}_2\right)$ represents a PNBS over U and V. Then $\left(\tilde{A}_1,\tilde{A}_2\right)$ is PNBNSG of a group $G_{(\tilde{A}_1,\tilde{A}_2)} = (U \cup V,*)$. Then $\left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$ is a normal subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ for all $\alpha \in (0,1]$, where $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(e) \succcurlyeq \alpha$ and e appears as the identity in $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Proof. Let $m \in \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$ and $g \in G_{(\tilde{A}_1,\tilde{A}_2)}$ be any element.

Then $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \succcurlyeq \alpha$. Also, $\left(\tilde{A}_1,\tilde{A}_2\right)$ is PNBNSG of a group $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Therefore, $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(g^{-1}mg) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m)$ for all $\in \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$, $g \in G_{(\tilde{A}_1,\tilde{A}_2)}$.

$$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(g^{-1}mg) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m) \succcurlyeq \alpha$$

$$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(g^{-1}mg) \succcurlyeq \alpha$$

$$\Rightarrow g^{-1}mg \in \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$$

Hence, $\left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$ is a normal subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Definition 3.6. If $\left(\tilde{A}_1,\tilde{A}_2\right)$ represents a PNBSG with structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = \left(G_{(\tilde{A}_1,\tilde{A}_2)},*\right)$ over U and V, then $\left(\tilde{A}_1,\tilde{A}_2\right)$ is called a pentapartitioned neutrosophic

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binary abelian subgroup (PNBASG) of $G_{(\tilde{A}_1,\tilde{A}_2)}$ if $\left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$ is an abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ for all $\alpha \in (0,1]$.

Theorem 3.7. If $G_{(\tilde{A}_1,\tilde{A}_2)}$ is an abelian group, then every PNBSG of $G_{(\tilde{A}_1,\tilde{A}_2)}$ is a PNBASG of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Proof. Let $\left(\tilde{A}_1,\tilde{A}_2\right)$ be a PNBSG of $G_{(\tilde{A}_1,\tilde{A}_2)}$ and given that $G_{(\tilde{A}_1,\tilde{A}_2)}$ is an abelian group.

$\Rightarrow \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$ is a subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ [By theorem]

$\Rightarrow \left(\tilde{A}_1,\tilde{A}_2\right)_\alpha$ is an abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ [By remark]

$\Rightarrow \left(\tilde{A}_1,\tilde{A}_2\right)$ is a PNBASG of $G_{(\tilde{A}_1,\tilde{A}_2)}$ [By definition]

Remark 3.8. The converse of Theorem 3.7 does not hold in general, as shown by the following counterexample:

Let $U = \{\pm 1, \pm i\}$ and $V = \{-1, \pm i, \pm j, \pm k\}$ be two sets under consideration. Therefore we get the combined set $G_{(\tilde{A}_1,\tilde{A}_2)} = \{U \cup V\}$. Clearly $(G_{(\tilde{A}_1,\tilde{A}_2)},*) = \{\pm 1, \pm i, \pm j, \pm k\}$ forms a group. Let $\left(\tilde{A}_1,\tilde{A}_2\right)$ be a PNB set defined over U and V as the following table:

for	$u = 1$	$u^2 = 1$, $u \neq 1$	$u^4 = 1$, $u \neq \pm 1$
$\mu_{\tilde{A}_1}(u)$	.2	.15	.1
$\sigma_{\tilde{A}_1}(u)$	.29	.1	.12
$\vartheta_{\tilde{A}_1}(u)$	.09	.1	.18
$\phi_{\tilde{A}_1}(u)$	.095	.65	.8
$\gamma_{\tilde{A}_1}(u)$	.06	.07	.95

for	$v^2 = 1$	$v^4 = 1$, $v \neq -1$
$\mu_{\tilde{A}_2}(v)$	.1	.12
$\sigma_{\tilde{A}_2}(v)$	.2	.19
$\vartheta_{\tilde{A}_2}(v)$	.25	.18
$\phi_{\tilde{A}_2}(v)$	.099	.7
$\gamma_{\tilde{A}_2}(v)$	.1	.93

The membership grade of combined PNB set is given by

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Clearly, $(\tilde{A}_1, \tilde{A}_2)$ is a PNBSG of $G_{(\tilde{A}_1,\tilde{A}_2)}$. Additionally, all $(\tilde{A}_1, \tilde{A}_2)_\alpha$ are abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ for any $\alpha \in (0,1]$. Hence, $(\tilde{A}_1, \tilde{A}_2)$ is a PNBASG of $G_{(\tilde{A}_1,\tilde{A}_2)}$, but $G_{(\tilde{A}_1,\tilde{A}_2)}$ is not an abelian group.

Theorem 3.9. If $(\tilde{A}_1, \tilde{A}_2)$ represents a PNBASG with structure $\mathcal{B}_{(\tilde{A}_1,\tilde{A}_2)} = (G_{(\tilde{A}_1,\tilde{A}_2)}, *)$ over U and V. Then $H_{(\tilde{A}_1,\tilde{A}_2)} = \{u \in G_{(\tilde{A}_1,\tilde{A}_2)} : \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(uv) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(vu) \ for\ all\ v \in G_{(\tilde{A}_1,\tilde{A}_2)}\}$ is an abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Proof. Let $(\tilde{A}_1, \tilde{A}_2)$ be a PNBASG of a group $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Then by definition, $(\tilde{A}_1, \tilde{A}_2)_\alpha$ is an abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$ for all $\alpha \in (0,1]$.

Clearly, $H_{(\tilde{A}_1,\tilde{A}_2)} \neq \emptyset$ as $e \in H_{(\tilde{A}_1,\tilde{A}_2)}$.

for	$m = 1$	$m^2 = 1, m \neq 1$	$m^4 = 1, m \neq 1$
$\mu_{(\tilde{A}_1,\tilde{A}_2)}(m)$	0.2	0.15	0.12
$\sigma_{(\tilde{A}_1,\tilde{A}_2)}(m)$	0.29	0.2	0.19
$\vartheta_{(\tilde{A}_1,\tilde{A}_2)}(m)$	0.09	0.1	0.18
$\phi_{(\tilde{A}_1,\tilde{A}_2)}(m)$	0.095	0.099	0.7
$\gamma_{(\tilde{A}_1,\tilde{A}_2)}(m)$	0.06	0.07	0.93

Let $m, n \in H_{(\tilde{A}_1,\tilde{A}_2)}$

$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(mu) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(um)$ and

$\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(nu) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(un)$ for all $u \in G_{(\tilde{A}_1,\tilde{A}_2)}$

Now, for $u \in G_{(\tilde{A}_1,\tilde{A}_2)}$, $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}((mn)u) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m(nu))$ [as $nu \in G_{(\tilde{A}_1,\tilde{A}_2)}$]

$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}((nu)m)$$
$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n(um))$$
$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n(mu))$$
$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}((nm)u)$$

Therefore, $m, n \in H_{(\tilde{A}_1,\tilde{A}_2)}$

Also, let $m \in H_{(\tilde{A}_1,\tilde{A}_2)}$

$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(mu) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(um)$ for all $u \in G_{(\tilde{A}_1,\tilde{A}_2)}$ ……(1)

By substituting $u = v^{-1}$ in (1), we get $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(mv^{-1}) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(v^{-1}m)$

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Now, $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m^{-1}v) = \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}((m^{-1}v)^{-1})$

$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(v^{-1}m)$$

$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(mv^{-1})$$

$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}((mv^{-1})^{-1})$$

$$= \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(vm^{-1}) \ \forall \ v \in G_{(\tilde{A}_1,\tilde{A}_2)}$$

Hence $m^{-1} \in H_{(\tilde{A}_1,\tilde{A}_2)}$. Therefore $H_{(\tilde{A}_1,\tilde{A}_2)}$ is a subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Now, to prove that $H_{(\tilde{A}_1,\tilde{A}_2)}$ is an abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

Let $m, n \in H_{(\tilde{A}_1,\tilde{A}_2)}$ be arbitrary. Without loss of generality let $\alpha_i < \alpha_j$ for $i \neq j$ such that $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(m)=\alpha_i$ and $\text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n) = \alpha_j$ where $\alpha_i, \alpha_j \in (0,1]$

Then $m \in (\tilde{A}_1, \tilde{A}_2)_{\alpha_i}$ and $n \in (\tilde{A}_1, \tilde{A}_2)_{\alpha_j}$

$$\Rightarrow \text{PNB}_{(\tilde{A}_1,\tilde{A}_2)}(n)=\alpha_j > \alpha_i$$

$$\Rightarrow \ n \in (\tilde{A}_1, \tilde{A}_2)_{\alpha_i}$$

Thus $m, n \in (\tilde{A}_1, \tilde{A}_2)_{\alpha_i}$ so that $mn = nm$

Hence $H_{(\tilde{A}_1,\tilde{A}_2)}$ is an abelian subgroup of $G_{(\tilde{A}_1,\tilde{A}_2)}$.

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A STUDY ON IRREDUNDANCE NUMBER IN CLUSTER HYPERGRAPHS

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Abstract

A dominating set S is a minimal dominating set of H if and only if for every vertex $x \in S, pn[x,S] \neq \emptyset$. ie, for every vertex $x \in S$, has at least one private neighbour in S. This article explores the concept of Irredundance Number in Cluster Hypergraphs. A set S is irredundant if for every vertex $x \in S,\ pn[x,S] \neq \emptyset$. An irredundant set S is called a Maximal Irredundant Set if no proper subset of S is irredundant. The minimum cardinality of a irredundant set is called the Irredundance Number and is denoted by $ir(H)$. The maximum cardinality of a irredundant set is called the Upper Irredundance Number and is denoted by $IR(H)$. It is proved that $ir(H) \leq IR(H)$, $ir(H) \leq \gamma(H)$ and $\Gamma(H) \leq IR(H)$. Also, some theorems and results related to the concept of Irredundance Number in Cluster Hypergraphs have been discussed and demonstrated in this article.

Keywords: cluster hypergraphs, irredundant set, maximal irredundant set, upper irredundance number

2020 Mathematics Subject Classification (AMS): 05C65

1. Introduction

The major research area in graph theory is the study of domination and related concepts such as independence, irredundance and covering. This article focuses mainly on irredundance number. The concept irredundance number was introduced by Cockayne, Hedetniemi and Miller. A set S is a irredundant set if for every vertex $v \in S, pn[v,S] \neq \emptyset$. An irredundant set S is called a maximal irredundant set if no proper subset of S is irredundant. The minimum cardinality of a maximal irredundant set in a graph G is called the irredundance number of G and is denoted by $ir(G)$. The maximum cardinality of a maximal irredundant set in a graph G is called the upper irredundance number of G and is denoted by $IR(G)$[1].

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A set S is a minimal dominating set in H if and only if for every vertex $y \in V_X(H)$ such that $N[y] \cap S = \{x\}$. The vertex y is called the private neighbour of x with respect to S. $pn[x, S]$ is called as the set all private neighbour of x with respect to S.

A dominating set S is a minimal dominating set in H if and only if for every vertex $x \in S,\ pn[x, S] \neq \emptyset$. Ie., for every vertex $x \in S,$ has atleast one private neighbour. This minimality condition for a dominating set explores another concept called irredundance. C. Mary Christal Flower and J. Befija Minnie together introduced the concept of Irredundance Number in Cluster Hypergraphs. Also, some theorems and results related to the concept of Irredundance Number in Cluster Hypergraphs have been discussed and demonstrated in this article.

2. Main Results

Definition 2.1. Let $H = (V_X, E)$ be a cluster hypergraph. A subset $S \subseteq V_X(H)$ is said to be independent if it does not contain an edge E in H with $|E| > 1$. The independence number or independent number $\alpha(H)$ of a cluster hypergraph H is defined as the maximum cardinality of a maximal independent set in H.

The set $S \subseteq V_X(H)$ is called a strongly independent set if no two vertices in S are adjacent. The maximum cardinality of a maximal strongly independent set is denoted by $\beta(H)$ and is called the strongly independence number or strongly independent number[2].

Definition 2.2. Let H be a cluster hypergraphs. A set $S \subseteq V_X(H)$ is called an irredundance set in H if for every vertex x in S has atleast one private neighbour with respect to S.

Theorem 2.3. A dominating set S of a cluster hypergraph H is a minimal dominating set in H if and only if S is both a dominating and a irredudant set.

Proof. Let H be a cluster hypergraph. Assume that, S is a minimal dominating set in H. Then by definition, S is both a irredundant and dominating set in H.

Conversely, assume that, S is both a irredundant and dominating set in H. To prove S is a minimal dominating set in H. Let $x \in S$, Since S is irredundant set in H, by definition $pn[x, S] \neq \phi$. Let $y \in pn[x, S]$. Then y is not adjacent to any vertex in $S \backslash \{x\}$ and so $S \backslash \{x\}$ is not a dominating set in H. It follows that S is a minimal dominating set of H.

Theorem 2.4. Every minimal dominating set S in a cluster hypergraph H is a maximal irredundant set of H.

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Proof. Let H be a cluster hypergraph and let S be a minimal dominating set in H. Then by theorem 3.2, S is an irredundant set in H. Therefore, it is enough to prove that S is a maximal dominating set of H. Suppose not, there exists a vertex $x \in V_X(H)\backslash S$ such that $S \cup \{x\}$ is a irredundant set in H. It follows that $pn[x, S \cup \{x\}] \neq \phi$. Let $y \in pn[x, S \cup \{x\}]$. Then no vertex in S is adjacent to y. This implies that, S is not a dominating set in H which gives a contradiction. Hence, S is a maximal irredundant set in H.

Definition 2.5. The minimum cardinalily of a maximal irredundant set in a cluster hypergraph H is called the Irredundance Number, and is denoted by $ir(H)$. The maximum cardinality of an irredundant set in a cluster hypergraph H is called the Upper Irredundance Number and is denoted by $IR(H)$.

Observation 2.6. For any cluster hypergraph H, $ir(H) \leq IR(H)$.

Observation 2.7. For any cluster hypergraph H, $ir(H) \leq \gamma(H)$ and $\Gamma(H) \leq IR(H)$.

Theorem 2.8. For any cluster hypergraphs H, $IR(H) + \delta(H) \leq |V_X(H)|$.

Proof. Let H be a cluster hypergraph and S be a maximal irredundant set with $|S| = IR(H)$. Suppose $x \in S$. Since S is an irredundant set in H, there exists a vertex y in H such that $y \in N[x]\backslash N[S\backslash\{x\}]$. Consider the following two cases.

Case (i) $x = y$.

In this case, the vertex x is not adjacent to every vertex in S and so it must have atleast $\delta(H)$ neighbours in $V_X(H)\backslash S$. Hence, $|V_X(H)| - IR(H) = |V_X(H)\backslash S| \geq \delta(H)$ and so, $IR(H) + \delta(H) \leq |V_X(H)|$.

Case (ii) $x \neq y$.

By the choice of y, $y \notin S$ and $N(y) \cap S = \{x\}$. Then $N[y]\backslash\{x\} \subset V_X(H)\backslash S$. It follows that, $|V_X(H)| - IR(H) = |V_X(H)\backslash S| \geq |N[y]\backslash\{x\}| \geq \delta(H)$ and hence, $IR(H) + \delta(H) \leq |V_X(H)|$.

Corollary 2.9. For any cluster hypergraph H, $\Gamma(H) + \delta(H) \leq |V_X(H)|$ and $\beta(H) + \delta(H) \leq |V_X(H)|$.

Theorem 2.10. For any cluster hypergraph H $IR(H) + \delta(H) = |V_X(H)|$ if and only if $\Gamma(H) + \delta(H) = |V_X(H)|$.

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Proof. Let H be a cluster hypergraph such that $\Gamma(H) + \delta(H) = |V_X(H)|$. By observation 2.6, $\Gamma(H) \leq IR(H)$.It follows that, $|V_X(H)| \leq IR(H) + \delta(H)$. By theorem 2.7, it is concluded that $IR(H) + \delta(H) = |V_X(H)|$.

Conversely, assume H be a cluster hypergraph such that $IR(H) + \delta(H) = |V_X(H)|$. To prove that $\Gamma(H) + \delta(H) = |V_X(H)|$. Let S be a maximal irredundant set in H with $IR(H) = |S|$. First to prove that, S itself is a dominating set in H. Suppose S is not a dominating set in H, then there is a vertex $y \in V_X(H)\backslash S$ such that y is not adjacent to any vertex in S. Thus, $N[y] \subseteq V_X(H)\backslash S$. But $|N[y]| = d(y) + 1 \geq \delta(H) + 1$. It follows that, $\delta(H) + 1 \leq |N[y]| \leq |V_X(H)\backslash S| = |V_X(H)| - IR(H)$. Here it is obtained that, $IR(H) + \delta(H) \leq |V_X(H)| - 1$. This implies that, $IR(H) - \delta(H) < |V_X(H)|$, which is a contradiction. Hence, S is a dominating set in H. Since S is an irredundant set, by theorem 3.2., S is a minimal dominating set in H. So, $\Gamma(H) \geq |S| = IR(H) \geq \Gamma(H)$ implies that $IR(H) = \Gamma(H)$. Hence, $\Gamma(H) + \delta(H) = |V_X(H)|$.

Theorem 2.11. Let H be any cluster hypergraph and let S be any dominating set in H. Then $|V_x(H)\backslash S| \leq \sum_{x \in S} d(x)$. Further, the inequality holds if and only if S is a strongly independent set in H and for every $x \in V_X(H)\backslash S$, there is unique vertex $y \in S$ such that $N(x) \cap S = \{y\}$.

Proof. Let H be a cluster hypergraph and let S be any dominating set in H. Then, by definition, every vertex in $V_X(H)\backslash S$ adds atleast one vertex to the degree of some vertex x in S. It follows that, $|V_X(H)\backslash S| \leq \sum_{x \in S} d(x)$.

Next, assume that $|V_X(H)\backslash S| = \sum_{x \in S} d(x)$. To prove that S is a strongly independent set in H. Suppose S is not a strongly independent set in H, then there exists vertices $x, y \in S$ such that x and y are adjacent. Since S is a dominating set in H, by definition, every vertex in $V_X(H)\backslash S$ is counted or added in the sum $\sum_{x \in S} d(x)$. Furthermore, the vertex x is counted or added in $d(y)$ and the vertex y is counted or added in $d(x)$. Also $x, y \in S$, implies that $x, y \notin V_X(H)\backslash S$. This shows that, $\sum_{x \in S} d(x) > |V_X(H)\backslash S| \geq |V_X(H)\backslash S| + 2$, implies that $\sum_{x \in S} d(x) > |V_X(H)\backslash S| + 1$, which is a contradiction. Thus, S is a strongly independent set in H. Now to demonstrate that, for every vertex $x \in V_X(H)\backslash S$, then there is a unique vertex $y \in S$ such that $N(x) \cap S = \{y\}$. Since, S is a dominating set in H, it is sufficient to show that $N(x) \cap S = \{y\}$. That is, $|N(x) \cap S| = 1$. Suppose, $|N(x) \cap S| \geq 2$. Let $y, z \in N(x) \cap S$. Then the sum $\sum_{x \in S} d(x)$ exceeds $|V_X(H)\backslash S|$ by atleast one, since the vertex x is counted or added atleast twice (one in $d(y)$ and one in $d(z)$). So $\sum_{x \in S} d(x) > |V_X(H)\backslash S|$, which is a contradiction. Thus $N(x) \cap S = \{y\}$.

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Conversely, if S is a strongly independent set in H. For every $x \in V_X(H)\setminus S$, there is a unique vertex $y \in S$ such that $N(x) \cap S = y$ then obviously the sum $\sum_{x\in S} d(x) = |V_X(H)\setminus S|$.

3. Conclusion

In this article, the concept Irredundance Number in Cluster Hypergraphs have been introduced and the same concept is extended to prove some theorems and results related to the Irredundance Number in Cluster Hypergraphs.

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SECURE MONOPHONIC DOMINATION NUMBER OF SOME SPECIAL GRAPHS

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Abstract

Let $G = (V, E)$ be a connected graph. A monophonic dominating set M is both a monophonic set and a dominating set. A monophonic dominating set M is said to be a secure monophonic dominating set S_m (abbreviated as SMD set) of G if for each $v \in V \backslash M$ there exists $u \in M$ such that v is adjacent to u and $S_m = (M \setminus \{u\}) \cup \{v\}$ is a monophonic dominating set.The minimum cardinality of a secure monophonic dominating set of G is the secure monophonic domination number of G and is denoted by $\gamma_{sm}(G)$. In this paper we investigate the secure monophonic domination number of special graph structures like Jellyfish graph, Ladder graph and Lollipop graph.

Key words : Monophonic path, monophonic domination number, secure domination number, secure monophonic domination number.

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1. Introduction

By a graph $G = (V, E)$, we mean a finite and undirected connected graph without loops or multiple edges. The vertex set and edge set of G are respectively denoted by $V(G)$ and $E(G)$. For basic graph theoretic terminology, we refer to [5]. A chord of a path P is an edge which connects two non-consecutive vertices of P. For two vertices u and v, the closed interval $J[u, v]$ consists of all the vertices lying in a $u - v$ monophonic path including the vertices u and v. If u and v are adjacent , then $J[u, v] = \{u, v\}$. For a set M of vertices, let $J[M] = \bigcup_{u,v \in M} J[u, v]$. Then certainly $M \subseteq J[M]$. A set $M \subseteq V(G)$is called a monophonic set of G if $J[M] = V$. In [8], Haynes et al introduced the concept of domination in graphs. A subset $D \subseteq V(G)$ is called a dominating set if every vertex $v \in V(G) \backslash D$is adjacent to a vertex $u \in D$. The domination

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number, $\gamma(G)$, of a graph G denotes the minimum cardinality of such dominating sets of G. For each $u \in V \backslash S$ there exists $v \in S$ such that v is adjacent to u and $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of G. In this case we say that u is $S-$ defended by v or v $S-$ defends u. A dominating set S in which every vertex in $V \backslash S$ is S - defended by a vertex in S is called a secure dominating set of G. The secure domination number $\gamma_s(G)$ is the minimum cardinality of a secure dominating set of G[11]. This concept was introduced by Cockayne et al in [7]. A subset M of V is said to be a monophonic dominating set of a graph G if M is both a monophonic set and a dominating set. The minimum of the cardinalities of monophonic dominating sets of G is called the monophonic domination number and is denoted by $\gamma_m(G)$. In 2012, John et al [10] introduced the concept of monophonic domination number of a graph. In this paper we introduce the concept secure monophonic domination number of graphs.

Definition 1.1. [7] A dominating set D is called a secure dominating set if for each $v \in V \backslash D$ there exists $u \in D$ such that v is adjacent to u and $S = (D \setminus \{u\}) \cup \{v\}$ is a dominating set.

Definition 1.2. [14] A chord of a path P is an edge which connects two non-adjacent vertices of P. An $u - v$ path is called a monophonic path if it is a chordless path. A monophonic set M of G is a set $M \subseteq V(G)$ such that every vertex of G is contained in a monophonic path joining some pair of vertices in M. A monophonic dominating set M is both a monophonic set and a dominating set. The minimum of the cardinalities of monophonic dominating sets of G is called the monophonic domination number and is denoted by $\gamma_m(G)$.

Definition 1.3. A monophonic dominating set M is said to be a secure monophonic dominating set S_m (abbreviated as SMD set) of G if for each $v \in V \backslash M$ there exists $u \in M$ such that v is adjacent to u and $S_m = (M \setminus \{u\}) \cup \{v\}$ is a monophonic dominating set.The minimum cardinality of a secure monophonic dominating set of G is the secure monophonic domination number of G and is denoted by $\gamma_{sm}(G)$.

2. Observation

i. Each end vertex of a connected graph G belongs to every SMD set of G.

3. Main Results

Theorem 3.1. For the Jellyfish graph $G = J_{m,n}$ with prime edge, $\gamma_{sm}(G) = m + n + 1$ $(m, n \geq 1)$.

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Proof. Let $G = J_{m,n}$ be Jellyfish graph obtained from 4-cycle with vertices f, f', g_0, g_0' including the prime edge connecting f and f'. Appending m pendant edges to g_0 and n pendant edges to g_0'. The resultant graph is $J_{m,n}$ whose vertex set $V(G) = \{f, f', g_0, g_0', g_i, g_j' \,/\, 1 \le i \le m,\ 1 \le j \le n\}$ and edge set $E(G) = \{ff', fg_0, fg_0', g_0f', g_0'f', g_0g_i, g_0'g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$ such that $|V(G)| = m + n + 4$ and $|E(G)| = m + n + 5$. Let $Z = \{g_i, g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$ be the $m + n$ end vertices of G. By observation, Z is a subset of every SMD set of G. Since the vertices f and f' are not dominated by any vertex of Z, Z is not a SMD set of G and so $\gamma_{sm}(G) \ge m + n + 1$. Let $Z' = Z \cup \{f\}$. Clearly monophonic path exists and $V(G) - Z'$ is dominated by atleast one element of Z'. Therefore Z' is a SMD set of G, so that $\gamma_{sm}(G) = m + n + 1$.

Theorem 3.2. For the Jellyfish graph without prime edge $G = {J^*}_{m,n}$, $\gamma_{sm}(G) = m + n + 1$ $(m, n \ge 1)$.

Proof. Let $J_{m,n}$ be Jellyfish graph with vertices $V(J_{m,n}) = \{f, f', g_0, g_0', g_i, g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$ and edges $E(J_{m,n}) = \{ff', fg_0, fg_0', g_0f', g_0'f', g_0g_i, g_0'g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$. Let $G = {J^*}_{m,n}$ be obtained by removing prime edge ff' from Jellyfish graph $J_{m,n}$ whose vertex set $V(G) = \{f, f', g_0, g_0', g_i, g_j' \,/\, 1 \le i \le m,\ 1 \le j \le n\}$ and edge set $E(G) = \{fg_0, fg_0', g_0f', g_0'f', g_0g_i, g_0'g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$ such that $|V(G)| = m + n + 4$ and $|E(G)| = m + n + 4$. Then by similar arguement as in theorem 1. Hence $\gamma_{sm}(G) = m + n + 1$.

Theorem 3.3. For the Extended Jellyfish graph $G = EJ_{m,n,l}$, $\gamma_{sm}(G) = m + n + l + 2$ $(m, n, l \ge 1)$.

Proof. Let ${J^*}_{m,n}$ be Jellyfish graph with vertices $V({J^*}_{m,n}) = \{f, f', g_0, g_0', g_i, g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$ and edges $E(J_{m,n}) = \{ff', fg_0, fg_0', g_0f', g_0'f', g_0g_i, g_0'g_j' \,/\, 1 \le i \le m, 1 \le j \le n\}$. Let $G = EJ_{m,n,l}$ be an extended Jellyfish graph from jellyfish graph ${J^*}_{m,n}$ without prime edge. Appending arbitrary l vertices($a_k, 1 \le k \le l$) in ${J^*}_{m,n}$ such a way that they all are connected to vertex f and f'. The resultant graph is $EJ_{m,n,l}$ whose vertex set $V(G) = \{f, f', g_0, g_0', g_i, g_j', a_k \,/\, 1 \le i \le m,\ 1 \le j \le n, 1 \le k \le l\}$ and edge set $E(G) = \{fg_0, fg_0', g_0f', g_0'f', g_0g_i, g_0'g_j', fa_k,\ f'a_k / 1 \le i \le m, 1 \le j \le n, 1 \le k \le l\}$ such that $|V(G)| = m + n + l + 4$ and $|E(G)| = m + n + 2l + 4$. Let $Z = \{g_i, g_j' \,/ 1 \le i \le m, 1 \le$

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$j \leq n\}$ be the $m+n$ end vertices of G. By observation, Z is a subset of every SMD set of G. Since the vertices f and $f', a_k(1 \leq k \leq l)$ are not dominated by any vertex of Z, Z is not a SMD set of G and so $\gamma_{sm}(G) \geq m+n+l+2$. Let $Z' = Z \cup \{f, f', a_k(1 \leq k \leq l)\}$. Clearly monophonic path exists and $V(G) - Z'$ is dominated by atleast one element of Z'. Therefore Z' is a SMD set of G, so that $\gamma_{sm}(G) = m+n+l+2$.

Theorem 3.4. For the Lollipop graph $G = Lp_{m,n}(m \geq 4, n \geq 7)$,

$$\gamma_{sm}(G) = \begin{cases} \left\lceil \frac{3n}{7} \right\rceil + m + 1 & if \quad n \equiv 1,3 \ (mod 7) \\ \left\lceil \frac{3n}{7} \right\rceil + m & if \quad n \equiv 0,2,4,5,6 (mod 7) \end{cases}$$

Proof. Let $\{f_i(1 \leq i \leq m), h_j(1 \leq j \leq n)\}$ be the vertices of G, whose edge set $E(G) = E(K_m) \cup \{f_m h_1, h_i h_{i+1}(1 \leq i \leq n-1)\}$ such that $|V(G)| = m+n$ and $|E(G)| = \frac{m(m-1)}{2} + n$. Let $m \geq 4, n \geq 7$. Consider the following cases.

Case a: Subcase (i): $n \equiv 0 (mod 7)$

Take $G = Lp_{m,7}(m \geq 4)$. Choose $S_m = \{f_i(1 \leq i \leq m-1),\ h_1, h_3, h_5, h_7\}$. Remove any vertex $x \in S_m$ and add another vertex $y \in V \backslash S_m$ to S_m such that x is adjacent to y. Hence the set S_m is again a secure dominating set of G. Also the monophonic path exists and it contain all the vertices of G. So $S_m = \{f_i(1 \leq i \leq m-1),\ h_1, h_3, h_5, h_7\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \leq i \leq m-1)\} \cup \{h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{3n}{7} \right\rceil + m$.

Subcase (ii): $n \equiv 2 (mod 7)$

Take $G = Lp_{m,9}(m \geq 4)$. Choose $S_m = \{f_i(1 \leq i \leq m-1),\ h_1, h_3, h_5, h_7, h_9\}$. Then by similar argument as in subcase (i), $S_m = \{f_i(1 \leq i \leq m-1),\ h_1, h_3, h_5, h_7, h_9\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \leq i \leq m-1)\} \cup \{h_{n-2}, h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{3n}{7} \right\rceil + m$.

Subcase (iii): $n \equiv 4 (mod 7)$

Take $G = Lp_{m,11}(m \geq 4)$. Choose $S_m = \{f_i(1 \leq i \leq m-1),\ h_1, h_3, h_5, h_7, h_9, h_{11}\}$. Then by similar argument as in subcase (i), $S_m = \{f_i(1 \leq i \leq m-1),\ h_1, h_3, h_5, h_7, h_9, h_{11}\}$ is a

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minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \le i \le m-1)\} \cup \{h_{n-4}, h_{n-2}, h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil\frac{3n}{7}\right\rceil + m$.

Subcase (iv): $n \equiv 5(mod7)$

Take $G = Lp_{m,12}(m \ge 4)$. Choose $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_8, h_{10}, h_{12}\}$. Then by similar argument as in subcase (i), $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_8, h_{10}, h_{12}\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \le i \le m-1)\} \cup \{h_{n-4}, h_{n-2}, h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil\frac{3n}{7}\right\rceil + m$.

Subcase (v): $n \equiv 6(mod7)$

Take $G = Lp_{m,13}(m \ge 4)$. Choose $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_8, h_{10}, h_{12}, h_{13}\}$. Then by similar argument as in subcase (i), $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_8, h_{10}, h_{12}, h_{13}\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \le i \le m-1)\} \cup \{h_{n-5}, h_{n-3}, h_{n-1}, h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil\frac{3n}{7}\right\rceil + m$.

Case b: Subcase (i): $n \equiv 1(mod7)$

Take $G = Lp_{m,8}(m \ge 4)$. Choose $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_7, h_8\}$. Then by similar argument as in subcase (i), $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_7, h_8\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \le i \le m-1), h_{n-1}, h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil\frac{3n}{7}\right\rceil + m + 1$.

Subcase (ii): $n \equiv 3(mod7)$

Take $G = Lp_{m,10}(m \ge 4)$. Choose $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_7, h_9, h_{10}\}$. Then by similar argument as in subcase (i), $S_m = \{f_i(1 \le i \le m-1),\ h_1, h_3, h_5, h_7, h_9, h_{10}\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \le i \le m-1), h_{n-3}, h_{n-1}, h_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil\frac{3n}{7}\right\rceil + m + 1$. From all the

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above cases, finally we conclude that $S_m = \bigcup_{j=0}^{k-1}\{h_{7j+1}, h_{7j+3}, h_{7j+5}\} \cup \{f_i(1 \leq i \leq m - 1)\} \cup \begin{Bmatrix} h_n & if & r = 0 \\ h_{n-1}, h_n & if & r = 1 \\ h_{n-2}, h_n & if & r = 2 \\ h_{n-3}, h_{n-1}, h_n & if & r = 3 \\ h_{n-4}, h_{n-2}, h_n & if & r = 4,5 \\ h_{n-5}, h_{n-3}, h_{n-1}, h_n & if & r = 6 \end{Bmatrix}$

Therefore $|S_m| = \begin{cases} \left\lceil \frac{3n}{7} \right\rceil + m + 1 & if \quad n \equiv 1,3\ (mod 7) \\ \left\lceil \frac{3n}{7} \right\rceil + m & if \quad n \equiv 0,2,4,5,6 (mod 7) \end{cases}$

Theorem 3.5. For the Ladder graph $G = Ld_n (n \geq 4)$,

$$\gamma_{sm}(G) = \begin{cases} n & if \quad n = 4,5,6 \\ \left\lceil \frac{6n}{7} \right\rceil + 1 & if \quad n \equiv 0,1,2,4,5,6\ (mod 7) \\ \left\lceil \frac{6n}{7} \right\rceil & if \quad n \equiv 3 (mod 7) \end{cases}$$

Proof. Let $\{f_i, g_i (1 \leq i \leq n)\}$ be the vertices of G, whose edge set $E(G) = \{f_i g_i (1 \leq i \leq n)\} \cup \{f_i f_{i+1}, g_i g_{i+1} (1 \leq i \leq n-1)\}$ such that $|V(G)| = 2n$ and $|E(G)| = 3n - 2$. If $n = 4$, then $G = Ld_4$, $S_m = \{f_1, f_3, g_2, g_4\}$ is minimum SMD set of G. Therefore $\gamma_{sm}(G) = 4$. If $n = 5$, then $G = Ld_5$, $S_m = \{f_1, f_3, f_5, g_2, g_4\}$ is minimum SMD set of G. Therefore $\gamma_{sm}(G) = 5$. If $n = 6$, then $G = Ld_6$, $S_m = \{f_1, f_3, f_5, g_2, g_4, g_6\}$ is minimum SMD set of G. Therefore $\gamma_{sm}(G) = 6$. Hence $\gamma_{sm}(G) = \{n \quad if \quad n = 4,5,6\}$. Let $n \geq 7$. Consider the following cases.

Case a: Subcase (i) $n \equiv 0 (mod 7)$

Take $G = Ld_7$. Choose $S_m = \{f_1, f_3, f_5, f_7, g_2, g_4, g_6\}$. Remove any vertex $x \in S_m$ and add another vertex $y \in V \backslash S_m$ to S_m such that x is adjacent to y. Hence the set S_m is again a secure dominating set of G. Also the monophonic path exists and it contain all the vertices of G. So $S_m = \{f_1, f_3, f_5, f_7, g_2, g_4, g_6\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil + 1$.

Subcase (ii): $n \equiv 1 (mod 7)$

Take $G = Ld_8$. Choose $S_m = \{f_1, f_3, f_5, f_7, g_2, g_4, g_6, g_8\}$. Then by similar argument as in subcase(i), $S_m = \{f_1, f_3, f_5, f_7, g_2, g_4, g_6, g_8\}$ is a minimum SMD set of G. In general $S_m =$

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$\bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_{n-1}, g_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil + 1$.

Subcase (iii): $n \equiv 2(mod 7)$

Take $G = Ld_9$. Choose $S_m = \{f_1, f_3, f_5, f_7, f_9, g_2, g_4, g_6, g_8\}$. Then by similar argument as in subcase(i), $S_m = \{f_1, f_3, f_5, f_7, f_9, g_2, g_4, g_6, g_8\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_{n-1}, f_n, g_{n-1}\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil + 1$.

Subcase (iv): $n \equiv 4(mod 7)$

Take $G = Ld_{11}$. Choose $S_m = \{f_1, f_3, f_5, f_8, f_{10}, g_2, g_4, g_6, g_9, g_{11}\}$. Then by similar argument as in subcase(i), $S_m = \{f_1, f_3, f_5, f_8, f_{10}, g_2, g_4, g_6, g_9, g_{11}\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_{n-3}, f_{n-1}, g_{n-2}, g_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil + 1$.

Subcase (v): $n \equiv 5(mod 7)$

Take $G = Ld_{12}$. Choose $S_m = \{f_1, f_3, f_5, f_8, f_{10}, f_{12}, g_2, g_4, g_6, g_9, g_{11}\}$. Then by similar argument as in subcase(i), $S_m = \{f_1, f_3, f_5, f_8, f_{10}, f_{12}, g_2, g_4, g_6, g_9, g_{11}\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_{n-4}, f_{n-2}, f_n, g_{n-3}, g_{n-1}\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil + 1$.

Subcase (vi): $n \equiv 6(mod 7)$

Take $G = Ld_{13}$. Choose $S_m = \{f_1, f_3, f_5, f_8, f_{10}, f_{12}, g_2, g_4, g_6, g_9, g_{11}, g_{13}\}$. Then by similar argument as in subcase(i), $S_m = \{f_1, f_3, f_5, f_8, f_{10}, f_{12}, g_2, g_4, g_6, g_9, g_{11}, g_{13}\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_{n-5}, f_{n-3}, f_{n-1}, g_{n-4}, g_{n-2}, g_n\}$ is a minimum SMD set of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil + 1$.

Case (b): $n \equiv 3(mod 7)$

Take $G = Ld_{10}$. Choose $S_m = \{f_1, f_3, f_5, f_8, f_{10}, g_2, g_4, g_6, g_9\}$. Then by similar argument as in subcase(i). So $S_m = \{f_1, f_3, f_5, f_8, f_{10}, g_2, g_4, g_6, g_9\}$ is a minimum SMD set of G. In general $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup \{f_{n-2}, f_n, g_{n-1}\}$ is a minimum SMD set

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of G. Therefore $|S_m| = \left\lceil \frac{6n}{7} \right\rceil$. From all the above cases, finally we conclude that $S_m = \bigcup_{j=0}^{k-1}\{f_{7j+1}, f_{7j+3}, f_{7j+5}, g_{7j+2}, g_{7j+4}, g_{7j+6}\} \cup$

$$\begin{Bmatrix} f_n & if & r = 0 \\ f_{n-1}, g_n & if & r = 1 \\ f_{n-2}, f_n, g_{n-1} & if & r = 2,3 \\ f_{n-3}, f_{n-1}, g_{n-2}, g_n & if & r = 4 \\ f_{n-4}, f_{n-2}, f_n, g_{n-3}, g_{n-1} & if & r = 5 \\ f_{n-5}, f_{n-3}, f_{n-1}, g_{n-4}, g_{n-2}, g_n & if & r = 6 \end{Bmatrix}$$

Therefore $|S_m| = \begin{cases} \left\lceil \frac{6n}{7} \right\rceil + 1 & if \quad n \equiv 0,1,2,4,5,6\ (mod7) \\ \left\lceil \frac{6n}{7} \right\rceil & if \quad n \equiv 3 (mod7) \end{cases}$

Hence $|S_m| = \begin{cases} n & if \quad n = 4,5,6 \\ \left\lceil \frac{6n}{7} \right\rceil + 1 & if \quad n \equiv 0,1,2,4,5,6\ (mod7) \\ \left\lceil \frac{6n}{7} \right\rceil & if \quad n \equiv 3 (mod7) \end{cases}$

4. Conclusion

In this paper, we investigated the secure monophonic domination number of Jellyfish graph $J_{m,n}$, Ladder graph Ld_n and Lollipop graph $Lp_{m,n}$.

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APPLICATION OF TOPOLOGICAL GRADIENT METHOD IN SKIN LESIONS USING EDGE DETECTION

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Abstract

We show in this paper the emerging Topological Gradient Method (TGM) which is a new way for modelling and we use it in detecting the skin lesions using edge detection. The irrelevant objects are destructed and the relevant objects are constructed with topological properties and objects are separated from the noisy background of an image. The developed pipeline integrates multiple stages of image processing to ensure high accuracy and reliability in lesion detection. The objective is to assist the clinicians in accurately extracting lesions from surrounding skin and enhancing subsequent diagnosis and treatment, it becomes feasible to monitor their progression over time using MATLAB.

Keywords: Image Processing, Topological Image Processing, Topological Gradient Method, Edge detection, Skin lesion, Noise Removal, MATLAB.

2020 Mathematics Subject Classification (AMS): 54H30

1. Introduction

Skin lesions are abnormal changes in the skin's colour, texture, or appearance. They can be classified based on their characteristics, causes, and whether they are primary (directly associated with a disease process) or secondary (arising from the progression of a primary lesion). The primary skin lesions are macule, plaque, wheal, tumor etc. The secondary skin lesions are crust, fissure, ulcer, scar etc. The common causes of skin lesions are infections, allergies, injuries, cancer and other conditions like acne or benign growth like moles or lipomas.

Topological image processing is an advanced technique in image analysis that uses principles from topology to understand and manipulate the structure and properties of an image. Unlike traditional methods that focus on pixel intensity or gradient-based features, topological approaches emphasize the connectivity, shape, and spatial relationships within an image.

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Applications of topological image processing are widespread, ranging from medical imaging, where it can identify complex anatomical structures, to material science and data visualization. Its strength lies in its resilience to noise and its ability to extract meaningful information about an image's global and local topological structure.

2. Preliminaries

2.1 Topological Gradient Method

The Topological Gradient is a method from topological image processing that can be used for edge detection and segmentation tasks. It uses the idea of topology (connectedness and local structure) to detect edges and boundaries in images. In topological image processing, the topological gradient is often applied in tasks like segmentation, where we look for regions of interest and boundaries by detecting abrupt changes in topological structures. The topological gradient is essentially a measure of how much a certain property (e.g., pixel intensity or gradient) changes when a small perturbation is made to a region of the image.[2]

2.2 Grayscale Transformation

Gray-scale transformation is a technique used to simplify image processing by converting images to grayscale, thus reducing computational load and focusing on essential features. In an RGB image, each pixel comprises red, green, and blue components. The grayscale conversion process involves combining these components into a single intensity value. This is typically done using a weighted sum of the red, green, and blue values, reflecting the human eye's sensitivity to different colours. For example, the formula:

$$Gray = \mathbf{0.2989} \times R + \mathbf{0.5870} \times G + \mathbf{0.1140} \times B$$

is commonly used, where R, G and B are the intensities of the red, green, and blue channels respectively. This ensures the grayscale image accurately represents the perceived brightness of the original RGB image.[5]

2.3 Histogram Equalization

To further enhance the quality of the grayscale images, histogram equalization is applied. This technique redistributes the brightness values to span the entire range of possible values, thereby enhancing the contrast of the image. Histogram equalization is particularly useful for making features more distinguishable, which is crucial in medical imaging where clear visibility of details is necessary for accurate diagnosis [5].

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2.4 Topological Gradient Approximation

The gradient of the image is computed using MATLAB's gradient() function. This computes the derivative of the image in the x and y directions, effectively capturing the rate of change in intensity across the image. The gradient magnitude is calculated by combining the gradients in the x and y directions using the formula:

$$\textbf{Original Gradient Magnitude} = \sqrt{(\nabla x)^2 + (\nabla y)^2}$$

This represents the magnitude of intensity change at each pixel, which is a good measure for detecting edges.

2.5 Thresholding

The gradient magnitude is normalized using mat2gray() to scale it between 0 and 1. Then, a threshold is applied to detect significant changes in intensity, which correspond to edges. We can adjust the threshold value to control the sensitivity of edge detection. A higher threshold will only highlight the most significant edges, while a lower threshold will include more subtle edges.

2.6 Noise Removal

During image pre processing, it is frequently essential to eliminate different forms of noise, such as hair and other unwanted elements in the images, which may disrupt the precise evaluation of skin lesions. The rationale behind this can be divided into three main reasons. First, it enhances the clarity of images by eliminating obstructions that may obscure important details, thereby reducing the risk of incorrect diagnoses or misinterpretations. Second, the elimination of noise improves image quality, which in turn enhances the performance of models used for tasks such as segmentation and classification. Lastly, it ensures consistency by providing uniform images that are free from artifacts, thereby maintaining the integrity of diagnostic procedures.[5]

2.7 Median Filtering

Median filtering replaces a pixel's intensity I(x, y) with the median value of its neighbourhood N(x, y). For a $k \times k$ kernel:

$$\mathbf{I(x, y) = median\{I(p, q) | (p, q) \in N}$$

is the filtered intensity. It effectively reduces salt-and-pepper noise while preserving edges.

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2.8 Gaussian Filtering

Smoothens the image to reduce Gaussian noise while introducing minimal blurring.

Applies a Gaussian Kernel

$$\mathbf{H(x,\ y)=}\frac{\mathbf{1}}{\mathbf{2\pi\sigma^2}}\,\boldsymbol{e}^{\frac{-x^2+y^2}{2\sigma^2}}$$

Where σ controls the smoothing intensity.[6]

2.9 Bilateral Filtering

The bilateral filter calculates the intensity of each pixel in the output image as a weighted average of the nearby pixels in the input image. The weights are based on the Euclidean distance between pixels, as well as the radiometric differences between them, such as colour intensity. The bilateral filter is effective for reducing noise and blocking artifacts.[6]

2.10 Edge Detection

Edge detection is crucial in image analysis across fields like medical imaging, industrial inspection, and computer vision. It identifies significant discontinuities in intensity levels, which represent edges. This process utilizes first- or second-order partial derivatives, with methods like the Sobel row-edge and Prewitt column-edge detectors, or the Laplacian of Gaussian detector, to detect changes in intensity. Advanced edge detection using the topological gradient method identifies boundaries by evaluating how small topological changes impact an energy functional representing image intensity. This approach robustly highlights edges by analyzing structural information globally rather than relying on local intensity alone, excelling in noise-prone scenarios and intricate boundary detection.

2.11 Overlaying Images

The overlayed image combines the original skin lesion image with highlighted edges (in red). This visualizes:

- The boundaries of the lesion.
- Areas of textural or structural change.
- Regions with sharp intensity gradients indicative of potential irregularities.

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The highlighted edges show the precise boundaries of a lesion, which can help clinicians assess the size, shape, and symmetry. Topological gradient and edge detection highlight areas with sudden intensity changes, pointing to irregular textures within the lesion.

3. Conceptual Connection to Topology

Topology studies properties of spaces that are preserved under continuous deformations, such as stretching or twisting, but not tearing or gluing. In image processing, these properties include connectedness, holes, and the number of components in a space, which can be used to extract meaningful features for analysis.

3.1. Topological Gradient: The topological gradient is closely related to how the image intensity changes locally when perturbed. In this case, the gradient magnitude approximates by calculating how the intensity changes in both the x and y directions.

3.2. Edge Detection as Topological Boundary Detection: The topological gradient can be seen as identifying "boundaries" or changes in the image, similar to identifying connected components or boundaries in topology. Significant intensity changes correspond to boundaries between different regions (objects or edges), which aligns with the concept of detecting boundaries in topological terms.

4. Application of Topological Gradient Method in Skin Lesions

Here we use a MATLAB Program to get the accurate image and assist the clinicians in accurately extracting lesions from surrounding skin and enhancing subsequent diagnosis and treatment, it becomes feasible to monitor their progression over time.

MATLAB code:

```
clc;
clear;
close all;

inputImage = imread('sk.jpeg');
grayImage = rgb2gray(inputImage);
equalizedImage = histeq(grayImage);
medianFiltered = medfilt2(equalizedImage, [3 3]);
h = fspecial('gaussian', [5 5], 1);
```


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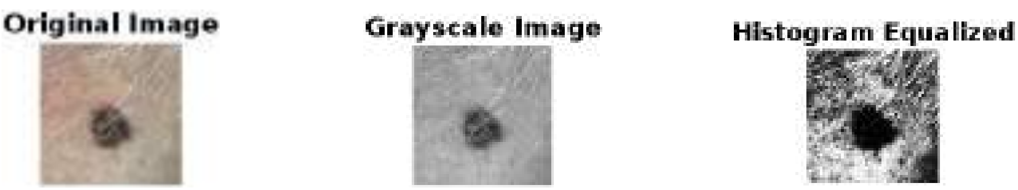
```
gaussianFiltered = imfilter(medianFiltered, h, 'replicate');
bilateralFiltered = imbilatfilt(gaussianFiltered);
cleanedImage = imnlmfilt(bilateralFiltered);
 [Gx, Gy] = imgradientxy(cleanedImage, 'sobel');
topologicalGradient = sqrt(Gx.^2 + Gy.^2);
thresholdValue = graythresh(topologicalGradient);
binaryImage = imbinarize(topologicalGradient, thresholdValue);
edges = edge(binaryImage, 'Canny');
se = strel('disk', 1);
cleanedEdges = imdilate(edges, se);
cleanedEdges = imerode(cleanedEdges, se);
overlayImage = inputImage;
if size(inputImage, 3) == 3
    overlayImage(:, :, 1) = uint8(cleanedEdges) * 255;
end
figure;
subplot(3, 3, 1); imshow(inputImage); title('Original Image');
subplot(3, 3, 2); imshow(grayImage); title('Grayscale Image');
subplot(3, 3, 3); imshow(equalizedImage); title('Histogram Equalized');
subplot(3, 3, 4); imshow(medianFiltered); title('Median Filtered');
subplot(3, 3, 5); imshow(gaussianFiltered); title('Gaussian Filtered');
subplot(3, 3, 6); imshow(bilateralFiltered); title('Bilateral Filtered');
subplot(3, 3, 7); imshow(topologicalGradient, []); title('Topological Gradient');
subplot(3, 3, 9); imshow(overlayImage); title('Edges Overlay on Original Image');
```

Output:



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5. Discussion

The analysis begins with loading the input image, which is then converted to grayscale using the **rgb2gray** function to simplify processing by reducing the image's dimensionality. Contrast enhancement is performed through **histeq** to redistribute pixel intensities, ensuring a uniform intensity distribution. Noise is sequentially removed using multiple filters: **medfilt2** for salt-and-pepper noise, **imfilter** with a Gaussian kernel for Gaussian noise, **imbilatfilt** for edge-preserving smoothing, and **imnlmfilt** for advanced non-local means denoising. The topological gradient of the denoised image is computed using the **Sobel operator** via **imgradientxy**, determining intensity changes along the x and y directions, with the gradient magnitude highlighting key features. Thresholding is performed using **graythresh** based on Otsu's method, and the gradient image is binarized using **imbinarize** to segment significant regions. The edges are refined using the **Canny edge detector** (**edge**), producing precise edge delineation. Further enhancement is achieved through morphological operations: **imdilate** fills gaps in the detected edges, and **imerode** refines their boundaries, ensuring smooth contours. The processed edges are then overlayed in red onto the original image by modifying the red channel, creating a clear visualization for clinical interpretation. The results, visualized using **subplot**, provide a step-by-step transformation of the image. The overlayed edges highlight lesion boundaries, aiding clinicians in identifying critical features such as size, shape, and texture. This pipeline effectively combines topological, morphological, and noise-removal techniques to deliver enhanced, accurate images for medical analysis.

6. How it Helps Clinicians

This program significantly enhances clinicians ability to assess skin lesions by providing clear visualizations and precise diagnostic tools. Overlayed edges, highlighted in red,

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delineate lesion boundaries, aiding in their clear identification. Grayscale mapping emphasizes intensity variations within the lesion, offering additional insights. By highlighting contrasts and edges, the program facilitates a detailed assessment of lesion shape, size, and texture, improving diagnostic accuracy. It supports both automated and manual segmentation, enabling precise feature measurements for analysis and classification. The colour mapping further enhances interpretation, with red representing detected edges and grayscale indicating intensity changes. This pre processing tool effectively extracts essential features from medical images, streamlining diagnostic workflows and contributing to advanced research in dermatological imaging.

7. Conclusion

This study introduces an integrated pipeline for enhancing skin lesion analysis using topological image processing techniques. The process begins by converting the lesion image to grayscale and enhancing contrast through histogram equalization. Advanced noise removal methods, including median, Gaussian, bilateral filtering, and non-local means denoising, refine the image while preserving critical features. A topological gradient approximation is applied to capture significant intensity variations, identifying boundaries and regions of interest within the lesion. Thresholding isolates key features, followed by the Canny edge detector, which ensures precise delineation of lesion edges. Morphological operations further enhance these boundaries, creating a refined binary representation. The final step overlays the detected edges onto the original image, highlighting lesion boundaries for clinical interpretation. This approach aids in analyzing lesion characteristics like size, shape, and texture, offering a robust visual aid for diagnostics. The pipeline's ability to handle noise effectively and extract features with high precision underscores its potential to support dermatological assessments and decision-making processes, advancing automated medical imaging techniques.

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APPLICATION OF GAUSS ELIMINATION METHOD AND KIRCHOFF`S THEROEM ON ELECTRICAL CIRCUIT USING PYTHON

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Abstract

Problem solving in engineering, science and other disciplines often requires complex analysis. A problem that requires effort is determining the value of electric current in a circuit. This research presents an alternative approach to determine the implementation of Gauss Elimination method in Electrical Circuits using PYTHON. This method is effective in finding unknown values in a system of linear equations through matrix operations. A deep understanding of currents in electrical circuits is essential in the design, analysis and maintenance of electrical systems. The application of the Gauss Elimination method becomes important in determining the value of current in complicated electrical circuits. Furthermore, this research also presents the results of identifying the voltage value of the circuit accurately and relevantly.

Keywords: Gauss Elimination, Electric Circuits, System of Linear Equations, Kirchoff`s Law.
2020 Mathematics Subject Classification (AMS): 65F45

1. Introduction

The application of mathematical models is essential for addressing problems across various domains, including the analysis of electrical circuits. When tackling such problems, it is crucial to understand that electrical circuits often comprise numerous components-such as resistors, capacitors, and inductors-connected through nodes. Calculating the current flowing through each component is vital, as it significantly influences the overall performance of an electrical circuit, including parameters such as voltage, power, and efficiency.

Several techniques can be employed for electrical circuit analysis [4], such as node analysis and super node analysis. Nodes are points where two or more components in a circuit intersect. These analytical methods are typically formulated into mathematical models, which are then

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represented numerically as Systems of Linear Equations (SLE). These models can become complex, often involving multiple equations, necessitating efficient solutions. The Gauss elimination method provides a systematic approach to solving such systems [1].

Gaussian elimination, widely recognized as the row reduction algorithm, is a mathematical method for solving systems of linear equations. This technique involves performing a sequence of operations on the coefficient matrix to derive solutions. Its versatility makes it suitable for various scenarios that can be modeled using SLEs [5, 2]. Practical applications of Gaussian elimination include solving problems related to nuclear fuel depletion.

This study introduces an alternative approach to determining the electric current in circuits using the Gauss Elimination method, facilitated by Python programming. The research emphasizes transforming electrical circuit problems into matrix form, demonstrating the process for efficiently obtaining accurate and relevant solutions to electrical engineering challenges.

2. Preliminaries

Electric Current

Electric current is the time rate of flow of electric charge through a circuit element or conductor.

Kirchhoff`s Law

Kirchhoff`s law are valid for all circuits and are considered essential tools for analyzing electrical circuits.

Current Law

Kirchhoff`s first law or the node law, "This law states that the total current entering a node in a circuit must equal the total current leaving the node". This is because charge is conserved

$$\Sigma\, I_{in} = \Sigma\, I_{out}$$

Voltage Law

"This law states that the total voltage around any closed loop in a circuit must equal zero". This is because energy is conserved

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$$\Sigma\varepsilon + \Sigma IR = 0$$

Augmented Matrix

A matrix obtained by appending a-dimensional column vector, on the right, as a further column to a-dimensional matrix.

Augmented matrix =[A/B]

3. Research Method

This study contains five processes (shown in Figure 1). These are initializing the research, Deriving Equation by Kirchoff`s law, transform equation to matrix, solved by Gauss elimination method and analyzing the result.

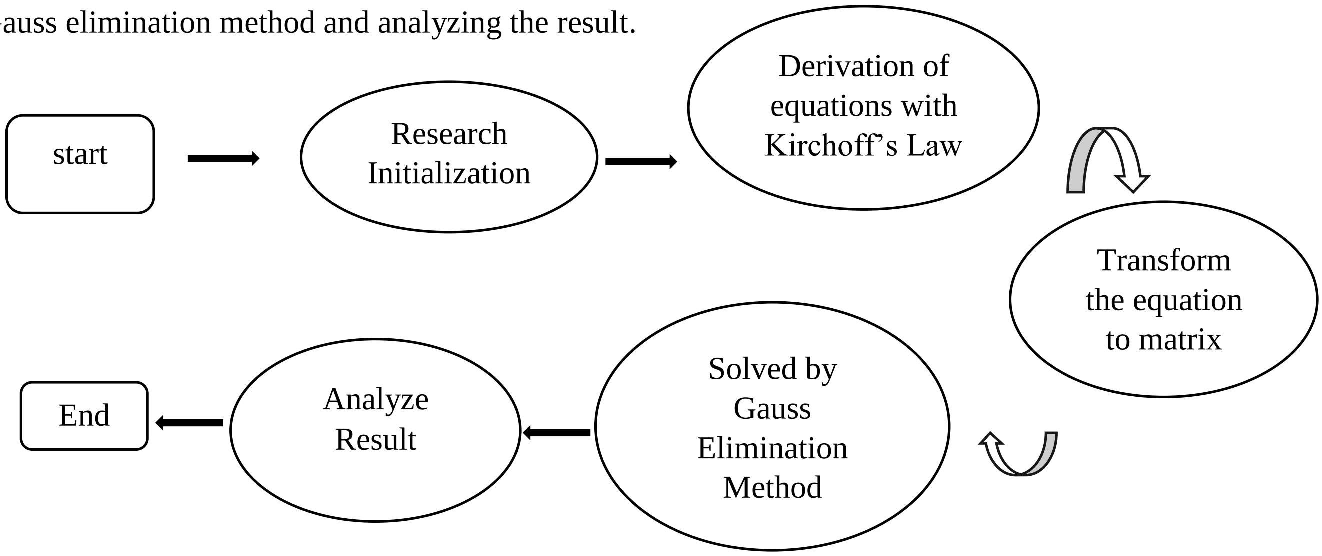

Fig 1: Research workflow

3.1. Research Initialization

At this stage, research defines its objectives, and studies literature to find simple electrical circuits to be used as simulation material. In addition, research determines the parameters to be studied, such as current and voltage on each component.

3.2. Kirchhoff`s law

Kirchhoff's first law declares, "The amount of electric current entering (Σ Iin) a node in a circuit must be equal to the amount of electric current leaving (ΣIout) the node", by equation (1).

Kirchhoff's second law relates to Kirchhoff`s voltage law or loop. The law states, "The

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sum of the voltage (R) drops in a loop in the circuit must be equal to the sum of voltage increases", by equation (2).

$\Sigma I_{in} = \Sigma I_{out}$ (1)

$\Sigma \varepsilon + \Sigma IR = 0$ (2)

3.3. Gauss Elimination Method

This study focuses on the Gaussian elimination method, a technique used to solve systems of linear equations by performing three types of matrix row operations on an augmented matrix. The process involves two main stages: forward elimination and back substitution.

3.3.1. Forward elimination

This step simplifies the matrix into its row echelon form. The primary objective here is to determine whether the system of equations has:

a. A single unique solution,

b. Infinitely many solutions, or

c. No solution at all. If the system is found to have no solution, further steps are unnecessary.

3.3.2. Back Substitution

If solutions are possible, this step is performed to further simplify the matrix into its reduced row echelon form.

The Gaussian elimination rules are the same as the rules for the three basic row operations, in other words, you can algebraically act on a matrix's rows in the following three ways:

a. Interchanging two rows, for example, $R2 \leftrightarrow R3$
b. Multiplying a row by a constant, for example, $R1 \rightarrow kR1$ where k is some nonzero number.
c. Adding a row to another row, for example, $R2 \rightarrow R2 + 3R1$.

4. Application of Gauss Elimination Method in Electrical Circuit using Python

The electrical installation was shown in an electrical circuit (Fig 2). The condition is an example of a real case to be simulated and determined the value of the voltage generated

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from each loop with the Gauss elimination method. There are four loops in the electrical circuit and four sources of electric current. Each loop is identified with the first Kirchhoff's law

Loop 1: $i_1 + 2i_2 - i_3 + i_4 = 6$

Loop 2: $-i_1 + i_2 + 2i_3 - i_4 = 3$

Loop 3: $2i_1 - i_2 + 2i_3 + 2i_4 = 14$

Loop 4: $i_1 + i_2 - i_3 + 2i_4 = 8$

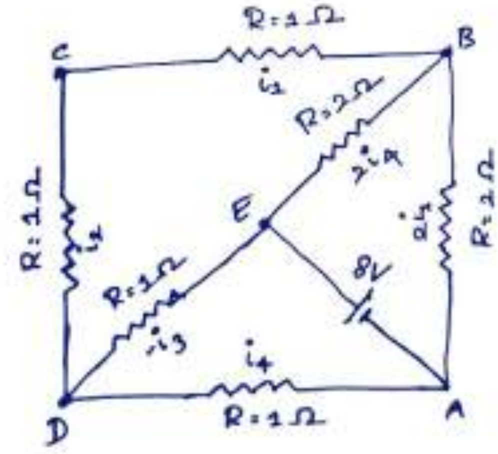

Fig 2: Example of electrical circuit

3.1 Gauss Elimination Result

Furthermore, the SPL of loop is transformed into a 4 x4 matrix, according to the number of electric current variables.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & -1 \\ 2 & -1 & 2 & 2 \\ 1 & 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 14 \\ 8 \end{bmatrix}$$

The Gauss elimination method is used to determine the value of each electric current. The solution steps are as follows: First iteration: operated (R2+R4) in other words adding row 2 and 4. So that the resulting equation

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & -1 \\ 2 & -1 & 2 & 2 \\ 1 & 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 14 \\ 8 \end{bmatrix} \xrightarrow{R_2 + R_4} \begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & -1 \\ 2 & -1 & 2 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 14 \\ 11 \end{bmatrix}$$

The second iteration operates on the first row such that all values of the first column (except diagonal 1) are 0. The third row is operated with 2R2+ $R3$ and the second row is operated with $R1$ + $R2$. Meanwhile, the fourth line is not operated because it is already 0.

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$$\begin{bmatrix}1 & 2 & -1 & 1\\ -1 & 1 & 2 & -1\\ 2 & -1 & 2 & 2\\ 0 & 2 & 1 & 1\end{bmatrix}\begin{bmatrix}6\\3\\14\\11\end{bmatrix}\xrightarrow{2R_2+R_3}\begin{bmatrix}1 & 2 & -1 & 1\\ -1 & 1 & 2 & -1\\ 0 & 1 & 6 & 0\\ 0 & 2 & 1 & 1\end{bmatrix}\begin{bmatrix}6\\3\\20\\11\end{bmatrix}$$

The third iteration operates on the second row so as to obtain the identity matrix component.

$$\begin{bmatrix}1 & 2 & -1 & 1\\ -1 & 1 & 2 & -1\\ 0 & 1 & 6 & 0\\ 0 & 2 & 1 & 1\end{bmatrix}\begin{bmatrix}6\\3\\20\\11\end{bmatrix}\xrightarrow{R_1+R_2}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 1 & 6 & 0\\ 0 & 2 & 1 & 1\end{bmatrix}\begin{bmatrix}6\\9\\20\\11\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 1 & 6 & 0\\ 0 & 2 & 1 & 1\end{bmatrix}\begin{bmatrix}6\\9\\20\\11\end{bmatrix}\xrightarrow{-2R_3+R_4}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 1 & 6 & 0\\ 0 & 0 & -11 & 1\end{bmatrix}\begin{bmatrix}6\\9\\20\\-29\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 1 & 6 & 0\\ 0 & 0 & -11 & 1\end{bmatrix}\begin{bmatrix}6\\9\\20\\-29\end{bmatrix}\xrightarrow{R_2-3R_3}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 0 & -17 & 0\\ 0 & 0 & -11 & 1\end{bmatrix}\begin{bmatrix}6\\9\\-51\\-29\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 0 & -17 & 0\\ 0 & 0 & -11 & 1\end{bmatrix}\begin{bmatrix}6\\9\\-51\\-29\end{bmatrix}\xrightarrow{-11R_3+17R_4}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 0 & -17 & 0\\ 0 & 0 & 0 & 17\end{bmatrix}\begin{bmatrix}6\\9\\-51\\68\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 3 & 1 & 0\\ 0 & 0 & -17 & 0\\ 0 & 0 & 0 & 17\end{bmatrix}\begin{bmatrix}6\\9\\-51\\68\end{bmatrix}\xrightarrow{1/3R_2}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 1 & 1/3 & 0\\ 0 & 0 & -17 & 0\\ 0 & 0 & 0 & 17\end{bmatrix}\begin{bmatrix}6\\3\\-51\\68\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 1 & 1/3 & 0\\ 0 & 0 & -17 & 0\\ 0 & 0 & 0 & 17\end{bmatrix}\begin{bmatrix}6\\3\\-51\\68\end{bmatrix}\xrightarrow{-1/17R_3}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 1 & 1/3 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 17\end{bmatrix}\begin{bmatrix}6\\3\\3\\68\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 1 & 1/3 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 17\end{bmatrix}\begin{bmatrix}6\\3\\3\\68\end{bmatrix}\xrightarrow{1/17R_4}\begin{bmatrix}1 & 2 & -1 & 1\\ 0 & 1 & 1/3 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}6\\3\\3\\4\end{bmatrix}$$

This research produces an identity matrix in the ninth iteration, the final result is the matrix below. At the ninth iteration, the strong current value in each loop is obtained. The current value in each loop $i_1 = 6, i_2 = 3, i_3 = 3, i_4 = 4$.

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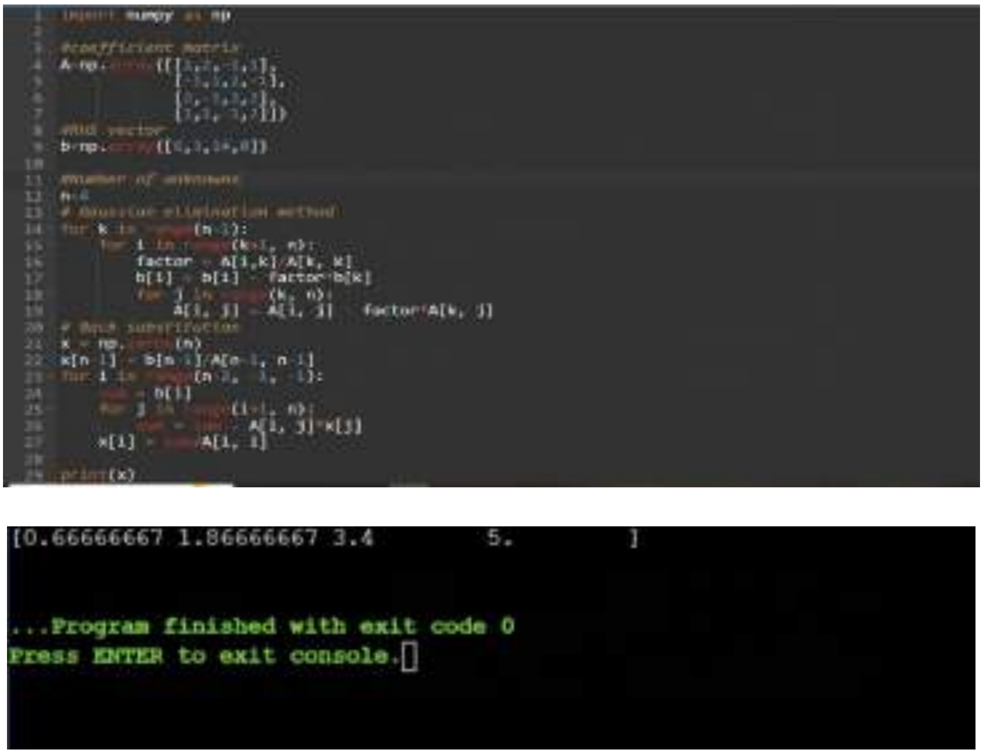
$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 3 \\ 4 \end{bmatrix}$$

This study not only manually applied the Gauss elimination method but also executed in algorithm using Python programming. To simplify calculations, the process of determining electric current values was effectively implemented in Python. The results from the two approaches showed only a small difference, ranging between 3-6%. Thus, it can be concluded that both methods produce nearly identical outcomes with no significant variance.



Gauss Elimination Using Python

4.2. Voltage Analysis

The next step involves analyzing the voltage (V) in each loop of the circuit. The voltage is calculated using Kirchhoff's second law ($V = \sum IR$). Analyzing voltage values in electrical circuits is essential for diagnosing issues and maintaining electrical systems in circuits and other applications. The voltage values for the electrical circuit are represented in the matrix below.

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$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & -1 \\ 2 & -1 & 2 & 2 \\ 1 & 1 & -1 & 2 \end{bmatrix} \text{x} \begin{bmatrix} 0.66 \\ 1.86 \\ 3.4 \\ 5 \end{bmatrix} = \begin{bmatrix} 5.98 \\ 2.96 \\ 16.3 \\ 9.12 \end{bmatrix}$$

5. Conclusion

From the identification, simulation, and analysis of the electrical circuit, it has been demonstrated that the Gauss elimination method is effective in determining the electric current in electrical circuits. To reduce the computational process, the Gauss elimination method can be executed using Python programming. In summary, this study successfully determined the current and voltage values in the analyzed electrical circuit. The mathematical approach is used to be efficient in addressing the problem, providing valuable insights into the characteristics and behavior of complex electrical circuits.

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APPLICATION OF LINEAR PROGRAMMING FOR OPTIMAL USE OF RAW MATERIALS IN MAKING A TRADITIONAL SWEET ADHIRASAM

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Abstract

This work utilized the concept of Simplex algorithm, an aspect of linear programming to allocate raw materials to competing variables (big size, small size, medium size) in making the traditional sweet Adhirasam for the purpose of profit maximization. The analysis was carried out and the result showed that 880 pieces of medium size, 500 pieces of small size and 0 pieces of big size should be produced respectively in order to make a profit of ₹20580. From the analysis, it was observed that medium size of pieces contribute objectively to the profit. Hence, more of medium size of pieces are needed to be produced and sold in order to maximize the profit.

Keywords: Linear programming model, Simplex method, Decision variables, Optimal result using PYTHON LAB.

2020 Mathematics Subject Classification (AMS): 90C05

1. Introduction

Linear programming is a family of mathematical programming that is concerned with or useful for allocation of scarce or limited resources to several competing activities on the basis of given criterion of optimality. In statistics, linear programming (LP) is a special techniques employed in operation research for the purpose of optimization of linear function subject to linear equality and inequality constraint. Linear programming determines the way to achieve best outcome, such as maximum profit or minimum cost in a given mathematical model and given some list of requirement as a linear equation. The technique of linear programming is used in a wide range as applications, including agriculture, industry, transportation, economics, health system, behavioural and social science and the military. Although many business organization see

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linear programming as a "new science" or recently development in mathematical history, but there is nothing new about the maximization of profit in any business organization, i.e in a production company or manufacturing company.

2. Literature review

The lack of good literature on the relationship between linear programmingutilization and optimization of raw materials in the breadbaking industry in Nigeria is another issue that has triggered this research work.To authenticate this, for instance, Akpan and Iwok (2016) investigated the application of linear programming to optimizeraw materials in a bakery. They found that a small loaf, followed by a big loaf, contributes objectively to the profit [1].

In their work titled "use of linear programming for optimal production" in Coca-Cola Company, they were able to applied linear programming in obtaining the optimal production process for Coca-Cola Company. In the course of formulating a linear programming model for the production process, they identified the decision variables to be the following Coke, Fanta, Schweppes, Fanta tonic, Krest soda etc. which some up to nine decision variables and the constraint were identified to be concentration of the drinks, sugar content, water volume and carbon (iv)oxide. The resulting model was solve using the simplex algorithm, after the data analysis they came to a conclusion that out of the nine product the company was producing only two contribute most to their profit maximization, that is Fanta orange 50cl and Coke 50cl with a specified quantity of 462,547 and 415,593 in order to obtain a maximum profit of N263,497,283. They advise the company to concentrate in the production of the two products in order not to run into high cost [2].

3. Preliminaries

Constraints are a series of equalities and inequalities that are a set of criteria necessary to satisfy when finding the optimal solution.

Optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model.

Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem.

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Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints.

Decision Variable are quantities that influence the objective function of a mathematical optimization model. They are represented by mathematical symbols and can take on any of a set of possible values.

4. Linear Programming Model

The general linear programming model with n decision variables and m constraints can be stated in the following form

Optimize (max or min) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n (\leq, =, \geq) b_1$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n (\leq, =, \geq) b_2$$

....

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n (\leq, =, \geq) b_m$$

The above model can also be expressed in a compact form as follows.

Optimize (max or min) Z = $\sum_{j=1}^{n} c_j x_i \ldots$(objective function)

Subject to the linear constraints

Z = $\sum_{j=1}^{n} a_{ij}\, x_j (\leq, =, \geq) b_{i,}$ i = 1,2,…, m and $x_j \geq 0$, j = 1,2,…n

Where $c_{1,} c_{2,\ldots,} c_n$ *represent the* each piece profit (or cost) of decision variables $x_{1,} x_{2,\ldots,} x_n$to the value of the objective function and $a_{11}, a_{12}, \ldots, a_{21,} a_{22,} \ldots, a_{m1,} a_{m2}, \ldots, a_{mn}$*repre*sent the amount of resource per unit of the decision variables. The b_i represents the total availability of the i^{th} resource. Z represent the measure of performance which can be either profit, or cost or reverence etc[1, 2, 3, 4].

4.1. Standard form of a Linear Programming Model

The use of the simplex method to solve a linear programming problem requires that the problem be converted into its standard form. For n decision variables and m constraints, the standard form of the linear programming model can be,

Optimize (max or min) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + 0s_1 + 0s_2 + \ldots + 0s_m$

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Subject to linear constraints $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + \ldots + s_1 = b_1$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + \ldots + s_2 = b_2$$

……. ………… …………

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + \ldots + s_m = b_m$$

$$x_1, x_2, ..x_n, s_1, s_2, ..s_m \geq 0$$

This can be stated in a more compact form as:

Optimize (max)Z = $\sum_{j=1}^{n} c_j x_i$ + $\sum_{j=1}^{m} 0 s_i$

Subject to the linear constraints

Z = $\sum_{j=1}^{n} a_{ij}\, x_j s_i = b_{i\,,}$ i = 1, 2,…, m and $x_j s_i \geq 0$, (for all i and j)

4.2 Data Presentation and Analysis

The data for this research project was collected from Marudham Adhirasam, Vadambacheri, Coimbatore, Tamilnadu. The data consist of total amount of raw materials (flour, jaggery and oil) available for daily production of three different sizes of Adhirasam (big size, small size, medium size) and profit contribution per each size of Adhirasam produced. The data analysis was carried out with PYTHON software. The content of each raw material per eachpiece of Adhirasam produced is shown below.

Flour

Total amount of flour available = 32kg

Each size of big piece requires 0.025kg of flour

Each size of small piece requires 0.02kg of flour

Each size of medium piece requires 0.025kg of flour

Jaggery

Total amount of jiggery available = 16kg

Each size of big piece requires 0.017kg of jaggery

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Each size of small piece requires 0.01kg of jaggery

Each size of medium piece requires 0.0125kg of jaggery

Oil

Total amount (volume) of oil available = 9L

Each size of big piece requires 0.0083L of oil

Each size of small piece requires 0.007L of oil

Each size of medium piece require 0.00625L of oil

Profit contribution per unit product (size) of adhirasam produced

Each unit of big size = ₹20

Each unit of small size = ₹13

Each unit of medium size = ₹16

The above data can be summarized in a tabular form.

Raw materials	**Product**			**Availability of raw materials**
	Big size	**Small size**	**Medium size**	
Flour (kg)	0.025	0.02	0.025	32
Jaggery (kg)	0.017	0.01	0.0125	16
Oil (L)	0.0083	0.007	0.00625	9
Profit (₹)	20	13	16	

4.3 Model formulation

Let the quantity of big size to be produce = x_1

Let the quantity of small size to be produce =x_2

Let the quantity of medium size to be produce = x_3

Let Z denote the profit to be maximize. The linear programming model for the above production data is given by

Max Z = 20 x_1 +13 $x_2 + 16x_3$

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Subject to constraints

$0.025x_1 + 0.02x_2 + 0.025\, x_3 \leq 32$

$0.017\, x_1 + 0.01x_2 + 0.0125\, x_3 \leq 16$

$0.0083x_1 + 0.007\, x_2 + 0.00625\, x_3 \leq 9$

$x_1,\ x_2, x_3 \geq 0$

Converting the model into its corresponding standard form;

Max $Z = 20x_1 + 13x_2 + 16\, x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to constraints

$0.025x_1 + 0.02x_2 + 0.025\, x_3 + s_1 = 32$

$0.017x_1 + 0.01x_2 + 0.0125\, x_3 + s_2 = 16$

$0.0083x_1 + 0.007x_2 + 0.00625\, x_3 + s_3 = 9$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

The above linear programming model was solved using PYTHON software, which gives an optimal solution of: $x_1 = 0$, $x_2 = 500$, $x_3 = 880$.Z = 20580.

4.4 Interpretation of Result

Based on the data collected the optimum result derived from the model indicates that two sizes of Adhirasam should be produced, small size and medium size. Their production quantities should be 500 and 880 respectively. This will produce a maximum profit of ₹20580.

4.5 Simplex Method using PYTHON

```
from scipy.optimize import linprog

# Define a large value for Big-M
M = 1e6  # Big-M penalty for artificial variables

# Objective function coefficients (Max Z -> Min -Z)
# Variables: x1, x2, x3, s1, s2, s3, a1, a2, a3
c = [-20, -13, -16, 0, 0, 0, M, M, M]

# Coefficients of the constraints (LHS)
A = [
    [0.025, 0.02, 0.025, 1, 0, 0, 1, 0, 0],  # Constraint 1
```


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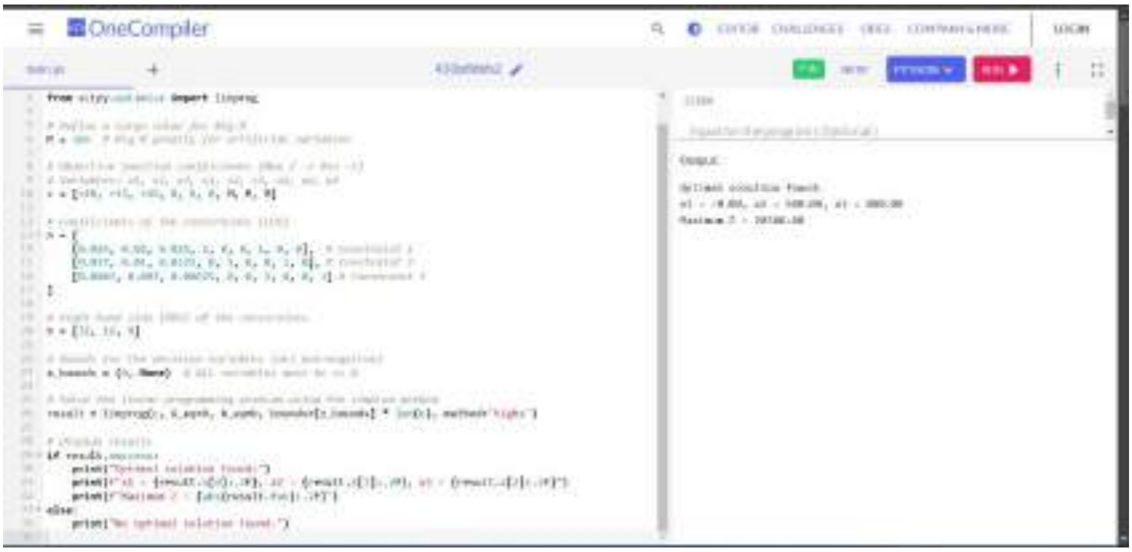
```
    [0.017, 0.01, 0.0125, 0, 1, 0, 0, 1, 0], # Constraint 2
    [0.0083, 0.007, 0.00625, 0, 0, 1, 0, 0, 1] # Constraint 3
]

# Right-hand side (RHS) of the constraints
b = [32, 16, 9]

# Bounds for the decision variables (all non-negative)
x_bounds = (0, None)  # All variables must be >= 0

# Solve the linear programming problem using the simplex method
result = linprog(c, A_eq=A, b_eq=b, bounds=[x_bounds] * len(c), method='highs')

# Display results
if result.success:
    print("Optimal solution found:")
    print(f"x1 = {result.x[0]:.2f}, x2 = {result.x[1]:.2f}, x3 = {result.x[2]:.2f}")
    print(f"Maximum Z = {abs(result.fun):.2f}")
else:
    print("No optimal solution found.")
```



5. Summary

The objective of this research work was to apply linear programming for optimal use of raw material in Adhirasam production. Marudham Adhirasam was used as our case study. The decision variables in this research work are the three different sizes of Adhirasam (big size, small size and medium size) produced by Marudham Adhirasam. The researcher focused mainly on three raw materials (flour, jaggery and oil)used in the production and the quantity of raw material required for each variable (Adhirasam size). The result shows that 0 piece of

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big size, 500 piece of small size and 880 piece of medium size should be produce respectively which will give a maximum profit of ₹20580.

6. Conclusion

Based on the analysis carried out in this research work and the result shown, Marudham Adhirasam should produce the three sizes of Adhirasam (big size, small size and medium size) in order to satisfy the customers. Also, more of medium size should be produce in order to attain maximum profit, because they contribute mostly to the profit earned by the company.

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OPERATIONS ON PYTHAGOREAN NEUTROSOPHIC FUZZY MAGIC GRAPHS

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Abstract

The Pythagorean Neutrosophic fuzzy magic graph is the combination of Pythagorean Neutrosophic Fuzzy set and graph model which is mainly applied across various fields. Pythagorean Neutrosophic set composed of elements with dependent Membership function ($\propto$), Non Membership function (γ) and independent indeterminancy function (β) with the condition that $0 \leq \propto^2 + \beta^2 + \gamma^2 \leq 2$. The concept of Pythagorean Neutrosophic fuzzy graph extended to Pythagorean Neutrosophic fuzzy magic graph. In this paper we define some operations that can be performed on Pythagorean Neutrosophic magic graph include Cartesian Product, Composition, Complement, Union, Intersection, and investigate their important properties.

Keywords: Pythagorean Neutrosophic fuzzy magic graph (PNFMG), Cartesian Product, Composition, Complement, Union, Intersection

2020 Mathematics Subject Classification (AMS): 05C72, 05C76, 05C78

1. Introduction

Fuzzy set was coined by Zadeh L.A in 1965. Atanassov presented the concept of intutionistic fuzzy set. A set which has only one component with degree of membership between 0 and 1 is a fuzzyset, while in intutionistic fuzzy set has two components namely, degree of membership and degree of non-membership value lies between 0 and 1. Thus this intutionistic fuzzy set was extended into neutrosophic set. To deal with intricate vagueness and ambiguity, Pythagorean fuzzy set was developed by Yager, which is sum of square of membership and non-membership value must lies between 0 and 1. Smarandache introduced the concept of neutrosophic fuzzy set, which is the generalisation of fuzzy and intutionistic fuzzy set includes three constraints as membership, non-membership and indeterminancy

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function and their sum must be lies between 0 and 3. In 2016, Smarandache developed the Pythagorean Neutrosophic set, which is the fusion of neutrosophic and Pythagorean set holding membership and non-membership grades as dependent and indeterminancy grade as independent components in which total square of membership, non-membership and indeterminancy value must lies between 0 and 2.

Pythagorean Neutrosophic fuzzy graph is the combination of Pythagorean Neutrosophic set and fuzzy graph theory. In recent years, some kinds of fuzzy graphs have been introduced. In this paper, we define some operations like Cartesian Product, Composition, Complement, Union and Intersection of Pythagorean Neutrosophic fuzzy Magic Graph and also investigate and discuss some of its properties.

2. Preliminaries

Definition 2.1. A fuzzy set A in X is defined as $A = \{(a, \mu_A(a))/\ a \in X\}$ where $\mu_A(a) \in [0, 1]$ is called the membership fuction for the fuzzy set A.

Definition 2.2. A fuzzy graph defined by $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ where, $\forall\ u\ v \in V$, $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$

Definition 2.3. A magic labeling on G will mean a one-to-one map λ from $V(G) \cup E(G)$ onto the integers 1,2,…,v+e, where $v = |V(G)|$ and $e = |E(G)|$, with the property that, given any edge (x, y), $\lambda(x) + \lambda(x, y) + \lambda(y) = k$, for some constant k.

Definition 2.4. A fuzzy graph G = (σ, μ) is said to be a fuzzy labeling graph, if $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ is bijective such that the membership value of edges and vertices are distinct and for all $u, v \in V$, $\mu(u, v) < \sigma(u) \wedge \sigma(v)$.

Definition 2.5. Pythagorean Neutrosophic Fuzzy Graph (PNFG) is $G = (V, E)$ where $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1, \beta_1\ and\ \sigma_1$ from $V\ to\ [0, 1]$ with $0 \leq \mu_1(v_i)^2 + \beta_1(v_i)^2 + \sigma_1(v_i)^2 \leq 2\ \forall\ v_i \in V$ signifies membership, indeterminacy and non-membership functions correspondingly and $E \subseteq V \times V$ where $\mu_2, \beta_2\ and\ \sigma_2$ from $V \times V\ to\ [0, 1]$ such that

$$\mu_2(v_i\ v_j) \leq\ \mu_1(v_i) \wedge \mu_1(v_j)$$

$$\beta_2(v_i\ v_j) \leq\ \beta_1(v_i) \wedge \beta_1(v_j)$$

$$\sigma_2(v_i\ v_j) \leq\ \sigma_1(v_i) \vee \sigma_1(v_j)$$

With $0 \leq \left(\mu_2(v_i\ v_j)\right)^2 + \left(\beta_2(v_i\ v_j)\right)^2 + \left(\sigma_2(v_i\ v_j)\right)^2 \leq 2\ \forall\ v_i v_j \in E$

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3. Pythagorean Neutrosophic Fuzzy Magic Labelling Graph

Definition 3.1. A Pythagorean Neutrosophic Fuzzy Graph $G = (V, E)$ is said to be Pythagorean Neutrosophic Fuzzy magic graph if there exist a magic graph M such that

$\mu(v_i) + \alpha(v_i\, v_j) + \mu(v_j)$ having a constant value denoted by m,

$\eta(v_i) + \beta(v_i\, v_j) + \eta(v_j)$ having a constant value denoted by m' and

$\delta(v_i) + \gamma(v_i\, v_j) + \delta(v_j)$ having a constant value denoted by $m'' \;\; \forall \;\; v_i, v_j \;\in V$

We denote a Pythagorean Neutrosophic Fuzzy Magic Constant by $M_0(G) = (m, m', m'')$

Example:

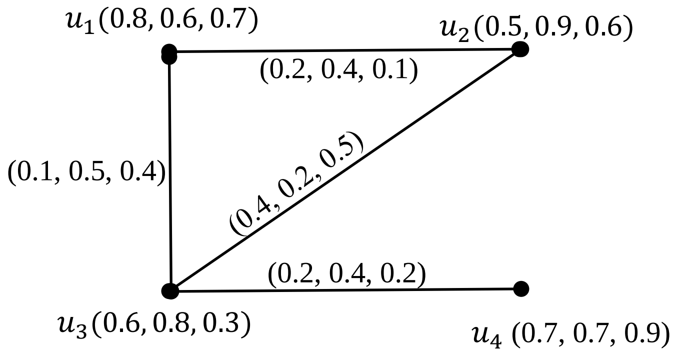

Figure 1

4. Main Results

Definition 4.1. Let $G' = (V_1, E_1)$ and $G'' = (V_2, E_2)$ be two PNFMG's where $V_1 = (\mu, \eta, \delta))$, $V_2 = \mu^*,\ \eta^*,\ \delta^*)$ and $E_1 = (\alpha, \beta, \gamma)), E_2 = \alpha^*,\ \beta^*,\ \gamma^*)$. The Cartesian Product $G' \times G'' = (V_1 \times V_2,\ E_1 \times E_2)$ is defined by

(i) $(\mu \times \mu^*)(u_1, u_2) = min\{\mu(u_1),\ \mu^*(u_2)\}$

$(\eta \times \eta^*)(u_1, u_2) = min\{\eta(u_1),\ \eta^*(u_2)\}$

$(\delta \times \delta^*)(u_1, u_2) = max\{\delta(u_1),\ \delta^*(u_2)\} \quad \forall\, u_1, u_2 \in V_1 \times V_2$

(ii) $(\alpha \times \alpha^*)\big((u, u_2), (u, v_2)\big) = min\{\mu(u),\ \alpha^*(u_2\, v_2)\}$

$(\beta \times \beta^*)\big((u, u_2), (u, v_2)\big) = min\{\eta(u),\ \beta^*(u_2\, v_2)\}$

$(\gamma \times \gamma^*)\big((u, u_2), (u, v_2)\big) = max\{\delta(u),\ \gamma^*(u_2\, v_2)\} \quad \forall\, u \in V_1\ and\ u_2 v_2 \in E_2$

(iii)$(\alpha \times \alpha^*)\big((u_1, w), (\,v_1, w)\big) = min\{\alpha(u_1\, v_1),\ \mu^*(w)\}$

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$$(\beta \times \beta^*)\big((u_1, w), (v_1, w)\big) = min\{\beta(u_1\, v_1),\ \eta^*(w)\}$$

$$(\gamma \times \gamma^*)\big((u_1, w), (v_1, w)\big) = max\{\gamma(u_1\, v_1),\ \delta^*(w)\} \quad \forall\, w \in V_2\ and\ u_1v_1 \in E_1$$

Theorem 4.2. Let $G'\ and\ G''$ be two Pythagorean Neutrosophic fuzzy Magic Labelled Graph then $G' \times G''$ is a Pythagorean Neutrosophic Fuzzy Graph

Proof. Let $u \in V_1\ and\ u_2v_2 \in E_2$. Then we get,

$$(\alpha \times \alpha^*)\big((u, u_2), (u, v_2)\big) = min\{\mu(u),\ \alpha^*(u_2\, v_2)\}$$

$$\leq min\left\{\mu(u), \Big(min\big(\ \mu^*(u_2),\ \mu^*(v_2)\big)\Big)\right\}$$

$$= \min\big\{\min\big(\mu(u),\ \mu^*(u_2)\big), min\big(\mu(u),\ \mu^*(v_2)\big)\big\}$$

$$= \min\{(\mu \times \mu^*)(u, u_2), (\mu \times \mu^*)(u, v_2)\}$$

$$(\beta \times \beta^*)\big((u, u_2), (u, v_2)\big) = min\{\eta(u),\ \beta^*(u_2\, v_2)\}$$

$$\leq min\left\{\eta(u), \Big(min\big(\ \eta^*(u_2),\ \eta^*(v_2)\big)\Big)\right\}$$

$$= \min\big\{\min\big(\eta(u),\ \eta^*(u_2)\big), min\big(\eta(u),\ \eta^*(v_2)\big)\big\}$$

$$= \min\{(\eta \times \eta^*)(u, u_2), (\eta \times \eta^*)(u, v_2)\}$$

$$(\gamma \times \gamma^*)\big((u, u_2), (u, v_2)\big) = max\{\delta(u),\ \gamma^*(u_2\, v_2)\}$$

$$\leq max\left\{\delta(u), \Big(max\big(\delta^*(u_2),\ \delta^*(v_2)\big)\Big)\right\}$$

$$= \max\big\{\max\big(\delta(u),\ \delta^*(u_2)\big), max\big(\delta(u),\ \delta^*(v_2)\big)\big\}$$

$$= \max\{(\delta \times \delta^*)(u, u_2), (\delta \times \delta^*)(u, v_2)\}$$

Let $w \in V_2\ and\ u_1v_1 \in E_1$. Then, we get

$$(\alpha \times \alpha^*)\big((u_1, w), (v_1, w)\big) = min\{\alpha(u_1\, v_1),\ \mu^*(w)\}$$

$$\leq min\left\{\Big(min\big(\ \mu(u_1), \mu(v_1)\big),\ \mu^*(w)\Big)\right\}$$

$$= \ \min\big\{\min\big(\mu(u_1),\ \mu^*(w)\big), min\big(\mu(v_1),\ \mu^*(w)\big)\big\}$$

$$= \min\{(\mu \times \mu^*)(\ u_1, w), (\mu \times \mu^*)(\ v_1, w)\}$$

$$(\beta \times \beta^*)\big((u_1, w), (v_1, w)\big) = min\{\beta(u_1\, v_1),\ \eta^*(w)\}$$

$$\leq min\left\{\Big(min\big(\ \eta(u_1), \mu(v_1)\big), \eta^*(w)\Big)\right\}$$

$$= \ \min\big\{\min\big(\ \eta(u_1),\ \eta^*(w)\big), min\big(\ \eta(v_1),\ \eta^*(w)\big)\big\}$$

$$= \min\{(\ \eta \times\ \eta^*)(\ u_1, w), (\ \eta \times\ \eta^*)(\ v_1, w)\}$$

$$(\gamma \times \gamma^*)\big((u_1, w), (v_1, w)\big) = max\{\gamma(u_1\, v_1),\ \delta^*(w)\}$$

$$\leq max\left\{\Big(max\big(\ \delta(u_1), \delta(v_1)\big),\ \delta^*(w)\Big)\right\}$$

$$= \max\big\{\max\big(\ \delta(u_1), \delta^*(w)\big), max\big(\ \ \delta(v_1), \delta^*(w)\big)\big\}$$

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$$= \max\{(\delta \times \delta^*)(u_1, w), (\delta \times \delta^*)(v_1, w)\}$$

This completes the proof

Definition 4.3. Let $G' = (V_1, E_1)$ and $G'' = (V_2, E_2)$ be two PNFMG's where $V_1 = (\mu, \eta, \delta))$, $V_2 = (\mu^*, \eta^*, \delta^*)$ and $E_1 = (\alpha, \beta, \gamma)), E_2 = \alpha^*, \beta^*, \gamma^*)$. The composition $G'[G''] = (V_1 \circ V_2, E_1 \circ E_2)$ is defined by

(i) $(\mu \circ \mu^*)(u_1, u_2) = min\{\mu(u_1), \mu^*(u_2)\}$

$(\eta \circ \eta^*)(u_1, u_2) = min\{\eta(u_1), \eta^*(u_2)\}$

$(\delta \circ \delta^*)(u_1, u_2) = max\{\delta(u_1), \delta^*(u_2)\} \quad \forall u_1, u_2 \in V_1 \times V_2$

(ii) $(\alpha \circ \alpha^*)\big((u, u_2), (u, v_2)\big) = min\{\mu(u), \alpha^*(u_2 v_2)\}$

$(\beta \circ \beta^*)\big((u, u_2), (u, v_2)\big) = min\{\eta(u), \beta^*(u_2 v_2)\}$

$(\gamma \circ \gamma^*)\big((u, u_2), (u, v_2)\big) = max\{\delta(u), \gamma^*(u_2 v_2)\} \quad \forall u \in V_1 \; and \; u_2 v_2 \in E_2$

(iii) $(\alpha \circ \alpha^*)\big((u_1, w), (v_1, w)\big) = min\{\alpha(u_1 v_1), \mu^*(w)\}$

$(\beta \circ \beta^*)\big((u_1, w), (v_1, w)\big) = min\{\beta(u_1 v_1), \eta^*(w)\}$

$(\gamma \circ \gamma^*)\big((u_1, w), (v_1, w)\big) = max\{\gamma(u_1 v_1), \delta^*(w)\} \quad \forall w \in V_2 \; and \; u_1 v_1 \in E_1$

(iv) $(\alpha \circ \alpha^*)\big((u_1, u_2), (v_1, v_2)\big) = min\{\alpha(u_1 v_1), \mu^*(u_2), \mu^*(v_2)\}$

$(\beta \circ \beta^*)\big((u_1, u_2), (v_1, v_2)\big) = min\{\beta(u_1 v_1), \eta^*(u_2), \eta^*(v_2)\}$

$(\gamma \circ \gamma^*)\big((u_1, u_2), (v_1, v_2)\big) = max\{\gamma(u_1 v_1), \delta^*(u_2), \delta^*(v_2)\}$

$\forall u_2, v_2 \in V_2 \; and \; u_1 v_1 \in E_1, u_2 \neq v_2$

Theorem 4.4. Let $G' \; and \; G''$ be two Pythagorean Neutrosophic fuzzy Magic Labelled Graph then $G'[G'']$ is a Pythagorean Neutrosophic Fuzzy Graph

Proof. Let $u \in V_1 \; and \; u_2 v_2 \in E_2$. Then we get,

$$(\alpha \circ \alpha^*)\big((u, u_2), (u, v_2)\big) = min\{\mu(u), \alpha^*(u_2 v_2)\}$$
$$\leq min\Big\{\mu(u), \Big(min\big(\mu^*(u_2), \mu^*(v_2)\big)\Big)\Big\}$$
$$= \min\big\{\min\big(\mu(u), \mu^*(u_2)\big), min\big(\mu(u), \mu^*(v_2)\big)\big\}$$
$$= \min\{(\mu \circ \mu^*)(u, u_2), (\mu \circ \mu^*)(u, v_2)\}$$

$$(\beta \circ \beta^*)\big((u, u_2), (u, v_2)\big) = min\{\eta(u), \beta^*(u_2 v_2)\}$$
$$\leq min\Big\{\eta(u), \Big(min\big(\eta^*(u_2), \eta^*(v_2)\big)\Big)\Big\}$$
$$= \min\big\{\min\big(\eta(u), \eta^*(u_2)\big), min\big(\eta(u), \eta^*(v_2)\big)\big\}$$
$$= \min\{(\eta \circ \eta^*)(u, u_2), (\eta \circ \eta^*)(u, v_2)\}$$

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$$(\gamma \circ \gamma^*)\big((u, u_2), (u, v_2)\big) = max\{\delta(u),\ \gamma^*(u_2\ v_2)\}$$
$$\leq max\left\{\delta(u), \left(max\big(\delta^*(u_2),\ \delta^*(v_2)\big)\right)\right\}$$
$$= \max\left\{\max\big(\delta(u),\ \delta^*(u_2)\big), max\big(\delta(u),\ \delta^*(v_2)\big)\right\}$$
$$= \max\{(\delta \circ \delta^*)(u, u_2), (\delta \circ \delta^*)(u, v_2)\}$$

Let $w \in V_2\ and\ u_1 v_1 \in E_1$. Then, we get

$$(\alpha \circ \alpha^*)\big((u_1, w), (\ v_1, w)\big) = min\{\alpha(u_1\ v_1),\ \mu^*(w)\}$$
$$\leq min\left\{\left(min\big(\ \mu(u_1), \mu(v_1)\big),\ \mu^*(w)\right)\right\}$$
$$= \ \min\left\{\min\big(\mu(u_1),\ \mu^*(w)\big), min\big(\mu(v_1),\ \mu^*(w)\big)\right\}$$
$$= \min\{(\mu \circ \mu^*)(\ u_1, w), (\mu \circ \mu^*)(\ v_1, w)\}$$

$$(\beta \circ \beta^*)\big((u_1, w), (\ v_1, w)\big) = min\{\beta(u_1\ v_1),\ \eta^*(w)\}$$
$$\leq min\left\{\left(min\big(\ \eta(u_1), \mu(v_1)\big), \eta^*(w)\right)\right\}$$
$$= \ \min\left\{\min\big(\ \eta(u_1),\ \eta^*(w)\big), min\big(\ \eta(v_1),\ \eta^*(w)\big)\right\}$$
$$= \min\{(\ \eta \circ\ \eta^*)(\ u_1, w), (\ \eta \circ\ \eta^*)(\ v_1, w)\}$$

$$(\gamma \circ \gamma^*)\big((u_1, w), (\ v_1, w)\big) = max\{\gamma(u_1\ v_1),\ \delta^*(w)\}$$
$$\leq max\left\{\left(max\big(\ \delta(u_1), \delta(v_1)\big),\ \delta^*(w)\right)\right\}$$
$$= \max\left\{\max\big(\ \delta(u_1),\ \delta^*(w)\big), max\big(\ \ \delta(v_1),\ \delta^*(w)\big)\right\}$$
$$= \max\{(\delta \circ\ \delta^*)(\ u_1, w), (\ \delta \circ\ \delta^*)(\ v_1, w)\}$$

Again, Let $u_2, v_2 \in V_2$, $u_1 v_1 \in E_1$ and $u_2 \neq\ v_2$

$$(\alpha \circ \alpha^*)\big((u_1, u_2), (\ v_1, v_2)\big) = min\{\alpha(u_1 v_1),\ \mu^*(u_2),\ \mu^*(v_2)\}$$
$$\leq min\left\{\left(min\big(\ \mu(u_1), \mu(v_1)\big),\ \mu^*(u_2),\ \mu^*(v_2)\right)\right\}$$
$$= \ \min\left\{\big(\ \mu(u_1), \mu(v_1)\big),\ \mu^*(u_2),\ \mu^*(v_2)\right\}$$
$$= \min\{(\mu \circ \mu^*)(\ u_1, u_2), (\mu \circ \mu^*)(\ v_1, v_2)\}$$

$$(\beta \circ \beta^*)\big((u_1, u_2), (\ v_1, v_2)\big) = min\{\beta(u_1 v_1),\ \eta^*(w)\}$$
$$\leq min\left\{\left(min\big(\ \eta(u_1), \eta(v_1)\big),\ \eta^*(u_2),\ \eta^*(v_2)\right)\right\}$$
$$= \ \min\left\{\big(\ \eta(u_1), \eta(v_1)\big),\ \eta^*(u_2), \eta^*(v_2)\right\}$$
$$= \min\{(\eta \circ \eta^*)(\ u_1, u_2), (\eta \circ \eta^*)(\ v_1, v_2)\}$$

$$(\gamma \circ \gamma^*)\big((u_1, u_2), (\ v_1, v_2)\big) = max\{\gamma(u_1\ v_1),\ \delta^*(w)\}$$

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$$\leq max\left\{\left(max\left(\ \delta(u_1), \delta(v_1)\right),\ \delta^*(u_2), \delta^*(v_2)\right)\right\}$$

$$=\ \max\left\{\left(\ \delta(u_1), \delta(v_1)\right),\ \delta^*(u_2),\ \delta^*(v_2)\right\}$$

$$= \max\{(\delta \circ \delta^*)(\ u_1, u_2), (\delta \circ \delta^*)(\ v_1, v_2)\}$$

This completes the proof

Definition 4.5. Let $G'\ and\ G''$ be two Pythagorean Neutrosophic fuzzy Magic Labelled Graph, the union of $G'\ and\ G''$ denoted as $G' \cup G'' = (V_1 \cup V_2, E_1 \cup E_2)$ is defined as

(i) $$(\mu \cup \mu^*)(u_1) = \begin{cases} \mu(u_1) & if\ u_1 \in V_1\ and\ u_1 \notin V_2 \\ \mu^*(u_1) & if\ u_1 \notin V_1\ and\ u_1 \in V_2 \\ max\{\mu(u_1), \mu^*(u_1)\} & if\ u_1 \in V_1 \cap V_2 \end{cases}$$

(ii) $$(\eta \cup \eta^*)(u_1) = \begin{cases} \eta(u_1) & if\ u_1 \in V_1\ and\ u_1 \notin V_2 \\ \eta^*(u_1) & if\ u_1 \notin V_1\ and\ u_1 \in V_2 \\ max\{\eta(u_1), \eta^*(u_1)\} & if\ u_1 \in V_1 \cap V_2 \end{cases}$$

(iii) $$(\delta \cup \delta^*)(u_1) = \begin{cases} \delta(u_1) & if\ u_1 \in V_1\ and\ u_1 \notin V_2 \\ \delta^*(u_1) & if\ u_1 \notin V_1\ and\ u_1 \in V_2 \\ min\ \{\delta(u_1), \delta^*(u_1)\} & if\ u_1 \in V_1 \cap V_2 \end{cases}$$

(iv) $$(\alpha \cup \alpha^*)(u_1u_2\ ) = \begin{cases} \alpha(u_1u_2\ ) & if\ u_1u_2 \in E_1\ and\ u_1u_2 \notin E_2 \\ \alpha^*(u_1u_2\ ) & if\ u_1u_2 \notin E_1\ and\ u_1u_2 \in E_2 \\ max\{\alpha(u_1u_2\ ), \alpha^*(u_1u_2\ )\} & if\ u_1u_2 \in E_1 \cap E_2 \end{cases}$$

(v) $$(\beta \cup \beta^*)(u_1u_2\ ) = \begin{cases} \beta(u_1u_2\ ) & if\ u_1u_2 \in E_1\ and\ u_1u_2 \notin E_2 \\ \beta^*(u_1u_2\ ) & if\ u_1u_2 \notin E_1\ and\ u_1u_2 \in E_2 \\ max\{\beta(u_1u_2\ ), \beta^*(u_1u_2\ )\} & if\ u_1u_2 \in E_1 \cap E_2 \end{cases}$$

(vi) $$(\gamma \cup \gamma^*)(u_1u_2\ ) = \begin{cases} \gamma(u_1u_2\ ) & if\ u_1 \in V_1\ and\ u_1 \notin V_2 \\ \gamma^*(u_1u_2\ ) & if\ u_1 \notin V_1\ and\ u_1 \in V_2 \\ min\{\gamma(u_1u_2\ ), \gamma^*(u_1u_2\ )\} & if\ u_1u_2 \in E_1 \cap E_2 \end{cases}$$

Theorem 4.6. Let $G'\ and\ G''$ be two Pythagorean Neutrosophic fuzzy Magic Labelled Graph then $G' \cup\ G''$ is a Pythagorean Neutrosophic Fuzzy Graph.

Definition 4.7. Let $G'\ and\ G''$ be two Pythagorean Neutrosophic fuzzy Magic Labelled Graph, the intersection of $G'\ and\ G''$ denoted as $G' \cap G'' = (V_1 \cap V_2, E_1 \cap E_2)$ is defined as

(i) $(\mu \cap \mu^*)(u_1) = min\{\mu(u_1), \mu^*(u_1)\ \}\quad if\ u_1 \in V_1 \cap V_2$

$(\eta \cap \eta^*)(u_1) = min\{\eta(u_1), \eta^*(u_1)\ \}\quad if\ u_1 \in V_1 \cap V_2$

$(\delta \cap \delta^*)(u_1) = max\{\mu(u_1), \mu^*(u_1)\ \}\quad if\ u_1 \in V_1 \cap V_2$

(ii) $(\alpha \cap \alpha^*)(u_1u_2\ ) = min\{\alpha(u_1u_2\ ), \alpha^*(u_1u_2\ )\}\quad if\ u_1u_2 \in E_1 \cap E_2$

$(\beta \cap \beta^*)(u_1u_2\ ) = min\{\beta(u_1u_2\ ), \beta^*(u_1u_2\ )\}\quad if\ u_1u_2 \in E_1 \cap E_2$

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$$(\gamma \cap \gamma^*)(u_1u_2) = max\{\gamma(u_1u_2), \gamma^*(u_1u_2)\} \quad if\ u_1u_2 \in E_1 \cap E_2$$

Theorem 4.8. Let G' and G'' be two Pythagorean Neutrosophic fuzzy Magic Labelled Graph then $G' \cap G''$ is a Pythagorean Neutrosophic Fuzzy Graph.

Definition 4.9. The complement of Pythagorean Neutrosophic fuzzy Magic Labelled Graph $G = (V, E)$ is denoted as $\bar{G}: \left(\overline{\mu, \eta, \delta,}, \overline{\alpha, \beta, \gamma}\right)$, where $\overline{\mu, \eta, \delta,} = (\mu, \eta, \delta)$ and $\overline{\alpha, \beta, \gamma} = \left(\bar{\alpha}, \bar{\beta}, \bar{\gamma}\right)$, where

$$\bar{\alpha}(u, v) = \min\{\mu(u), \mu(v)\} - \alpha(u, v)$$
$$\bar{\beta}(u, v) = \min\{\eta(u), \eta(v)\} - \beta(u, v)$$
$$\bar{\gamma}(u, v) = \min\{\delta(u), \delta(v)\} - \gamma(u, v)$$

Theorem 4.10. The complement of complement of a Pythagorean Neutrosophic Fuzzy Magic Graph G is G i.e., $\bar{\bar{G}} = G$

Proof. Let $G = (V, E)$ be a fuzzy magic Graph. Then the complement of Fuzzy magic Graph is $\bar{G}: \left(\overline{\mu, \eta, \delta,}, \overline{\alpha, \beta, \gamma}\right)$, where $\overline{\mu, \eta, \delta,} = (\mu, \eta, \delta)$ and $\overline{\alpha, \beta, \gamma} = \left(\bar{\alpha}, \bar{\beta}, \bar{\gamma}\right)$, where

$$\bar{\alpha}(u, v) = \min\{\mu(u), \mu(v)\} - \alpha(u, v)\ \forall\ u, v \in V$$

Now $\bar{\bar{\mu}} = \bar{\mu} = \mu$ and $\bar{\alpha}(u, v) = \min\{\bar{\mu}(u), \bar{\mu}(v)\} - \bar{\alpha}(u, v)$

$$= \min\{\mu(u), \mu(v)\} - [\min\{\mu(u), \mu(v)\} - \alpha(u, v)]$$
$$= \alpha(u, v)\ for\ all\ u, v \in V$$

Similarly, $\bar{\bar{\eta}} = \bar{\eta} = \eta$ and $\bar{\beta}(u, v) = \beta(u, v)\ for\ all\ u, v \in V$ and

$\bar{\bar{\delta}} = \bar{\delta} = \delta$ and $\bar{\gamma}(u, v) = \gamma(u, v)\ for\ all\ u, v \in V$

Hence, $\bar{\bar{G}} = G$

5. Conclusion

Pythagorean Neutrosophic fuzzy graph is the fussion of graph theory and Pythagorean neutrosophic set. The notion of Pythagorean neutrosophic fuzzy graph extended to Pythagorean neutrosophic fuzzy magic graph. In this article, we define some operations like cartesion product, composition, complement union and intersection and also investigate their properties. In future we develop a model using this defined graph and applies it in real life decision making problems.

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CERTIFIED DOMINATION POLYNOMIAL OF TRIANGULAR BOOK GRAPH $B(3,n)$

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Abstract

Let $G = (V, E)$ be a simple graph. A dominating set S is a certified dominating set of G if S has either zero or at least two neighbours in $V - S$. Let $B(3, n)$ be the triangular book graph with $n + 2$ vertices. Let $D_{cer}(B(3, n), i)$ denote the family of all certified dominating sets with cardinality i of $B(3, n)$. Let $d_{cer}(B(3, n), i) = |D_{cer}(B(3, n), i)|$. In this paper, we obtain a exact formula for $d_{cer}(B(3, n), i)$. Using this formula, we construct the polynomial, $D_{cer}(B(3, n), x) = \sum_{1}^{n+2} d_{cer}(B(3, n), i)\, x^i$, which we call the certified domination polynomial of $B(3, n)$ and also obtain some properties of this polynomials.

Key words: Certified domination number, Certified domination polynomial.

2020 Mathematics Subject Classification (AMS): 05C69,05C31,05

1. Introduction

Let $G = (V, E)$ be a simple graph of order $|V| = n$. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V \,/\, uv \in E\}$ and the closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$.

The concept of certified domination in graphs was introduced by Dettlaf et al., 2020. A set $S \subseteq V$ is a dominating set of G, if every vertex in $V - S$ is adjacent to at least one vertex in S. A dominating set S is a certified dominating set of G if S has either zero or at least two neighbours in $V - S$. The certified domination number $\gamma_{cer}(G)$ of G is the minimum cardinality of certified dominating set.

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Triangular book with n pages is defined as n copies of cycle C_3 sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B(3,n)$. Let $B(3,n)$ be the triangular book graph with $n+2$ vertices and $V(B(3,n)) = X \cup Y$, where $X = \{x_i/i = 1,2\}$ and $Y = \{y_i/1 \leq i \leq n\}$ and $E(B(3,n)) = \{x_1x_2\} \cup \{x_iy_j/1 \leq i \leq n\}$.

In the next sections, we construct the certified domination polynomials of $B(3,n)$.

Definition 1.1. Let G be a simple connected graph. Let $D_{cer}(G,i)$ be a family of all certified dominating sets of G with cardinality i and let $d_{cer}(G,i) = |D_{cer}(G,i)|$. Then the certified domination polynomial $D_{cer}(G,x)$ is defined as

$$D_{cer}(G,x) = \sum_{i=\gamma_{cer}(G)}^{|V(G)|} d_{cer}(G,i)\, x^i$$

Where $\gamma_{cer}(G)$ is the certified domination number of G.

2. Certified domination polynomial of $B(3,n)$

Observation 2.1. For a triangular book graph $\gamma_{cer}(B(3,n)) = 1$, for all $n \in N$

Lemma 2.2. Let G be the graph with n vertices. Then

(i) $d_{cer}(G,n) = 1$

(ii) $d_{cer}(G,i) = 0$ if and only if $i < \gamma_{cer}(G)$ or $i = n-1$ or $i > n$

(iii) $D_{cer}(G,x)$ has no constant term.

Lemma 2.3. Let $B(3,n)$ be a triangular book graph with $n+2$ vertices, then for all $n \geq 2$,

$$d_{cer}(B(3,n),i) = \begin{cases} 2, & if\ \ i = 1 \\ \binom{n}{i-2}, & if\ \ 2 \leq i \leq n-1 \text{ and } i = n+2 \\ \binom{n}{i-2} + 1, & if\ \ i = n \\ 0\ , & if\ \ i = n+1 \end{cases}$$

Proof. Let $B(3,n)$ be a triangular book graph with $n+2$ vertices. Let $V(B(3,n)) = X \cup Y$, where $X = \{x_i/i = 1,2\}$ and $Y = \{y_i/1 \leq i \leq n\}$

When $i = 1$,

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$\{x_1\}$ and $\{x_2\}$ are the only certified dominating sets. Therefore $d_{cer}(B(3,n),1) = 2$

When $2 \leq i \leq n-1$ and $i = n+2$,

for the certified dominating set, we need to select all vertices from X and $i-2$ vertices from Y. This means there are $\binom{n}{i-2}$sets.

When $i = n$,

for the certified dominating set, we need to select all vertices from X and $i-2$ vertices from Y. In addition to that Y is also a certified dominating set. This means there are $\binom{n}{i-2} + 1$sets

When $i = n+1$,

by Lemma 2.2, $d_{cer}(B(3,n), n+1) = 0$

Lemma 2.4.

(i) $d_{cer}(B(3,n), i) = d_{cer}(B(3,n), n-i+4)$, where $n \geq 7$ and $i = 2, n+2, 5$ to $n-1$

(ii) For all $n \geq 5$,

$$d_{cer}(B(3,n), i) = \begin{cases} d_{cer}(B(3,n-1), i-1) + d_{cer}(B(3,n-1), i) & if\ 3 \leq i \leq n-2 \\ d_{cer}(B(3,n-1), i-1) + d_{cer}(B(3,n-1), i) - 1 & if\ \ i = n-1 \\ d_{cer}(B(3,n-1), i-1) + (n-1) & if\ \ i = n \end{cases}$$

Proof.

(i) $d_{cer}(B(3,n), i) = \binom{n}{i-2}$ (by Lemma 2.3)

$= \binom{n}{n-i+2}$ $(nC_r = nC_{n-r})$

$= d_{cer}(B(3,n), n-i+4)$

(ii) When $3 \leq i \leq n-2$,

$d_{cer}(B(3,n-1), i) = \binom{n}{i-2}$ (by Lemma 2.3)

$= \binom{n-1}{i-3} + \binom{n-1}{i-2}$ $\left(\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}\right)$

$= d_{cer}(B(3,n-1), i-1) + d_{cer}(B(3,n-1), i)$

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When $i = n - 1$,

$$d_{cer}(Bt_n, n-1) = \binom{n}{n-3}$$

$$= \binom{n-1}{n-4} + \left\{\binom{n-1}{n-3} + 1\right\} - 1$$

$$= d_{cer}(B(3, n-1), n-2) + d_{cer}(B(3, n-1), n-1) - 1$$

When $i = n$,

$$d_{cer}(Bt_n, n) = \binom{n}{n-2} + 1$$

$$= \binom{n-1}{n-2} + \left\{\binom{n-1}{n-3} + 1\right\}$$

$$= d_{cer}(B(3, n-1), n-1) + (n-1)$$

Theorem 2.5. For the triangular book graph $B(3, n)$, $D_{cer}(B(3,n), x) = x^2[(1+x)^n - nx^{n-1}] + (x^n + 2x)$, for all n.

Proof.

$$D_{cer}(B(3,n), x) = d_{cer}(B(3,n), 1)x + d_{cer}(B(3,n), 2)x^2 + \cdots + d_{cer}(B(3,n), n+2)\, x^{n+2}$$

$$= 2x + \binom{n}{0}x^2 + \binom{n}{1}x^3 + \cdots + \left\{\binom{n}{n-2} + 1\right\}x^n + 0 + \binom{n}{n}x^{n+2}$$

$$= 2x + x^2\left\{\sum_{r=0}^{n}\binom{n}{r}x^r - \binom{n}{n-1}x^{n-1}\right\} + x^n$$

$$= x^2[(1+x)^n - nx^{n-1}] + (x^n + 2x)$$

Remark 2.6. Sum of co-efficients of a certified dominating polynomial of the triangular book graph is $2^n - (n-3)$, for all $n \geq 3$.

n/i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	3	0	1									
2	2	2	0	1								
3	2	1	4	0	1							
4	2	1	4	7	0	1						
5	2	1	5	10	11	0	1					

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6	2	1	6	15	20	16	0	1				
7	2	1	7	21	35	35	22	0	1			
8	2	1	8	28	56	70	56	29	0	1		
9	2	1	9	36	84	126	126	84	37	0	1	
10	2	1	10	45	120	210	252	210	120	46	0	1

Table 1: $d_{cer}(B(3,n),i)$, the number of certified dominating sets of Bt_n with cardinality i.

3. Conclusion

In this paper, we have derived the important relation of $d_{cer}(B(3,n),i)$. Using this relation we have to found out the certified domination polynomial of the triangular book graph $B(3,n)$.

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NEW APPROACH TO ASSIGNMENT PROBLEM USING PLAY FAIR CIPHER METHOD

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Abstract

Assignment model comes under the class of linear programming model, which looks alike with the transportation model with an objective function of minimizing the time or cost of manufacturing the products by allocating one job to one machine or one machine to one job or one destination to one origin or one origin to one destination only. Basically assignment model is a minimization model. In this paper to introduce a new approach to assignment problem namely an approach to assignment using play fair cipher method and the result is verified using PYTHON.

Keywords: Assignment problem, Hungarian method, Cost matrix, Optimization, Cryptography, Play fair cipher, Encryption.

2020 Mathematics Subject Classification (AMS): 90C05

1. Introduction

The assignment problem is a special structure of transportation problem, in which number of jobs (tasks) is equal to number of persons (facilities). Thus the objective of the problem is how the assignment should be made to archieved allocation. In the assignment model worker represent source and jobs represent destination. The supply amount at each source exactly 1. For example if $n = 3$ person can assigned to 3 jobs. Then the possible ways is 3!=6. These allocation will take large time. There are many methods to develop such problem. Hungarian is one of them [1, 3].

Cryptography is the study of mathematical techniques related to aspects of information security such as confidentiality, data integrity, entity authentication, and data origin authentication. In this, play fair cipher is a classic cryptographic method developed in 1854 by Charles Wheatstone [9]. It is a systematic encryption technique that encrypts text using a 5x5 matrix of letters based on the key, offering a secure and systematic approach to data

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confidentiality. This paper explores the intersection of these two areas by addressing the assignment problem using the fair play cipher-based method.

2. Hungarian method for solving assignment problem:

2.1 Numerical example

Three men are available to do three different jobs. From past records, the time (in hours), that each man takes to do each jobs is known and is given in the following table.

JOB/MAN	A	B	C
1	8	7	6
2	5	7	8
3	6	8	7

Solution:

Step 1: Subtract the least element from every column of each row.

JOB/MAN	A	B	C
1	2	1	0
2	0	2	3
3	0	2	1

Step 2: Subtract the least element from every column of each column.

JOB/MAN	A	B	C
1	2	0	0
2	0	1	3
3	0	1	1

Step 3: Assigning the zeros to the matrix in regular manner, the matrix is reduced. Since the row 3 and column 3 has no assignment, we proceed with the minimal number of lines being drawn.

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JOB/MAN	A	B	C
1	2	(0)	0
2	(0)	1	3
3	0	1	1

Step 4: The least number that do not contain lines is 1. Subtracting the number 1 from elements that do not contain line and adding to intersection of lines, We make the zero assignment. Hence the optimal solution is obtained.

JOB/MAN	A	B	C
1	2	(0)	0
2	(0)	0	2
3	0	0	(0)

The obtained assignment is given as $1 \to B, 2 \to A, 3 \to C$

Minimal assignment = 7+5+7= 19.

3. Approach to assignment problem using play fair cipher

The new algorithm is as follows:

1. Subtract the smallest element of each row from every element of the correspondingrow.
2. Subtract the smallest element of each column from every element of the corresponding column.
3. Consider the location of the zero at each row. If there is no zero in any of the row, then subtract the least number from the particular row. Now consider the number next to zero in each row and each column. And consider the least number from the each row and each column. Here rows and columns are the two conditions.
4. Subtract the least element in the first row and first column and it forms the matrix.Then again consider the least element from the second row and second column fromthe obtained matrix.
5. Repeat the process for all rows and columns that we have taken.

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6. Now we assign the zeros. If there is no assignment , then we proceed the minimal line condition of the assignment problem.
7. Now we obtain the encrypted optimal solution.
8. Here now we use the keyword to decrypt the optimal solution that we obtained by assignment method. The keyword varies for each problem according to obtained optimal solution.

3.1 Numerical Problem

Three men are available to do three different jobs. From past records, the time (in hour) that each man takes to do each job is known and is given in the following table.

JOB/MAN	A	B	C
1	8	7	6
2	5	7	8
3	6	8	7

Solution:

Step 1: Subtract the least element row each column of every element.

JOB/MAN	A	B	C
1	2	0	0
2	0	1	3
3	0	1	1

Step 2: Subtract the least element from each column of every element.

JOB/MAN	A	B	C
1	2	0	0
2	0	1	3
3	0	1	1

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Step 3: Now we consider the numbers next to zeros in each row and column. Here row and column are the two conditions.

ROW	**COLUMN**
1→ 0,2	A→0,2
2→1	B→1
3→1	C→3

Step 4: Now we consider least number in the first row and the first column. Here the least element is 0, subtract the number in first row and first column

JOB/MAN	A	B	C
1	2	0	0
2	0	1	3
3	0	1	1

Step 5: Consider the least number in second row and second column. Here the least number is 1. Subtract the number in second row and second column. Repeat the same process for the third row and column. The process takes upto the number of columns and the row that we have taken.

JOB/MAN	A	B	C
1	2	1	0
2	1	0	2
3	0	0	1

JOB/MAN	A	B	C
1	2	1	1
2	1	0	1

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3	1	1	0

Step 6: Since the first row does not have any zeros, subtract the least number from all other elements in the row. Assign zeros to the matrix.

JOB/MAN	A	B	C
1	1	~~0~~	(0)
2	1	(0)	1
3	1	1	~~0~~

Step 7: Since the third row does not have the assignment, we draw minimal number of lines. Then the encrypted optimal solution is obtained. The encrypted optimal solution is given as 1→C, 2→B, 3→A

JOB/MAN	A	B	C
1	~~0~~	~~0~~	(0)
2	1	(0)	2
3	(0)	~~0~~	~~0~~

The encrypted optimal solution is given as 1→C, 2→B, 3→A

Minimal solution for encrypted problem = 6+7+6 = 19

Here the encrypted keyword is taken as 2n. The value of n is taken as number that gets repeated maximum times in the final assigning step.

Here there is no repeatation, the n value is zero. Hence by decrypting the optimal value is 5.

4. Result verification by Python

```
import numpy as np
from scipy.optimize import linear_sum_assignment
#Define the cost matrix
cost_matrix = np.array ([
    [8, 7, 6],
    [5, 7, 8],
    [6, 8, 7],
```


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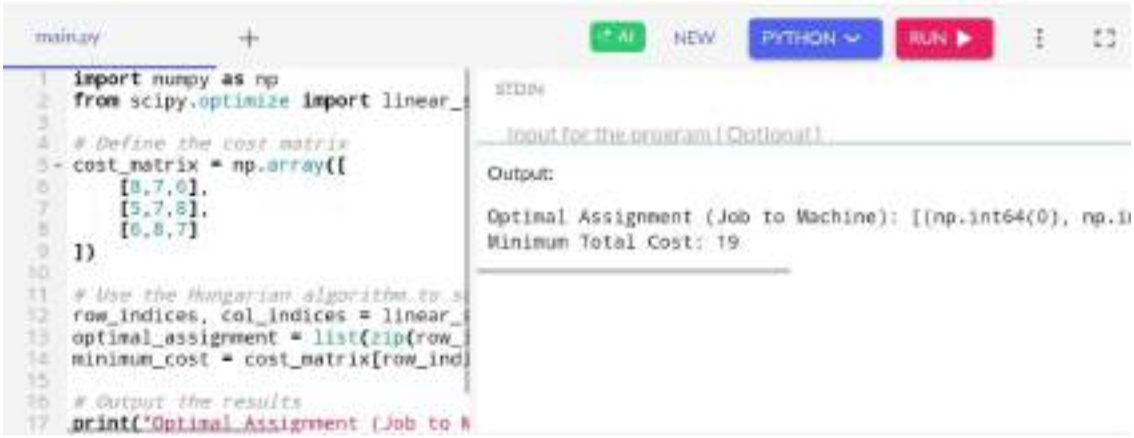
```
])

# Use the Hungarian algorithm to solve the assignment problem
row_indices,col_indices=linear_sum_assignment(cost_matrix)
optimal_assignment = list(zip(row_indices, col_indices))
minimum_cost = cost_matrix[row_indices, col_indices].sum()
# Output the results
print("Optimal Assignment(Job to Machine):",optimal_assignment)
print("Minimum Total Cost:", minimum_cost)
```



5. Conclusion

In this paper, the use of play fair cipher method in the assignment has provided valuable insights into the application of the polygraphic encryption techniques. By employing a 5x5 matrix for digraph substitution, the play fair cipher enhances the security of the plaintext beyond basic substitution ciphers, offering a more complex approach of encryption. Throughout the assignment, the process of constructing the matrix, and applying cipher encryption and decryption is explored in detailed. This assignment has helped reinforce the importance of cryptographic methods in information security and play fair cipher provides a valuable educational tool for learning the fundamentals of encryption and cryptanalysis.

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GRAPH BASED SECURE COMMUNICATION IN CRYPTOGRAPHY

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Abstract

Graph theory is one of the approaches used to secure data protection and message transmission, which is one of the most crucial methods used in cryptography. Many techniques are available to encrypt and decrypt the info[13]. Cryptography is especially used to make the text unintelligible and non-readable so that the opponents cannot understand the meaning of the text. Cryptography provides privacy and security for the key information by hiding it. It is done through mathematical technique. This paper provides the concept of secure communication by using graph theory in cryptography.

Keywords: Cryptography, Substitution, Adjacent Matrix, Data Encryption.

2020 Mathematics Subject Classification (AMS): 94A60

1. Introduction

Cryptography is the study of techniques to secure communication by making it unreadable to unauthorized parties. It deals with protecting sensitive information from hackers and ensuring the privacy, integrity, and authenticity of data. The main goal of cryptography is to enable secure data transmission over insecure channels. This is achieved through encryption, which converts plaintext (readable data) into ciphertext (unreadable data). Only authorized parties can decrypt the ciphertext to retrieve the original message.

Graph theory, a branch of mathematics, plays a crucial role in cryptography. It involves the study of graphs, which are used to model relationships between objects. Graph theory has been successfully applied to develop stronger encryption algorithms that are resistant to hacking. This paper presents a new cryptosystem that combines cryptography and graph theory principles to provide high security and efficient data processing.

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2. Preliminaries

Plain text- Plain text, also known as cleartext or plaintext, refers to unencrypted and human-readable text data. It is text that has not been encrypted or encoded in any way, making it easily readable and understandable by anyone who has access to it.

Cipher text- Cipher text is encrypted text that has been transformed from plain text using an encryption algorithm and a secret key. The resulting cipher text is unreadable and unintelligible to anyone without the corresponding decryption key or algorithm.

Key- In the context of cryptography and encryption, a key is a unique string of characters, numbers, or symbols used to encrypt and decrypt data. Keys are used to control the encryption and decryption processes, ensuring that only authorized parties can access the encrypted data.

Encipher- Encipher, also known as encrypt, is the process of converting plaintext (readable data) into ciphertext (unreadable data) using an encryption algorithm and a secret key.

Decipher- Decipher, also known as decrypt, is the process of converting ciphertext (unreadable data) back using a decryption algorithm and a secret key.

Encryption and Decryption- The process of encoding a message using some key or method so that it is the meaning is not easily understood. The reverse process of the ciphertext conversion encryption method into plain text is decryption.

Brute Force Attack- A brute force attack is a type of cyber attack where an attacker attempts to guess or crack a password, encryption key, or other type of secret information by systematically trying all possible combinations.

Graph- A graph is a non-linear data structure consisting of nodes or vertices connected by edges. Each nodes represents an entity, and the edges represent the relationships or connections between these entities.

Cycle- A cycle is a path in a graph that starts and ends at the same node, and passes through at least one edge. In other words, a cycle is a closed path in a graph.

2.1 Adjacency matrix

This is a matrix representation of the graph. It is used in computer processing. In graph

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theory and computer science, an adjacency matrix is a square matrix used to represent a finite graph. The matrix elements indicate whether a pair of vertices in the graph are adjacent or not.

3. Proposed Algorithm

Use the proposed algorithm to encrypt and decrypt data. (Send key2 in the form of graph)

3.1 Encryption Algorithm

This algorithm is used to convert plain text to cipher text.

- Input Message: Receive a message from the user to be encrypted.
- Shift Characters: Use a secret key (Key 1) to shift each character in the message.
- Encrypt Message: Replace each letter with the shifted character based on Key 1.
- Matrix Formation: Arrange the encrypted message in a matrix format, with dimensions (n-1) x n, where n is the number of digits in Key 2.
- Column Permutation: Read the matrix row by row and rearrange the columns according to a predetermined permutation.
- Re-Matrix Formation: Rearrange the permuted columns into a new matrix.
- Cipher Text Generation: Read the final matrix row by row to obtain the encrypted cipher text.

3.2 Decryption Algorithm

This algorithm is used to convert cipher text to plain text.

- Cipher Text Input: Receive the encrypted cipher text.
- Matrix Formation: Use Key 2 to arrange the cipher text in a matrix format, with dimensions (n-1) x n, where n is the number of digits in Key 2.
- Column Rearrangement: Rearrange the matrix columns using Key 2.
- Row-by-Row Read: Read the rearranged matrix row by row to obtain the intermediate decrypted text.
- Re-Matrix Formation: Rearrange the intermediate decrypted text in a matrix format, column by column, using Key 2.
- Decryption with Key 1: Use Key 1 to decrypt the rearranged matrix, reversing the initial encryption process.
- Plain Text Recovery: Obtain the original plain text after decryption.

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3.3 Example: Encryption

First take a message or plain text from user which we have to encrypt. For ex. TEACHERS INSPIRE YOUNG MINDS DAILY.

- Use key1 to shift character.
- Suppose key 1 = +4
- Encrypt the message by replacing each letter by decided key 1.
- YJFHMJWXNSXUNWJDTZSLRNSLRNSIXIFNQD
- Write encrypted message in the form of matrix(where (n-1) x n where n = number of digits in key2) which is decided by sender and receiver.
- Key2 is shared in the form of adjacent graph, sender and receiver have to calculate key2 from the given graph

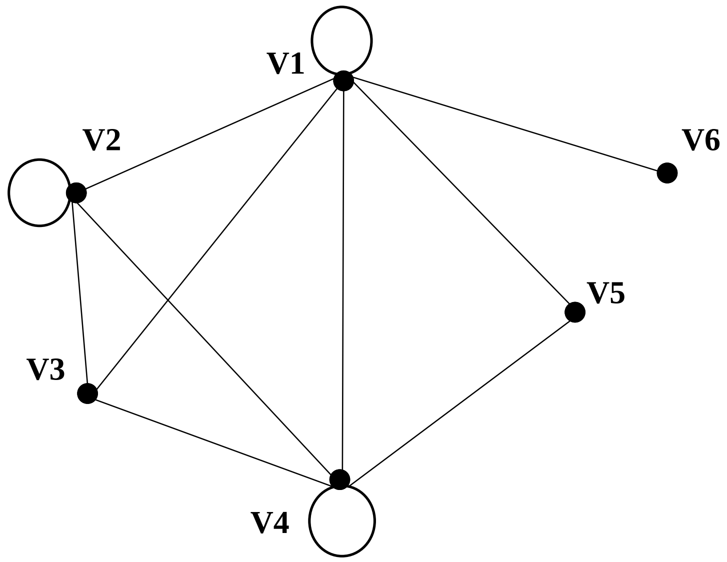

Fig.1: Graph for the calculation for key2

Convert the above graph into adjacent matrix which is used as key 2.

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Table 1: Adjacent matrix of key2

	V1	V2	V3	V4	V5	V6
V1	1	1	1	1	1	1
V2	1	1	1	1	0	0
V3	1	1	0	1	0	0
V4	1	1	1	1	1	0
V5	1	0	0	1	0	0
V6	1	0	0	0	0	0

Now the key2 is **6 4 3 5 2 1**

Table 2(a): Message creation from key2

6	4	3	5	2	1
Y	J	F	H	M	J
W	X	N	S	X	U
N	W	J	D	T	Z
S	L	R	N	S	I
X	I	F	N	Q	D

Read off the message row by row and permute the order of column JUZIDMXTSQFNJRFJXWLIHSDNNYWNSX.

The output of step 5, write in matrix form again and read row by row.

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Table 2(b): Message creation from key2

6	4	3	5	2	1
J	U	Z	I	D	M
X	T	S	Q	F	N
J	R	F	J	X	W
L	I	H	S	D	N
N	Y	W	N	S	X

After reading row by row, we get our cipher text.

MNWNXDFXDSZSFHWUTRIYIQJSNJXJLN(cipher text to be sent)

3.4 Decryption

It takes the cipher text and use key2 to write cipher text in the form of matrix (where (n-1) x n, where n = number of digits in key2) which is decided by sender and receiver.

Received cipher text is: - MNWNXDFXDSZSFHWUTRIYIQJSNJXJLN

Arrange the cipher in matrix form column by column using key2.

Table 3(a): Cipher text in matrix form

6	4	3	5	2	1
J	U	Z	I	D	M
X	T	S	Q	F	N
J	R	F	J	X	W
L	I	H	S	D	N
N	Y	W	N	S	X

Read message row by row. JUZIDMXTSQFNJRFJXWLIHSDNNYWNSX.

Again, arrange the cipher of step 3 in matrix form column by column using key 2

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Table 3(b): Cipher text in matrix form

6	4	3	5	2	1
Y	J	F	H	M	J
W	X	N	S	X	U
N	W	J	D	T	Z
S	L	R	N	S	I
X	I	F	N	Q	D

Received cipher text is: - YJFHMJWXNSXUNWJDTZSLRNSLRNSIXIFNQD

Now decrypt the message with key1. Key1= (-4)

Finally, we get plain text.

Result: Teachers inspire young minds daily.

4. Conclusion

The Double Transposition Column method, which leverages graph theory as the key, offers significant advantages over basic algorithms. By incorporating a graph to generate the key, cryptanalysis becomes more complex, enhancing security and making the plaintext output nearly impossible to crack. This approach eliminates the possibility of brute-force attacks, thereby overcoming the limitations of the traditional Caesar cipher. Furthermore, the algorithm is adaptable and can easily be integrated into new applications. It supports the creation of multiple keys, making it ideal for secure applications like online banking, e-commerce, and electronic voting. However, due to the use of graph theory, the implementation may require additional memory, making the simple Caesar cipher more challenging to implement.

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SIGNAL DOMINATION IN MIDDLE GRAPHS

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Abstract

The term "Domination" refers to the act of exerting control or influence over someone or something. In graph theory, the concept of domination has spread all over the world in various fields and has a special impact in maximizing the efficiency with minimum input. Along with distance parameter, the concept of domination has been improvised in many aspects by providing solutions to unsolved real – life problems. Middle graphs, though primarily a theoretical construction in graph theory, have several uses and applications in various areas of research and practical problem-solving. It provides a direct way to model these edge-to-edge relationships. This can be useful in problems related to symmetry breaking such as crystallography. In this paper, we calculate the signal domination number of middle graph of some common graph families.

Keywords: Signal number, signal domination number, middle graph.

2020 Mathematics Subject Classification (AMS): 05C12, 05C69, 05C76

1. Introduction

In the domains of mathematics and computer science, graph theory is the study of graphs, which concerns the relationships among vertices and edges. A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. Formally, a graph G is a non-empty finite undirected graph with no multiple edges or loops. It is widely used in designing circuit connections, representing data organization, networks of communication, flow of computation, molecular and chemical structures, as well as in algorithms such as Kruskal's, Prim's, and Dijkstra's, among many other branches of biological and applied sciences. For a better understanding on graphs, refer [4].

The study of domination in graphs originated from the 8×8 chessboard, where the minimum number of queens required covering all 64 squares was investigated. This problem of dominating the squares of a chessboard can be formulated as the problem of dominating the vertices of a graph. The theory of domination has a wide range of applications in computer

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networking, communication networks, and also in the fields of transportation. For a detailed study, refer [8].

Throughout this paper, we consider G to be a connected graph. A subset D of vertices in a graph G is a dominating set if each vertex of G that is not in D is adjacent to at least one vertex of D. The size of the dominating set with minimum number of elements among all dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$. For a vertex $u \in V(G)$, the open neighborhood $N(u)$ is the set of all vertices that are adjacent to u, and $N[u] = N(u) \cup \{u\}$ is the closed neighborhood of u. The degree of a vertex v is defined by $deg(v) = |N(v)|$. A $x-y$ path of length $d(x,y)$ in a graph is called $x-y$ geodesic. A vertex v is said to be an internal vertex of $x-y$ path if it lies in the $x-y$ path. The shortest distance between any two vertices u and v in a graph G is known as geodesic distance. For a detailed study on distances in graphs, refer [3].

In the year 2010, Kathiresan introduced a distance parameter called signal distance refer to [6]. Through research on signal distance, Sethu Ramalingam and Balamurugan introduced the concept of the signal number [7]. Furthermore, Balamurugan and Antony Doss worked on the signal number, and the signal chain was constructed [2]. In this paper, we estimate the values of signal domination number of middle graphs of some common families of graphs.

2. Preliminaries

Definition 2.1. [6] The signal distance between a pair of vertices u and v in a graph G is defined as $\min\{d(u,v) + (\deg(u) - 1) + (\deg(v) - 1) + \sum_{w \in u-v} (\deg(w) - 2)\}$, where $d(u,v)$ is the length of the $u - v$ path and w is the internal vertices of $u - v$ path The signal path between u and v is called as the geosig path.

Definition 2.2. [7] The subset $S \subseteq V$ is called the signal set of G if every vertex u in G lies in a geosig path between the vertices in S and the minimum cardinality of the set S is called as the signal number of a graph. It is denoted by $sn(G)$.

Definition 2.3. [5] A set $S \subseteq V$ is called a signal dominating set of a graph G if S is a dominating set of G as well as a signal set of G. The minimum cardinality of the signal dominating set is called the signal domination number and it is denoted by $\gamma_{sn}(G)$.

Definition 2.4. [1] The middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ if one of the following conditions holds.

1. $x, y \in E(G)$ and x, y are adjacent in G.
2. $x \in V(G), y \in E(G)$ and x, y are incident in G.

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3. Main Results

Proposition 3.1. For a path P_n, $\gamma_{sn}\big(M(P_n)\big) = n$.

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of a path graph P_n whose edge set is $\{v_1, v_2, \dots, v_{n-1}\}$. By the definition of middle graph, P_n is transformed into $M(P_n)$, where $(u_i v_i), (v_j v_{j+1}), (u_{i+1} v_i)$ with $1 \le i \le n-1$ and $1 \le j \le n-2$ forms and edge set in $M(P_n)$ and $|V(M(P_n))| = 2n-1$. Since u_1 and u_n are pendant vertices, $2 \le sn\big(M(P_n)\big)$. Since the signal distance between the pendant vertices is $3n-4$, the $u_1 - u_n$ geosig path covers v_j $(1 \le j \le n-1)$ while u_i $(2 \le i \le n-1)$ are not covered by the geosig path, Hence the set $\{u_i \,/\, 1 \le i \le n\}$ forms a signal basis of $M(P_n)$. Furthermore, the signal basis set dominates every vertices of $M(P_n)$. So $\gamma_{sn}\big(M(P_n)\big) = n$.

Proposition 3.2. For a cycle C_n, with $n \ge 3$, we have $\gamma_{sn}\big(M(C_n)\big) = n$.

Proof. Let $\{x_1, x_2, \dots, x_n\}$ be the vertex set of a cycle C_n with the corresponding edge set $\{y_1, y_2, \dots, y_n\}$. According to the definition of middle graph, C_n is transformed into $M(C_n)$ whose vertex set is $V(M(C_n)) = \{x_i, y_i \,/\, 1 \le i \le n\}$ with $\big|V\big(M(C_n)\big)\big| = 2n$ and the edge set is formed in such a way that $y_1, y_2, \dots, y_n$ induces a cycle of length n and $N(y_i) = \{y_{i+1}, y_{i-1}, x_i, x_{i+1}\}$ for $1 < i < n$ with $E\big(M(C_n)\big) = 3n$. Since x_i and x_j are not adjacent in $M(C_n)$ for every $i \ne j$, the geosig path formed by every distinct pair of x_i and x_j can cover $V\big(M(C_n)\big)$ and so $sn\big(M(C_n)\big) \le n$. Suppose $sn\big(M(C_n)\big) = n-1$, then there exist a vertex x_k in $M(C_n)$ such that x_k is not in $sn\big(M(C_n)\big)$. However $x_{i-1} - x_{i+1}$ geosig path only covers y_{i-1} and y_i leaving x_i behind which leads to a contradiction. So $sn\big(M(C_n)\big) = n$. In addition, the sn- set of $M(C_n)$ dominates every vertices of $M(C_n)$. Therefore $\gamma_{sn}\big(M(C_n)\big) = n$.

Proposition 3.3. For a complete graph K_n, $\gamma_{sn}\big(M(K_n)\big) = n$.

Proof. Let K_n be a complete graph of order n whose vertex set is $\{u_1, u_2, \dots, u_n\}$ and the edge set is $\left\{e_1, e_2, \dots, e_{\frac{n(n-1)}{2}}\right\}$. By the definition of middle graph, $|V(M(K_n))| = \frac{n(n+1)}{2}$. Since it is a complete graph, the signal distance between u_i and u_j with $i \ne j$ in $M(K_n)$ is $4n-6$ and $d\big(u_i, u_j\big) = 2$. Clearly the geosig path formed by every pair of u_i's covers every vertices of $M(K_n)$ and forms a signal basis of $M(K_n)$. Therefore, $sn\big(M(K_n)\big) = n$. Moreover, the set $\{u_1, u_2, \dots, u_n\}$ dominates every vertices of $M(K_n)$. So $\gamma_{sn}\big(M(K_n)\big) = n$.

Corollary 3.4. $\gamma_{sn}\big(M(K_n)\big) = \gamma_{sn}(K_n)$.

The proof is obvious.

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Proposition 3.5. For a star graph $K_{1,n}$, $\gamma_{sn}\left(M(K_{1,n})\right) = n+1$.

Proof. Let $K_{1,n}$ be a star graph of order $n+1$ whose vertices are $x, x_1, x_2, \dots, x_n$ where, x is the central vertex and let $e_i = xx_i$ $(1 \le i \le n)$ be the edges of $K_{1,n}$. Upon transforming $K_{1,n}$ to obtain its middle graph $M(K_{1,n})$, we get $|V(M(K_{1,n}))| = 2n+1$ and e_ie_j becomes an edge in $M(K_{1,n})$ where i and j are distinct. It is obvious that all the pendant vertices are contained in the signal set and so $sn\left(M(K_{1,n})\right) \ge n$. Since e_ie_j $(i \ne j)$ is an edge in $M(K_{1,n})$, the geosig path formed by any pair of x_i's does not cover x. So we conclude that $sn\left(M(K_{1,n})\right) = n+1$. Furthermore, the set $\{x_1, x_2, \dots, x_n\}$ forms a dominating set of $M(K_{1,n})$. Hence $\gamma_{sn}\left(M(K_{1,n})\right) = n+1$.

Corollary 3.6. $\gamma_{sn}\left(M(K_{1,n})\right) = \gamma_{sn}(K_{1,n}) + \gamma(K_{1,n})$.

The proof is obvious.

Theorem 3.7. For any connected graph G, $\gamma_{sn}(G) \le \gamma_{sn}(M(G))$.

Proof. Let G be a connected graph of order n and size m. Let S_1 and S_2 be the minimum signal dominating sets of G and $M(G)$ respectively. Clearly, $S_1 \le n$. Since $M(G)$ contains $n+m$ vertices, $S_2 \le n+m$. If $S_1 = n$, then the result is obvious. Suppose not, let $S_1 = n-a$ where a is any positive integer with $a < n$. Then $S_2 \le S_1 + a + m$. Since $a+m$ is positive, we conclude that $\gamma_{sn}(G) \le \gamma_{sn}(M(G))$.

4. Conclusion

Continued research into this area will likely unveil more sophisticated methods and applications, advancing our knowledge of graph theory and its practical implications. Since middle graph is constructed based on edge to edge relation, we can have a better understanding regarding the relationship between vertices and the potential for optimization in network design, resource allocation, and other applied fields. Moving forward, further research into the nuances of signal domination in middle graphs will deepen our understanding of their structure, ultimately advancing both theoretical frameworks and practical strategies for solving complex problems in diverse domains.

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TEMPERATURE ANALYSIS USING INVERSE FUZZY NUMBER

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Abstract

Meteorological forecasting is one of the most underappreciated and challenging operational roles of worldwide meteorological agencies. Temperature is the most important factor in all seasonal processes in life for humans. Several approaches for predicting temperature distribution have been presented using fuzzy time series data, but accuracy remains a serious challenge. The goal of this study is to compare the performance of several fuzzy time series approaches. In this work, 42 years of temperature data from the Chennai region using the statistical tools Mean Squared Error (MSE) and Mean Absolute Error (MAE).

Keywords: Inverse Fuzzy Number, Forecasting, Temperature, Fuzzy Time Series.

2020 Mathematics Subject Classification (AMS): 62M10, 62A86, 90C70.

1. Introduction

Song and Chissom proposed a fuzzy time series in 1993. Forecasting is anticipating future events, in which decision-makers examine connected facts and graphs to make the best judgments for the future. The model specifies the collection of inaccurate data at equally spaced discrete time intervals, which are then characterized as fuzzy variables. The collection of discrete fuzzy data constitutes a fuzzy time series, and it also specifies that chronological sequences of imprecise data are called time series with fuzzy data.

A discrete domain is a set of input values that are either a finite or countable infinite set of numbers. Discrete domains are used to represent signals that are not continuous functions of

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a variable. Discrete domains can be used to represent data that is disconnected or separate. Discrete domains are used in probability distributions, such as Bernoulli, Poisson, Binomial, and Multinomial. A Fuzzy number A is a fuzzy set on the real line R, must satisfy the following conditions:

i) $\mu_A(x_0)$ is precewise continuous.
ii) There exists atleast one $x_0 \in R$ with $\mu_A(x_0) = 1$.
iii) A must be normal and convex.

Inverse fuzzy number is the inverse of fuzzy number. If A is a fuzzy number and if A * B = I, then B is the inverse of A, where * is any binary operator. A time series including fuzzy data is known as a fuzzy time series. Several fuzzy time series (FTS) models have been studied in the scientific literature for the past twenty years. This article proposes a forecasting model different from fuzzy time series forecasting approaches. Chennai district temperature statistics over the previous 42 years (1981–2022) demonstrate the suggested approach's forecasting procedure. Mean Squared Error (MSE) and Mean Absolute Error (MAE) are the two statistical criteria used to analyze the comparative data.

2. Methodology

The data used in this investigation is yearly temperature data, taken over 42 years for the Chennai district.

2.1 Construction of Fuzzy Time Series Model

Step 1: Let D be the discrete domain

$$D = E_i - E_{i-1}$$

Where, E_i are the historical data.

Step 2: Compute the inverse fuzzy number

$$v_\alpha = \frac{1+0.0001}{\frac{1}{d_\alpha}+\frac{0.0001}{d_{\alpha+1}}}$$

$$v_\alpha = \frac{0.0001+1+0.0001}{\frac{0.0001}{d_{\alpha-1}}+\frac{1}{d_\alpha}+\frac{0.0001}{d_{\alpha+1}}}, 1983 \leq \alpha \leq 2022$$

$$v_\alpha = \frac{0.0001+1}{\frac{0.0001}{d_{\alpha-1}}+\frac{1}{d_\alpha}}$$

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Step 3: Compute the forecasted value using formula

$$F_{\propto} = E_{\propto-1} - v_{\propto}$$

Where,

$F_{\propto}$, $E_{\propto-1}$ *and* $v_{\propto}$ are forecasting data, historical data and inverse fuzzy numbers respectively.

Step 4: The accuracy error of the fitted model is measured by using the following formula.

Mean Absolute Error (MAE) = $\sum_{i=1}^{n} \frac{|Forecasted\ value - Actual\ vale|}{n}$

Mean Squared Error (MSE) = $\sum_{i=1}^{n} \frac{(Forecasted\ value - Actual\ vale)^2}{n}$

3. Result and Discussion

3.1 Fuzzy Time Series Model

Step 1: The discrete domain D are calculated using formula $D = E_i - E_{i-1}$

Where, E_i are the historical data.

Discrete domain	Discrete domain
$d_{1982} = 0.17$	$d_{2003} = 0.11$
$d_{1983} = 0.49$	$d_{2004} = -0.74$
$d_{1984} = -0.58$	$d_{2005} = 0.41$
$d_{1985} = -0.08$	$d_{2006} = -0.08$
$d_{1986} = 0.49$	$d_{2007} = -0.17$
$d_{1987} = 0.2$	$d_{2008} = 0.02$
$d_{1988} = -0.21$	$d_{2009} = 0.58$
$d_{1989} = -0.1$	$d_{2010} = -0.4$
$d_{1990} = 0.13$	$d_{2011} = -0.1$
$d_{1991} = 0.17$	$d_{2012} = 0.25$
$d_{1992} - 0.19$	$d_{2013} = -0.19$
$d_{1993} = -0.17$	$d_{2014} = 0.19$
$d_{1994} = 0.07$	$d_{2015} = -0.11$
$d_{1995} = -0.52$	$d_{2016} = 0.24$

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$d_{1996} = 0.25$	$d_{2017} = 0.02$
$d_{1997} = 0.59$	$d_{2018} = -0.05$
$d_{1998} = 0.21$	$d_{2019} = 0.29$
$d_{1999} = -0.62$	$d_{2020} = -0.47$
$d_{2000} = 0.12$	$d_{2021} = -0.28$
$d_{2001} = 0.33$	$d_{2022} = 0.03$
$d_{2002} = -0.03$	

Table 1 Discrete Domain of Temperature

Step 2: We have computed the inverse fuzzy numbers using above mentioned formula

$$v_{1982} = \frac{1+0.0001}{\frac{1}{d_{1982}}+\frac{0.0001}{d_{1983}}}$$

$$v_{1982} = \frac{1+0.0001}{\frac{1}{0.17}+\frac{0.0001}{0.49}}$$

$$v_{1982} = \frac{1.0002}{5.88255702}$$

$$v_{1982} = 0.170011$$

$$v_{1983} = \frac{1.0002}{\frac{0.0001}{d_{1982}}+\frac{1}{d_{1983}}+\frac{0.0001}{d_{1984}}}$$

$$v_{1983} = \frac{1.0002}{\frac{0.0001}{0.17}+\frac{1}{0.49}+\frac{0.0001}{-0.58}}$$

$$v_{1983} = \frac{1.0002}{2.04123215}$$

$$v_{1983} = 0.489998$$

$$\dots\dots\dots$$

$$\dots\dots\dots$$

$$\dots\dots\dots$$

$$v_{2022} = \frac{1+0.0001}{\frac{0.0001}{d_{2021}}+\frac{1}{d_{2022}}}$$

$$v_{2022} = \frac{1.0002}{\frac{0.0001}{-0.28}+\frac{1}{0.03}}$$

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$$v_{2022} = \frac{1.0002}{33.3329762}$$

$$v_{2022} = 0.030003$$

Step 3: We have computed the forecasting value using the above-mentioned formula and the values are given in the table below 3.2.

Year	E_i	F_i	Year	E_i	F_i
1981	27.4	-	2002	28.12	28.11999
1982	27.57	27.57001	2003	28.23	28.23006
1983	28.06	28.06	2004	27.49	27.48928
1984	27.48	27.48024	2005	27.9	27.90032
1985	27.4	27.39998	2006	27.82	27.81999
1986	27.89	27.89028	2007	27.63	27.62983
1987	28.09	28.09005	2008	27.65	27.65
1988	27.88	27.87998	2009	28.23	28.22852
1989	27.78	27.77998	2010	27.83	27.83005
1990	27.91	27.91003	2011	27.73	27.72998
1991	28.08	28.08003	2012	27.98	27.98015
1992	27.89	27.88996	2013	27.79	27.78993
1993	27.72	27.71994	2014	27.98	27.98009
1994	27.79	27.79002	2015	27.87	27.86887
1995	27.27	27.2694	2016	28.11	28.10981
1996	27.52	27.52005	2017	28.13	28.13
1997	28.11	28.10981	2018	28.08	28.07998
1998	28.32	28.32004	2019	28.37	28.37024
1999	27.7	27.69937	2020	27.9	27.89991
2000	27.82	27.82002	2021	27.62	27.6197
2001	28.15	28.15034	2022	27.65	27.65

Table 2 Observed and Estimated Value of Temperature

Step 4:

Mean Absolute Error = 0.652543

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$$\text{Mean Squared Error} = 17.87524$$

4. Conclusion

This research aims to improve prediction accuracy by eliminating detected outliers from the dataset. In this paper, we calculated and compared the forecasted values of the Chennai district's temperature data using the forecasting models and fuzzy inverse numbers. The experimental findings indicate that the forecasting error for mean absolute error is 0.652543, and the mean squared error is 17.87524.

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RADIO CONTRA HARMONIC MEAN D-DISTANCE GRAPHS ON DUPLICATION PARAMETERS

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Abstract

The field of graph theory is vast and evolving quickly. Duplication is the process of generating a new graph in graph theory by appending vertices or edges to an already existing graph. In this research, we study certain duplication parameters in relation to the radio contra harmonic mean D-distance graph, including duplication of a vertex, duplicate graph and anti-duplication of vertex. The radio contra harmonic mean D-distance labeling process establishes several outcomes on the resulting graphs.

Keywords: D-distance, D-diameter, $rchm^D n(G)$, Duplication.
2020 Mathematics Subject Classification (AMS): 05C07, 05C12, 05C78.

1. Introduction

By a graph G we mean a finite undirected simple graph. The concept of radio contra harmonic mean D-distance of graphs was introduced by Ashika T S and Dr. Asha S and also they calculated its radio contra harmonic mean D-distance number [1]. Sampathkumar E introduced the notation Duplication of graphs [5]. Jayasekaran C, Ashwin Shijo M introduced the concept Anti-duplication of a vertex in graphs and investigated some properties of the resultant graphs [2]. Thulukkanam K, Vijayakumar P, and Thirusangu K studied some kinds of duplication parameters like extended duplication of graph in the paper titled Various Harmonious Labeling in Some Duplicate Graphs [6]. Throughout this paper for some basic graphs we referred Gallian [3].

2. Preliminaries

Definition 2.1. [4] Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$.

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Definition 2.2. [5] Let V' be a set such that $V \cap V' = \phi$, $|V| = |V'|$ and $f: V \to V'$ be bijective. For $a \in V$ we write $f(a)$ as a' for convenience. Consider the graph DG on the vertex set $V \cup V'$, whose edges are given as follows: In the graph G, ab is an edge if and only if both ab' and $a'b$ are edges in DG. The graph DG is called the duplicate of G.

Theorem 2.3. [5] For a connected graph G

- **(i)** DG is connected if and only if G contains an odd cycle.
- **(ii)** $DG = 2G$ iff G has no odd cycle.

Definition 2.4. [2] Anti duplication of a vertex v in G produces a new graph G' by adding a new vertex v' such that $N_{G'}(v') = [N_G[v]]^c$. The graph obtained from G by anti-duplication of the vertex v is denoted by $AD(vG)$.

Definition 2.5. [1] The Radio contra harmonic mean D-distance labeling of a connected graph G is an injective function $f: V(G) \to \mathbb{Z}^+$ such that for any two distinct vertices u, v

$$d^D(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq 1 + diam^D(G) \forall\, u, v \in V(G) \quad \text{……………. (1) or}$$

$$d^D(u,v) + \left\lfloor \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rfloor \geq 1 + diam^D(G)\ \forall\, u, v \in V(G)$$

where $d^D(u,v)$ denote D-distance or D-length between u and v of G and $diam^D(G)$ denote the D-diameter of G, then G is a radio contra harmonic mean D-distance graph. The radio contra harmonic mean D-distance number of f is represented as $rchm^D n(f)$ is the highest integer allocated to any vertex $v \in V(G)$ under the mapping f. Further, the radio contra harmonic mean D-distance number of G is presented as $rchm^D n(G)$, which is the smallest span of $rchm^D n(f)$ taken across every radio contra harmonic mean D-distance labeling of G. If $rchm^D n(G) = |V(G)|$ then G is called radio contra harmonic mean D-distance graceful graph. Further, if u and v are vertices of connected graph G, the D-length of a connected $u - v$ path s is defined as $d^D(u,v) = min\{l^D(s)\}$, $l^D(s) = l(s) + deg(v) + deg\,(u) + \Sigma\, deg(w)$, where the sum runs over all intermediate vertices w in s of G and $diam^D(G) = \max\{d^D(u,v)\}$.

3. Main Results

Theorem 3.1. Every connected graph is radio contra harmonic mean D-distance graph.

Proof. Let G be a connected graph. Then for every distinct vertices u and v there exist $u - v$ path. Therefore for every distinct pair of vertices (u, v) we can obtain D-distance as $d^D(u,v) =$

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$min\{l^D(s)\}$, $l^D(s) = l(s) + deg(v) + deg\,(u) + \Sigma\, deg(w)$, where degree of all intermediate vertices w in s of G are added together and $diam^D(G) = \max\{d^D(u,v)\}$. Thus G admits radio contra harmonic mean D-distance labeling.

Hence G is a contra harmonic mean D-distance graph.

Theorem 3.2. For any connected graph G with $|V(G)| \geq 2$ and $diam(G) < 3$ then $rchm^D n(G) = |V(G)|$ if any one of the following hold

(i) $G \cong K_{|V(G)|}$

(ii) G is an acyclic graph.

Proof. Let G be a connected graph with $|V(G)| \geq 2$ and $diam(G) < 3$.

To prove $rchm^D n(G) = |V(G)|$

Case (i): $G \cong K_{|V(G)|}$, then $diam(G) = 1$.

Let $|V(G)| = n$ such that G be a complete graph K_n with vertices $v_1, v_2, \ldots, v_n$ and $diam^D(K_n) = 2n - 1$.

For $n \geq 2$, $diam^D(K_n) = 2n - 1$. Therefore equation (1) reduces to

$$d^D(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq 2n \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Define a function $f: V(K_n) \to Z^+$ such that $f(v_i) = i, 1 \leq i \leq n$. Also for every distinct pair of vertices $(v_i, v_j), d^D(v_i, v_j) = 2n - 1$ for $1 \leq i,j \leq n, i \neq j$ such that $\left\lceil \frac{(i)^2+(j)^2}{i+j} \right\rceil \geq 1$.

Clearly f is a one to one mapping and every distinct pair of vertices and will hold (2) and the largest integer assigned to the vertex v_n is n. Therefore $rchm^D n(K_n) = n$.

Thus $rchm^D n(G) = |V(G)|$.

Case (ii): G is an acyclic graph

Without loss of generality let G be a graph with diameter 2 such that there exist a $u - v$ path such that $deg(u) = deg(v) = 1$ and $d(u,v) = 2$. Also the intermediate vertex of u and v is denoted by w. Define a function $f: V(G) \to Z^+$ such that $f(u) = 1$, $f(v) = 2$ and $f(w) = 3$.

Subcase (i): $deg(w) = 2$

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Then $d^D(u,v) = 6$ and also which is the $diam^D(G)$ and RHS of inequality (1) reduces to 7. Therefore for the pair (u,v) we get $6 + \left\lceil \frac{(1)^2+(2)^2}{1+2} \right\rceil \geq 7$ is obvious. Also for the pair (u,w) we get $4 + \left\lceil \frac{(1)^2+(3)^2}{1+3} \right\rceil \geq 7$ and similarly for the pair (v,w), we get $4 + \left\lceil \frac{(2)^2+(3)^2}{2+3} \right\rceil \geq 7$ which satisfies the radio contra harmonic mean D-distance condition and $3 = |V(G)|$ is the largest label assigned to the vertex w. Hence $rchm^D n(G) = |V(G)|$.

Subcase (ii): $deg(w) \geq 2$

Then w have adjacency with some vertices $v_i, 1 \leq i \leq n$(say) other than u and v and which are disjoint with u and v since G is acyclic and $diam(G) = 2$. Thus $deg(w) = n + 2$ and $\{u, v, v_i : 1 \leq i \leq n\}$ are the vertices with degree one. Now assign $f(v_i) = 3 + i, 1 \leq i \leq n$ under the defined mapping f and $diam^D(G) = n + 6$, RHS of inequality (1) reduces to $n + 7$.

Therefore for the pair (w,x), where x may be either u or v or $v_i : 1 \leq i \leq n$, then we get $n + 4 + \left\lceil \frac{(3)^2+(x)^2}{3+x} \right\rceil \geq n + 7$ is obvious. Also for the pair (x,y), where x may be either u or v or $v_i : 1 \leq i \leq n$, $x \neq y$ then we get $n + 6 + \left\lceil \frac{(x)^2+(y)^2}{x+y} \right\rceil \geq n + 7$ will hold. Here $n + 3 = |V(G)|$ is the largest label assigned to the vertex v_n. Hence the proof.

Theorem 3.3. If $rchm^D n(G) = |V(G)|$ then $rchm^D n(G) < rchm^D n(G')$.

Proof. Assume that $rchm^D n(G) = |V(G)|$.

Let $f: V(G) \to \mathbb{Z}^+$ be the function defined at which the graph G attains its least upper bound with respect to radio contra harmonic mean D-distance labeling. Let $v \in V$ be the vertex which is to be duplicated and v' is the resultant vertex obtained by duplicating the vertex v. Now $|V(G')| = |V(G)| + 1$.

Therefore $rchm^D n(G') \geq |V(G')| = |V(G)| + 1$

$$> |V(G)|$$

$$= rchm^D n(G)$$

Hence $rchm^D n(G) < rchm^D n(G')$

Corollary 3.4. The converse of the above theorem is not true.

Result 3.5. If $rchm^D n(G) = |V(G)|$ then

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(i) $rchm^D n(G) < rchm^D n(ED(G))$

(ii) $rchm^D n(G) < rchm^D n(DG)$

Theorem 3.6. If G is a $n-1$ regular graph with n vertices then $rchm^D n(G') = 2rchm^D n(G) - 1$.

Proof. Let G be a $n-1$ regular graph with $|V(G)| = n$ and $V(G) = v_1, v_2, \dots, v_n$. Let v_1' be a vertex which exist due to the duplication of any vertex $v_1(say)$ and G' be the resultant graph. Then $diam(G') = 3n$. Therefore equation (1) reduces to

$$d^D(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq 3n+1 \text{…………………} (3)$$

Define a function $f: V(G') \to Z^+$ such that $f(v_1') = n-1$ and $f(v_i) = n-1+i, 1 \leq i \leq n$.

For the pair of vertices $(v_i, v_j), d^D(v_i, v_j) \geq 2n$ for $1 \leq i,j \leq n, i \neq j$, $\left\lceil \frac{(n-1+i)^2+(n-1+j)^2}{2n-2+i+j} \right\rceil \geq 3n+1-d^D(v_i,v_j)$.

For the pair of vertices $(v_i, v_i'), d^D(v_n, v_n') = 2n$ for $1 \leq i \leq n$, $\left\lceil \frac{(n-1+i)^2+(n-1)^2}{2n-2+i} \right\rceil \geq n+1$.

Clearly f is a one to one mapping and every distinct pair of vertices and will hold (3) and the largest integer assigned is $2n-1$ to the vertex v_n and $rchm^D n(G') = 2n-1$. By Theorem 2.2, $rchm^D n(G) = n$. Hence $rchm^D n(G') = 2rchm^D n(G) - 1$.

Theorem 3.7. Let G be a connected graph with n vertices. If $v \in V(G)$ then there exist $rchm^D n(AD(vG))$ if and only if $\deg(v) < n-1$.

Proof. Let $|V(G)| = n$.

Assume that $\deg(v) < n-1$. To prove $rchm^D n(AD(vG))$ exist.

Let $v \in V$ be the vertex which is chosen for anti-duplication in a graph G and v' is the resultant vertex occurred by anti-duplicating the vertex v and let the graph be $AD(vG)$. By our assumption, $1 \leq \deg(v) \leq n-2$. Since $|V(G)| = n$, there is atleast one vertex which is not adjacent to v in G. Then by the definition of anti-duplication of vertex, the graph still remains connected.

By theorem 4.1, $rchm^D n(AD(vG))$ exist.

Conversely, assume that $rchm^D n(AD(vG))$ exist. To prove $\deg(v) < n-1$.

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Suppose that $\deg(v) = n - 1$. Then v is adjacent to every vertices in the graph G. By anti-duplication of vertex v the resultant vertex is non adjacent to any vertex of G. Thus v' is an isolated vertex. Then $AD(vG)$ is a graph with two components. Therefore, $AD(vG)$ is an disconnected graph and hence $rchm^D n(AD(vG))$ will not exist which is contradiction to our assumption.

Hence $\deg(v) < n - 1$.

Theorem 3.8. If G is a graph with $n + 1$ vertices and n pendant edges then $rchm^D n(AD(v_n G)) = rchm^D n(G) + n$.

Proof. Let G be a graph with $n + 1$ vertices and n pendant edges. Since there are n pendant edges, there must be n pendant vertices $v_1, v_2, \dots, v_n$ and let the vertex with degree n be u. Let v_n' be the new vertex by anti-duplication of the vertex v_n of G and the resultant graph is $AD(v_n G)$ with $diam(AD(v_n G)) = 2n + 5$. Therefore equation (1) reduces to

$$d^D(u, v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u) + f(v)} \right\rceil \geq 2n + 6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Define a function $f: V(AD(v_n G)) \to Z^+$ such that $f(u) = 2n$, $f(v_n') = 2n + 1$ and $f(v_i) = n - 1 + i, 1 \leq i \leq n$.

For the pair of vertices $(v_i, v_j), d^D(v_i, v_j) \geq n + 5$ for $1 \leq i, j \leq n, i \neq j$, $\left\lceil \frac{(n-1+i)^2 + (n-1+j)^2}{2n-2+i+j} \right\rceil \geq 2n + 6 - d^D(v_i, v_j)$.

For the pair of vertices $(u, v_i), d^D(u, v_i) \geq n + 2$ for $1 \leq i, j \leq n, i \neq j$, $\left\lceil \frac{(2n)^2 + (n-1+i)^2}{3n-1+i} \right\rceil \geq 2n + 6 - d^D(v, v_i)$.

For the pair of vertices $(v_i, v_n'), d^D(v_i, v_n') \geq n + 2$ for $1 \leq i, j \leq n$, $\left\lceil \frac{(n-1+i)^2 + (2n+1)^2}{3n+i} \right\rceil \geq 2n + 6 - d^D(v_i, v_n')$.

For the pair of vertices $(u, v_n'), d^D(u, v_n') = 2n + 3$, $\left\lceil \frac{(2n)^2 + (2n+1)^2}{4n+1} \right\rceil \geq 3$.

Clearly f is a one to one mapping and every distinct pair of vertices will hold (4) and the largest integer assigned is $2n + 1$ to the vertex v_n' and hence $rchm^D n(AD(v_n G)) = 2n + 1$. By theorem 3.2, $rchm^D n(G) = n + 1$.

Hence $rchm^D n(AD(v_n G)) = rchm^D n(G) + n$.

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Theorem 3.9. For a connected graph G, $rchm^D n(DG)$ exist if and only if G contains an odd cycle.

Proof. Let G be a connected graph.

Assume that $rchm^D n(DG)$ exist. To prove G contains an odd cycle.

Suppose G contains no odd cycle then by theorem 2.3 (ii), $DG = 2G$. Thus DG is a disconnected graph with two components which contradicts the existence of $rchm^D n(DG)$.

Hence G contains an odd cycle.

Conversely, assume that G contains an odd cycle.

To prove the existence of $rchm^D n(DG)$.

Since G contains an odd cycle, by theorem 2.3(i), DG is connected. By theorem 3.1, DG is a radio contra harmonic mean D-distance graph.

Hence, $rchm^D n(DG)$ will exist.

4. Conclusion

In this study, we investigated the labeling of graphs with radio contra harmonic mean D-distance under different duplication parameters. The resulting graph's radio contra harmonic mean D-distance number is computed based on the criteria. These kinds of outcomes can be extended to triplicate parameters and applied to other radio mean labeling parameters.

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CONNECTED DOMINATION POLYNOMIAL OF ZERO-DIVISOR GRAPHS OF COMMUTATIVE RINGS

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Abstract

The connected domination polynomial of a graph G of order n is the generating function of the number of connected dominating sets of G of any size. Let $D_c(G,i)$ be the family of connected dominating sets of a graph G with cardinality i and Let $d_c(G,i) = |D_c(G,i)|$. Then the connected domination polynomial $D_c(G,x)$ of G is defined as $D_c(G,x) = \sum_{i=\gamma_c(G)}^{|V(G)|} D_c(G,i)x^i$, where $\gamma_c(G)$ is the connected domination number of G. In this paper, we study the connected domination polynomials of zero-divisor graphs of ring $\mathbb{Z}_n$, where $n \in \{2p, p^2, pq\}$ for distinct prime numbers p and q, and $p > q > 2$.

Keywords: Connected Domination Polynomial, Connected Domination set, Zero-divisor graph.

2020 Mathematics Subject Classification (AMS): 05C69, 05C25.

1. Introduction

The domination polynomial of a graph is introduced by Saeid Alikhani and Yee-hock Peng in the year 2009 [7]. While extending the concept of domination polynomial in view of connected dominating set, we came across with many interesting relations among the connected domination polynomials of different graphs, which is defined by Sampathkumar and H.B Wlikar in the year 1979 [8].

Let $G = (V,E)$ be a simple graph. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V : uv \in E\}$ and the closed neighbourhood of v is the set

$N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \cup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ is a dominating set of G, if

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$N[S] = V$, or equivalently every vertex in $V \backslash S$ is adjacent to atleast one vertex in S. The domination number $\Upsilon(G)$ is the minimum cardinality of a dominating set in G [7].

Let G be a simple connected graph of order n. A connected domination set (cd-set) of G is a set S of vertices of G such that every vertex in $V \backslash S$ is adjacent to some vertex in S and the induced subgraph $< S >$ is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set in G [8].

Zero-divisor graph of a commutative ring was introduced in the work of Beck. Beck was interested in colouring of rings and the vertex set of graph consists of all elements of the ring in his definition. Later, the definition of zero-divisor graph of a commutative ring has been modified by Anderson and Livingston [4]. They defined the zero-divisor graph of a commutative ring on nonzero zero-divisor elements of the ring [2].

In recent years, the study of zero-divisor graphs has grown in various directions. Actually, it is the interplay between the ring theoretic properties of a ring R and the graph theoretic properties of its zero-divisor graph [1,2]. There are many papers which studied some parameters and topological indices of the zero-divisor graphs. Recently, Gursoy, Ulker and Gursoy in [6] have studied the independent domination polynomials of some zero-divisor graphs of the rings.

This paper consists of four sections. In Section 2, we give some notions. In section 3, we collect the basic definitions that are needed for the subsequent sections. In section 4, we investigate the connected domination polynomial of some zero-divisor graphs of the rings $\mathbb{Z}_n$ for $p > q > 2$ where p, q are distinct prime numbers.

2. Notation

- $D_c(G, x)$: Connected domination polynomial of a graph G
- $d_c(G, i)$: Number of connected dominating sets of G of cardinality i
- $\gamma_c(G)$: Connected domination number of G
- $N[v]$: Closed neighbourhood of the vertex v of a graph G
- $N(v)$: Open neighbourhood of the vertex v of a graph G
- $D(G, x)$: Domination polynomial of a graph G
- cd-set : Connected dominating set
- cd- polynomial: Connected domination polynomial

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3. Preliminaries

Definition 3.1. [8] Let $D_c(G,i)$ be the family of connected dominating sets of a graph G with cardinality i and Let $d_c(G,i) = |D_c(G,i)|$. Then the connected domination polynomial $D_c(G,x)$ of G is defined as $D_c(G,x) = \sum_{i=\gamma_c(G)}^{|V(G)|} d_c(G,i)x^i$, where $\gamma_c(G)$ is the connected domination number of G.

Definition 3.2. [2] Let $\mathbb{Z}_n$ be the ring of integers modulo n. The zero-divisor graph $\Gamma(\mathbb{Z}_n)$ is the simple undirected graph without loops which has its vertex set coincides with the nonzero zero-divisors of $\mathbb{Z}_n$ and two distinct vertices u and v in $\Gamma(\mathbb{Z}_n)$ are adjacent whenever $uv = 0$ in $\mathbb{Z}_n$.

Example 3.3. [6] For the graph $\Gamma(\mathbb{Z}_{75})$, we have $\left|V\left(\Gamma(\mathbb{Z}_{75})\right)\right| = 34$ and $\left|E\left(\Gamma(\mathbb{Z}_{75})\right)\right| = 86$.

An integer d is called a proper divisor of n if $1 < d < n$ and $d|n$. Let $d_1, \dots, d_k$ be the distinct proper divisors of n. For $1 \le i \le k$, consider the following sets:

$$V_{d_i} = \{x \epsilon \mathbb{Z}_n : \gcd(x,n) = d_i\}.$$

The sets $V_{d_1}, \dots, V_{d_k}$ are pairwise disjoint and we can partition the vertex set of $\Gamma(\mathbb{Z}_n)$

$$V\big(\Gamma(\mathbb{Z}_n)\big) = \bigcup_{i=1}^{k} V_{d_i}.$$

The following lemma gives the cardinalities of each vertex subset of $\Gamma(\mathbb{Z}_n)$.

Lemma 3.4. [2] Let n be a positive integer with distinct divisors $d_1, d_2, \dots, d_r$. If $V_{d_i} = \{x \epsilon \mathbb{Z}_n : \gcd(x,n) = d_i\}$ for $= 1,2, \dots, r$, then $|V_{d_i}| = \phi(n/d_i)$ where ϕ is the Euler's Totient function.

Lemma 3.5. [5] For $i,j \in \{1, \dots, k\}$, a vertex of V_{d_i} is adjacent to a vertex of V_{d_j} in $\Gamma(\mathbb{Z}_n)$ if and only if n divides $d_i d_j$.

Corollary 3.6. [5]

i. For $i \in \{1, \dots, k\}$, the induced subgraph $\Gamma(V_{d_i})$ of $\Gamma(\mathbb{Z}_n)$ on the vertex set V_{d_i} is either the complete graph $K_{\phi(n/d_i)}$ or its complement graph $\overline{K}_{\phi(n/d_i)}$. Indeed, $\Gamma(V_{d_i})$ is $K_{\phi(n/d_i)}$ if and only if n divides d_i^2.

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ii. For $i, j \in \{1, \dots, k\}$, with $i \neq j$ a vertex of V_{d_i} is adjacent to either all or none of the vertices of V_{d_j} in $\Gamma(\mathbb{Z}_n)$.

4. Main results

In this section, we study connected domination polynomial of zero-divisor graphs of rings $\mathbb{Z}_n$, where $n \in \{2p, p^2, pq\}$ for distinct prime numbers p and q.

Theorem 4.1. For prime p, $D_c(\Gamma(\mathbb{Z}_{p^2}), x) = (1 + x)^{\phi(p)} - 1$

Proof. Given p is a prime number, then integer p is only proper divisor of p^2. By corollary 3.6 $\Gamma(\mathbb{Z}_{p^2})$ is the complete graph $K_{\phi(p)}$, where ϕ is the Euler's totient function. Let K_n be the complete graph on n vertices. $D(K_n, x) = (1 + x)^n$-1, Also $D_c(K_n, x) = (1 + x)^n$-1 and $\phi = p - 1$, so

$$D_c(\Gamma(\mathbb{Z}_{p^2}), x) = \binom{p-1}{p-1} x^{p-1} + \binom{p-1}{p-2} x^{p-2} + \binom{p-1}{p-3} x^{p-3} + \cdots + \binom{p-1}{2} x^2 + \binom{p-1}{1} x$$

$$= \sum_{i=1}^{p-1} \binom{p-1}{i} x^i$$

$$= (1 + x)^{\phi(p)} - 1$$

Hence the result.

Theorem 4.2. For prime p, $D_c(\Gamma(\mathbb{Z}_{2p}), x) = x(1 + x)^{\phi(p)}$

Proof. Given p is a prime number. The vertex set of the graph can be partitioned into two distinct subsets as

$$V_p = \{p\}$$

$$V_2 = \{2x : x = 1, \dots, p - 1\}$$

Since the integers 2 and p are the proper divisors of 2p. By corollary 3.6, $\Gamma(\mathbb{Z}_{2p})$ is the star graph $K_{1,\phi(p)}$.

$$D_c(\Gamma(\mathbb{Z}_{2p}), x) = \binom{p-1}{p-1} x^p + \binom{p-1}{p-2} x^{p-1} + \binom{p-1}{p-3} x^{p-2} + \cdots + \binom{p-1}{2} x^3 + \binom{p-1}{1} x^2 + \binom{p-1}{0} x$$

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$$=x\left\{\binom{p-1}{p-1}x^{p-1}+\binom{p-1}{p-2}x^{p-2}+\binom{p-1}{p-3}x^{p-3}+\cdots+\binom{p-1}{2}x^{2}+\binom{p-1}{1}x+1\right\}$$

$$=x\sum_{i=0}^{p-1}\binom{p-1}{i}x^{i}$$

$$=x(1+x)^{\phi(p)}$$

Hence the result.

Theorem 4.3. If $p>q>2$ are prime numbers, then, $D_c(\Gamma(\mathbb{Z}_{pq}),x)=[(1+x)^{p-1}-1]\ [(1+x)^{q-1}-1]$

Proof. Given $p>q>2$ **,** where p and q are prime numbers. The vertex set of the graph can be partitioned into distinct subsets as,

$V_p=\{px: x=1,\dots,q-1\}$ and

$V_q=\{qx: x=1,\dots,p-1\}$

Since the integers p and q are the proper divisors of pq. Consequently, by corollary 3.6, $\Gamma(\mathbb{Z}_{pq})$ is the complete bipartite graph $K_{p-1,q-1}$.

since $\phi(p)>\phi(q)>1$ so $\gamma_c\left(\Gamma(\mathbb{Z}_{pq})\right)=2$.

Any dominating set of size i is a connected dominating set if there are connected between some of the vertices of the V_p and V_q subsets.

Dominating sets that are not related are the subsets V_p and V_q, which have cardinality $q-1$ and $p-1$, respectively

For $i\geq 2$,

$$d_c(\Gamma(\mathbb{Z}_{pq}),j)=\begin{cases} d(\Gamma(\mathbb{Z}_{pq}),j)-1 & if\ j=q-1\ or\ p-1 \\ d(\Gamma(\mathbb{Z}_{pq}),j) & otherwise\end{cases}$$

Since, $D_c(K_{m,n},x)=[(1+x)^m-1][(1+x)^n-1]$ and $\phi(p)=p-1,\phi(q)=q-1$

$$D_c(\Gamma(\mathbb{Z}_{pq}),x)=\binom{p-1}{1}\binom{q-1}{1}x^2+\binom{p-1}{1}\binom{q-1}{2}x^3+\binom{p-1}{2}\binom{q-1}{1}x^3+\cdots+\left[\binom{p-1}{1}\binom{q-1}{p+q-3}+\cdots+\binom{p-1}{p+q-3}\binom{q-1}{1}\right]x^{p+q-3}$$

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$= \binom{p-1}{1}\binom{q-1}{1}x^2 + \binom{p-1}{1}\binom{q-1}{2}x^3 + \binom{p-1}{2}\binom{q-1}{1}x^3 + \cdots + [\binom{p-1}{1}\binom{q-1}{p-1+q-1-1} + \cdots + \binom{p-1}{p-1+q-1-1}\binom{q-1}{1}]x^{p-1+q-1-1}$

$= [\binom{p-1}{1}x + \binom{p-1}{2}x^2 + \cdots + \binom{p-1}{p-1}x^{p-1}]$

$[\binom{q-1}{1}x + \binom{q-1}{2}x^2 + \cdots + \binom{q-1}{q-1}x^{q-1}]$

$= \sum_{j=0}^{p-1}\binom{p-1}{j}x^j \sum_{j=0}^{q-1}\binom{q-1}{j}x^j$

$= [(1+x)^{p-1} - 1][(1+x)^{q-1} - 1]$

Hence the result.

5. Conclusion

In this paper, the connected domination polynomials of the zero-divisor graphs of rings $\mathbb{Z}_n$, where $n \in \{2p, p^2, pq\}$ for distinct prime numbers p and q has been derived by identifying its connected dominating sets. Further we can generalize connected domination polynomial of zero-divisor graphs of rings $\mathbb{Z}_n$, where $n \in \{p^2q, pqr, p^\alpha\}$ for distinct prime numbers p, q and r.

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THE CHROMATIC RESTRAINED DOMINATION NUMBER ON DIRECT PRODUCT OF GRAPHS

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Abstract

Let $G = (V, E)$ be a graph. A subset D of $V(G)$ is said to be a chromatic restrained dominating set (or crd-set) of G if D is a restrained dominating set of G and $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal chromatic restrained dominating sets is called the chromatic restrained domination number of G and is denoted by $\gamma_r^c(G)$. In this paper, the chromatic restrained domination number on the direct product of certain standard graphs were obtained.

Keywords: Domination, Restrained Domination, Chromatic Number, Direct Product

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1. Introduction

All the graphs $G = (V, E) = (n, m)$ considered here are simple, finite and undirected, with neither loops nor multiple edges. For $D \subseteq V$, the subgraph induced by D is denoted by $\langle D \rangle$. A $k-$vertex coloring of a graph, or simply a $k-$coloring, is an assignment of $k-$colors to its vertices. The coloring is proper if no two adjacent vertices are assigned the same color. A coloring in which $k-$ colors are used is a $k-$coloring. A graph is $k-$colorable if it has a proper $k-$coloring. The minimum k for which a graph G is $k-$colorable is called its chromatic number and denoted by $\chi(G)$. Graph Theory terminologies which are not defined here can be seen in [2] and [7].

A set $D \subseteq V$ of vertices in a graph G is called a dominating set if every vertex $u \in V$ is either an element of D or is adjacent to an element of D. The minimum cardinality taken over all minimal dominating sets is called the domination number of G and is denoted by $\gamma(G)$. A set $D \subseteq V$ is a restrained dominating set if every vertex in $V - D$ is adjacent to a vertex in D

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and another vertex in $V - D$ [4]. The minimal cardinality taken over all minimal restrained dominating sets is called the restrained domination number of G and is denoted by $\gamma_r(G)$. A set D is a γ_r −set if D is a restrained dominating set of cardinality $\gamma_r(G)$.

For graphs G and H, the direct product $G \times H$ (also known as the tensor product, cross product, cardinal product, kronecker product) is the graph with vertex set $V(G) \times V(H)$ where two vertices (x, y) and (v, w) are adjacent if and only if $xv \in E(G)$ and $yw \in E(H)$ [3].

A set $D \subseteq V$ is a chromatic preserving set or a cp-set if $\chi(< D >) = \chi(G)$ and the minimum cardinality taken over all cp-set in G is called the chromatic preserving number or cp-number of G, denoted by $\text{cpn}(G)$ [5]. A subset D of V is said to be a dom-chromatic set (or dc-set) if D is a dominating set and $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal dom-chromatic sets in G is called the dom-chromatic number and is denoted by $\gamma_{ch}(G)$ [6]. In this paper, the chromatic restrained domination number on the direct product of some standard graphs are obtained.

2. Main Results

In this section, we obtained the chromatic restrained domination number for the direct product of some standard graphs.

Definition 2.1. Let $G = (V, E)$ be a graph. A subset D of V is said to be a chromatic restrained dominating set (or crd-set) if D is a restrained dominating set and $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal chromatic restrained dominating sets is called **chromatic restrained domination number** and is denoted by γ_r^c (G).

Observation 2.2. $\gamma_r^c(K_2 \times K_n) = \begin{cases} 4 & if \;\; n = 2,3 \\ 3 & otherwise \end{cases}$.

Theorem 2.3. For any $m, n \geq 3$, $\gamma_r^c(K_m \times K_n) = min\{m, n\}$.

Proof. Let $V(K_m) = \{u_1, u_2, u_3, \ldots, u_m\}$ and $V(K_n) = \{v_1, v_2, v_3, \ldots, v_n\}$ with $|V(K_m)| = m$ and $|V(K_n)| = n$. Then $V(K_m \times K_n) = \{(u_i, v_j)/1 \leq i \leq m, 1 \leq j \leq n\}$ with cardinality mn. Each vertex in $K_m \times K_n$, say (u_1, v_1) is adjacent to all the vertices except $(u_1, v_j), 2 \leq j \leq n$ and $(u_i, v_1), 2 \leq i \leq m$ and similar adjacency holds for every vertex of $K_m \times K_n$. Let $m \leq n$. Then, for any $1 \leq j \leq n, (u_1, v_j)$ can be colored with color 1, (u_2, v_j) can be colored with color 2,..., and (u_m, v_j) can be colored with color m. Thus, $\chi(K_m \times K_n) = m$. Let $D = \{(u_1, v_1), (u_2, v_2), (u_3, v_3), \ldots, (u_m, v_m)\}$. Clearly, D is a restrained dominating set of $K_m \times$

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K_n. Since, each vertex of D is adjacent to all the other vertices of D, $\langle D \rangle$ forms a complete graph on m vertices and $\chi(\langle D \rangle) = m = \chi(K_m \times K_n)$. This implies that, D is a chromatic restrained dominating set of $K_m \times K_n$ with cardinality m. Therefore, $\gamma_r^c(K_m \times K_n) \leq m$. Since $\chi(K_m \times K_n) = m$, any chromatic restrained dominating set of $K_m \times K_n$ must contain a minimum of m vertices and so, $\gamma_r^c(K_m \times K_n) \geq m$. Therefore, $\gamma_r^c(K_m \times K_n) = m$. Similarly for $n \leq m, \gamma_r^c(K_m \times K_n) = n$. Thus, $\gamma_r^c(K_m \times K_n) = min\{m, n\}$.

Observation 2.4. $\gamma_r^c(K_m \times K_{1,n}) = \begin{cases} 2(n+1) & if \;\; m = 2 \\ n+2 & if \;\; m = 3 \end{cases}$.

Theorem 2.5. For $m \geq 4$, $\gamma_r^c(K_m \times K_{1,n}) = 3$.

Proof. Let $V(K_m) = \{u_1, u_2, u_3, \ldots, u_m\}$ and $V(K_{1,n}) = \{v_0, v_1, v_2, \ldots, v_n\}$ where v_0 is the full degree vertex of $K_{1,n}$. Then, $V(K_m \times K_{1,n}) = \{(u_i, v_j)/1 \leq i \leq m, 0 \leq j \leq n\}$ with cardinality $(n+1)m$. *T*he vertex set of $K_m \times K_{1,n}$ can be partitioned into two partite sets V_1, V_2 where $V_1 = \{(u_i, v_0)/1 \leq i \leq m\}$ and $V_2 = V(K_m \times K_{1,n}) \backslash V_1$. Also, no two vertices of V_1 and no two vertices of V_2 are adjacent. Thus, $\chi(K_m \times K_{1,n}) = 2$. Furthermore, every vertex (u_i, v_0) in V_1 is adjacent to all the vertices of V_2 except (u_i, v_j),$1 \leq i \leq m, 1 \leq j \leq n$. Also, each vertex (u_i, v_j),$1 \leq i \leq m, 1 \leq j \leq n$ of V_2 is adjacent to all the vertices of V_1 except the vertex with same i. Let $D = \{(u_1, v_0), (u_2, v_0), (u_1, v_1)\}$. Then D is a restrained dominating set of $K_m \times K_{1,n}$ and $\gamma_r(K_m \times K_{1,n}) \leq |D| = 3$. Since $K_m \times K_{1,n}$ is a bipartite graph, every restrained dominating set must contain a vertex from V_1 and another vertex from V_2. Let $x \in V_1$, $y \in V_2$ and $xy \notin E(K_m \times K_{1,n})$. Then $x = (u_1, v_0)$ and $y = (u_1, v_j)$ for any $j \neq 0$, and (u_1, v_j) for remaining $j's$ is not dominated by x and y. Thus, choosing another vertex from V_1 dominates all the remaining vertices which are not dominated. Suppose $xy \in E(K_m \times K_{1,n})$. Then, $x = (u_1, v_0)$ and $y = (u_2, v_1)$ dominates all the vertices except $(u_2, v_0) \in V_1$ and $(u_1, v_j) \in V_2$. Thus x, y and (u_2, v_0) forms a restrained dominating set. From both the cases, $\gamma_r(K_m \times K_{1,n}) \geq 3$. Therefore, $\gamma_r(K_m \times K_{1,n}) = 3$. Since $(u_2, v_0)(u_1, v_1) \in E(K_m \times K_{1,n})$, $\chi(\langle D \rangle) = 2 = \chi(K_m \times K_{1,n})$. Therefore, D is a chromatic restrained dominating set of $K_m \times K_{1,n}$ and $\gamma_r^c(K_m \times K_{1,n}) = |D| = 3$.

Observation 2.6. For any $n \geq 3$, $\gamma_r^c(K_3 \times P_n) = \begin{cases} \frac{2n}{3} + 2 & if \; n \equiv 0 (mod\; 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor + 2 & if \; n \equiv 1 (mod\; 3) \\ 2\left\lceil \frac{n}{3} \right\rceil + 1 & if \; n \equiv 2 (mod\; 3) \end{cases}$.

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Theorem 2.7. For any $m > 3$ and $n \geq 2$, $\gamma_r^c(K_m \times P_n) = \begin{cases} \frac{2n}{3} + 1 & if\ n \equiv 0(mod\ 3) \\ 2\left\lfloor\frac{n}{3}\right\rfloor + 2 & if\ n \equiv 1(mod\ 3) \\ 2\left\lceil\frac{n}{3}\right\rceil + 1 & if\ n \equiv 2(mod\ 3) \end{cases}$

Proof. Let $V(K_m) = \{u_i/1 \leq i \leq m\}$ and $V(P_n) = \{v_j/1 \leq j \leq n\}$. Then $V(K_m \times P_n) = \{(u_i, v_j)/1 \leq i \leq m, 1 \leq j \leq n\}$ and $|V(K_m \times P_n)| = mn$. Now, the vertex set can be divided into two subsets V_1, V_2 where $V_1 = \{(u_i, v_j)/1 \leq i \leq m, 1 \leq j \leq n\ and\ \ j\ \ is\ \ odd\ \}$ and $V_2 = \{(u_i, v_j)/1 \leq i \leq m, 2 \leq j \leq n\ and\ \ j\ \ is\ \ even\ \}$. Also, $V_1 \cap V_2 = \Phi$. Furthermore, no two vertices of V_1 and no two vertices of V_2 are adjacent. Then, $\chi(\langle V_1 \rangle) = 1$ and $\chi(\langle V_2 \rangle) = 1$. This implies that, $\chi(K_m \times P_n) = 2$. Moreover, $\{(u_1, v_j)/1 \leq j \leq n\}$ represents the first row of $K_m \times P_n$ and $\{(u_i, v_1)/1 \leq i \leq m\}$ represents the first column of $K_m \times P_n$. Likewise, $K_m \times P_n$ contains m rows and n columns. Now, let us consider the following three cases.

Case (i): $n \equiv 0(mod\ 3)$

Let $D_1 = \{(u_1, v_1), (u_1, v_2), (u_2, v_4), (u_2, v_5), (u_1, v_7), (u_1, v_8), \dots,\ (u_k, v_{n-2}),\ (u_k, v_{n-1}), (u_l, v_{n-1})\}$ where $k = 1$, $l = 2$ if n is odd and $k = 2$, $l = 1$ if n is even. Then, D_1 is a dominating set of $K_m \times P_n$, since the vertices of $V(K_m \times P_n)\backslash D_1$ which belongs to the first row are dominated by a vertex of D_1 belonging to second row, the vertices of $V(K_m \times P_n)\backslash D_1$ which belongs to the second row are adjacent to a vertex of D_1 belonging to the first row and all the vertices in the remaining rows are adjacent to some vertex of D_1. Since $\langle V(K_m \times P_n) - D_1 \rangle$ has no isolated vertices, D_1 is a restrained dominating set of $K_m \times P_n$ and $\gamma_r(K_m \times P_n) \leq |D_1| = \frac{2n}{3} + 1$. Since $\gamma(K_m \times P_n) = \frac{2n}{3} + 1$, $\gamma_r(K_m \times P_n) \geq \frac{2n}{3} + 1$. Therefore, $\gamma_r(K_m \times P_n) = \frac{2n}{3} + 1$. Since at least two vertices of any minimum restrained dominating set of $K_m \times P_n$ is adjacent, $\chi(\langle D_1 \rangle) = 2 = \chi(K_m \times P_n)$. Therefore, D_1 is a chromatic restrained dominating set of $K_m \times P_n$ and $\gamma_r^c(K_m \times P_n) = |D_1| = \frac{2n}{3} + 1$.

Case (ii): $n \equiv 1(mod\ 3)$

Let $D_2 = \{(u_1, v_1),\ (u_1, v_2),\ (u_2, v_4),\ (u_2, v_5),\ (u_1, v_7),\ (u_1, v_8), \dots, (u_k, v_{n-3}),\ (u_k, v_{n-2}), (u_l, v_{n-1}), (u_l, v_n)\}$ where $k = 1, l = 2$ if n is even and $k = 2, l = 1$ if n is odd. Then, D_2 is a dominating set, since every vertex in $V(K_m \times P_n)\backslash D_2$ is adjacent to at least one vertex of D_2. Also, every vertex of $V(K_m \times P_n)\backslash D_2$ is adjacent to at least one another vertex of $V(K_m \times P_n)\backslash D_2$ and so D_2 is a restrained dominating set. Thus, $\gamma_r(K_m \times P_n) \leq |D_2| = 2\left\lfloor\frac{n}{3}\right\rfloor +$

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2. Since there exists no restrained dominating set with cardinality less than $2\left\lfloor\frac{n}{3}\right\rfloor+2$, $\gamma_r(K_m \times P_n) = 2\left\lfloor\frac{n}{3}\right\rfloor+2$. Also, $(u_k, v_{n-2})(u_l, v_{n-1}) \in E(K_m \times P_n)$, $\chi(\langle D_2\rangle) = 2 = \chi(K_m \times P_n)$. Therefore, D_2 is a chromatic restrained dominating set of $K_m \times P_n$ and $\gamma_r^c(K_m \times P_n) = |D_2| = 2\left\lfloor\frac{n}{3}\right\rfloor+2$.

Case (iii): $n \equiv 2(mod\ 3)$

Let $D_3 = \{(u_1, v_1), (u_1, v_2), (u_2, v_4), (u_2, v_5), (u_1, v_7), (u_1, v_8), \ldots, (u_k, v_{n-1}), (u_k, v_n)\}$ where $k = 1$ if n is even and $k = 2$ if n is odd where $|D_3| = 2\left\lceil\frac{n}{3}\right\rceil$. Clearly, D_3 is a restrained dominating set of $K_m \times P_n$ since every vertex of $V(K_m \times P_n)\backslash D_3$ is adjacent to at least one vertex of D_3 and is adjacent to at least one other vertex of $V(K_m \times P_n)\backslash D_3$. Therefore, $\gamma_r(K_m \times P_n) \le |D_3| = 2\left\lceil\frac{n}{3}\right\rceil$. Since $\gamma(K_m \times P_n) = 2\left\lceil\frac{n}{3}\right\rceil$, $\gamma_r(K_m \times P_n) \ge 2\left\lceil\frac{n}{3}\right\rceil$. Thus, $\gamma_r(K_m \times P_n) = 2\left\lceil\frac{n}{3}\right\rceil$. Since any subgraph induced by a minimum restrained dominating set contains only isolated vertices, $\chi(\langle D_3\rangle) = 1 \ne \chi(K_m \times P_n)$. Therefore, D_3 is not a chromatic restrained dominating set of $K_m \times P_n$. Consider $D_4 = D_3 \cup \{u_l, v_n\}$ where $l = 1$ $if\,k = 2$ and $l = 2$ if $k = 1$. Since $(u_k, v_{n-1})(u_l, v_n) \in E(K_m \times P_n), \chi(\langle D_4\rangle) = 2 = \chi(K_m \times P_n)$. Also, D_4 is a restrained dominating set of $K_m \times P_n$. Therefore, D_4 is a chromatic restrained dominating set of $K_m \times P_n$ and $\gamma_r^c(K_m \times P_n) \le |D_4| = |D_3| + 1 = 2\left\lceil\frac{n}{3}\right\rceil + 1$. Suppose, there exists a chromatic restrained dominating set S such that $|S| < 2\left\lceil\frac{n}{3}\right\rceil + 1$. Then $|D_3| < |S| < |D_4| = |D_3| + 1$, which is not possible. Therefore, $\gamma_r^c(K_m \times P_n) = 2\left\lceil\frac{n}{3}\right\rceil + 1$, where $n \equiv 2(mod\ 3)$.

Theorem 2.8. Let $|V(K_2 \times C_m)| = n$ and $n = 2m$. Then

(i) if m is odd, $\gamma_r^c(K_2 \times C_m) = \begin{cases} \left\lceil\frac{n}{3}\right\rceil & if\ n \equiv 1(mod\ 3) \\ \left\lceil\frac{n}{3}\right\rceil + 1 & if\ n \equiv 2(mod\ 3) \\ \frac{n}{3} + 2 & if\ n \equiv 0(mod\ 3) \end{cases}$

(ii) if m is even, $\gamma_r^c(K_2 \times C_m) = \begin{cases} 2\left\lceil\frac{m}{3}\right\rceil & if\ m \equiv 1(mod\ 3) \\ 2\left(\left\lceil\frac{m}{3}\right\rceil + 1\right) & if\ m \equiv 2(mod\ 3). \\ 2\left(\frac{m}{3} + 1\right) & if\ m \equiv 0(mod\ 3) \end{cases}$

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Proof. Let $V(K_2) = \{u_1, u_2\}$ and $V(C_m) = \{v_1, v_2, v_3, \ldots, v_m\}$. Then $V(K_2 \times C_m) = \{(u_1, v_i), (u_2, v_i)/1 \le i \le m\}$ and $|V(K_2 \times C_m)| = 2m$ where $n = 2m$. Now, the vertex set $V(K_2 \times C_m)$ can be bipartitioned into disjoint subsets $V_1 = \{(u_1, v_i\}/1 \le i \le m\}$ and $V_2 = \{(u_2, v_i)/1 \le i \le m\}$. Since, no two vertices of V_1 and no two vertices of V_2 are adjacent, $\chi(\langle V_1 \rangle) = 1 = \chi(\langle V_2 \rangle)$. Then, $\chi(K_2 \times C_m) = 2$. Clearly, $K_2 \times C_m$ contains two rows and m columns.

Case (i): m is odd

Then $K_2 \times C_m$ is a cycle on $2m$ vertices i.e,C_{2m} and $\gamma_r^c(K_2 \times C_m) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & if\ n \equiv 1(mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n \equiv 2(mod\ 3). \\ \frac{n}{3} + 2 & if\ n \equiv 0(mod\ 3) \end{cases}$

Case (ii): m is even

Then $K_2 \times C_m$ is the union of two cycle graphs C_m. Thus, $\gamma_r^c(K_2 \times C_m) = \gamma_r(K_2 \times C_m) + \gamma_r^c(K_2 \times C_m)$.

Subcase (i): $m \equiv 1(mod\ 3)$

Then, $\gamma_r^c(C_m) = \gamma_r(C_m) = \left\lceil \frac{m}{3} \right\rceil$. Therefore, $\gamma_r^c(K_2 \times C_m) = 2\left\lceil \frac{m}{3} \right\rceil$.

Subcase (ii): $m \equiv 2(mod\ 3)$

Then $\gamma_r^c(C_m) = \gamma_r(C_m) = \left\lceil \frac{m}{3} \right\rceil + 1$. Therefore, $\gamma_r^c(K_2 \times C_m) = 2\left(\left\lceil \frac{m}{3} \right\rceil + 1\right)$.

Subcase (iii): $m \equiv 0(mod\ 3)$

Then $\gamma_r(C_m) = \frac{m}{3}$ and $\gamma_r^c(C_m) = \frac{m}{3} + 2$. Therefore, $\gamma_r^c(K_2 \times C_m) = \frac{m}{3} + \frac{m}{3} + 2 = 2\left(\frac{m}{3} + 1\right)$.

Theorem 2.9. $\gamma_r^c(P_2 \times P_n) = 2\left\lceil \frac{n-2}{3} \right\rceil + 4$.

Proof. Let the vertex set of P_2 be $V(P_2) = \{u_1, u_2\}$ and the vertex set of P_n be $\{v_1, v_2, v_3, \ldots,$

$v_n\}$. Then, $V(P_2 \times P_n) = \{(u_1, v_1), (u_1, v_2), (u_1, v_3), \ldots, (u_1, v_n), (u_2, v_1), (u_2, v_2), \ldots, (u_2, v_n)\}$ where cardinality of $P_2 \times P_n$ is $2n$. Clearly, there exists two disjoint partitions V_1, V_2 on the vertex set of $P_2 \times P_n$ where $V_1 = \{(u_1, v_1), (u_1, v_2), (u_1, v_3), \ldots, (u_1, v_n)\}$ and $V_2 = \{(u_2, v_1), (u_2, v_2), (u_2, v_3), \ldots, (u_2, v_n)\}$. Since V_1 and V_2 are independent, $\chi(\langle V_1 \rangle) =$

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$\chi(\langle V_2 \rangle) = 1$. Then, $\chi(P_2 \times P_n) = 2$ and $P_2 \times P_n$ is a bipartite graph containing n columns. Now, $P_2 \times P_n$ is the disjoint union of two path graphs P_1 and P_2 where $|V(P_1)| = |V(P_2)| = n$ and $\gamma_r^c(P_2 \times P_n) = \gamma_r^c(P_n) + \gamma_r(P_n)$. Now, the following three cases can be considered.

Case (i): $n \equiv 0(mod\ 3)$

Then $\gamma_r^c(P_n) = \gamma_r(P_n) = \frac{n}{3} + 2$ and $\gamma_r^c(P_2 \times P_n) = \left(\frac{n}{3} + 2\right) + \left(\frac{n}{3} + 2\right) = 2\left(\frac{n}{3}\right) + 4$. Therefore, $\gamma_r^c(P_2 \times P_n) = 2\left\lceil\frac{n-2}{3}\right\rceil + 4$.

Case(ii): $n \equiv 1(mod\ 3)$

Then $\gamma_r(P_n) = \left\lceil\frac{n}{3}\right\rceil$ and $\gamma_r^c(P_n) = \left\lceil\frac{n}{3}\right\rceil + 2$. Now, $\gamma_r^c(P_2 \times P_n) = \left\lceil\frac{n}{3}\right\rceil + \left\lceil\frac{n}{3}\right\rceil + 2 = 2\left\lceil\frac{n}{3}\right\rceil + 2$. Therefore, $\gamma_r^c(P_2 \times P_n) = 2\left\lceil\frac{n-2}{3}\right\rceil + 4$.

Case (iii): $n \equiv 2(mod\ 3)$

Then $\gamma_r(P_n) = \gamma_r^c(P_n) = \left\lceil\frac{n}{3}\right\rceil + 1$ and $\gamma_r^c(P_2 \times P_n) = 2\left\lceil\frac{n}{3}\right\rceil + 2$. Therefore, $\gamma_r^c(P_2 \times P_n) = 2\left\lceil\frac{n-2}{3}\right\rceil + 4$.

From all the cases, $\gamma_r^c(P_2 \times P_n) = 2\left\lceil\frac{n-2}{3}\right\rceil + 4$.

Theorem 2.10. Let $n \geq 5$. Then $\gamma_r^c(P_4 \times P_n) = \begin{cases} n+5 & if\ n \equiv 1,3(mod\ 4) \\ n+4 & if\ n \equiv 0,2(mod\ 4) \end{cases}$.

Proof. Let $V(P_4) = \{u_1, u_2, u_3, u_4\}$ and $V(P_n) = \{v_i / 1 \leq i \leq n\}$ where $|V(P_n)| = n$. Then $V(P_4 \times P_n) = \{(u_1, v_i), (u_2, v_i), (u_3, v_i), (u_4, v_i) / 1 \leq i \leq n\}$ with cardinality $4n$. Now, the vertex set can be divided into four disjoint subsets V_1, V_2, V_3, V_4 where $V_1 = \{(u_1, v_i)/1 \leq i \leq n\}$, $V_2 = \{(u_2, v_i)/1 \leq i \leq n\}$, $V_3 = \{(u_3, v_i)/1 \leq i \leq n\}$ and $V_4 = \{(u_4, v_i)/1 \leq i \leq n\}$ where the vertices of each subset represents a row. Clearly, V_1, V_2, V_3 and V_4 are independent and so, $\chi(\langle V_j \rangle) = 1, j = 1$ to 4. Also, there exists adjacency between vertices of *(i)* V_1, V_2, *(ii)* V_2, V_3 and *(iii)* V_3, V_4. Thus, $\chi(P_4 \times P_n) = 2$. Moreover, $P_4 \times P_n$ contains n columns $\{(\mathrm{u_j}, \mathrm{v_1}), (\mathrm{u_j}, \mathrm{v_2}), (\mathrm{u_j}, \mathrm{v_3}), \ldots, (\mathrm{u_j}, \mathrm{v_n}) / 1 \leq \mathrm{j} \leq 4\}$ where each j represents a column.

Case (i): $n \equiv 0(mod\ 4)$

Let $D_1 = \{(u_2, v_4), (u_3, v_4), (u_2, v_5), (u_3, v_5), (u_2, v_8), (u_3, v_8), (u_2, v_9), (u_3, v_9), \ldots,$

$(u_2, v_{n-4}), (u_3, v_{n-4}), (u_2, v_{n-3}), (u_3, v_{n-3}) \cup \{(u_1, v_1), (u_2, v_1), (u_3, v_1), (u_4, v_1), (u_1, v_n),$

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$(u_2, v_n), (u_3, v_n), (u_4, v_n)\}$ and $|D_1| = 4\left(\frac{n-4}{4}\right) + 8 = n + 4$. Clearly, D_1 is a dominating set of $P_4 \times P_n$ where the vertices of $V(P_4 \times P_n)\backslash D_1$ which belongs to the first and third row are dominated by the elements of D_1 belonging to second row, and the vertices of $V(P_4 \times P_n)\backslash D_1$ belonging to the second and fourth row are dominated by the elements of D_1 that belongs to third row. Also, for each vertex of $V(P_4 \times P_n)\backslash D_1$, there exists an adjacent vertex in $V(P_4 \times P_n)\backslash D_1$ and so D_1 is a restrained dominating set. Thus, $\gamma_r(P_4 \times P_n) \leq |D_1| = n + 4$. On removing any single vertex of D_1, there exists at least one vertex in $V(P_4 \times P_n)\backslash D_1$ which is not adjacent to any vertex of D_1 and any other minimum restrained dominating set is of cardinality $n + 4$. Thus, $\gamma_r(P_4 \times P_n) = n + 4$. Since $\langle D_1 \rangle$ contains path on two vertices, $\chi(\langle D_1 \rangle) = 2 = \chi(P_4 \times P_n)$. Therefore, D_1 is a chromatic restrained dominating set of $P_4 \times P_n$ and $\gamma_r^c(P_4 \times P_n) = n + 4$ where $n \equiv 0(mod\ 4)$.

Case (ii): $n \equiv 1(\text{mod } 4)$

Let $D_2 = \left\{(u_2, v_{4j}),\ (u_3, v_{4j}),\ (u_2, v_{4j+1}),\ (u_3, v_{4j+1})\ /1 \leq j \leq \left\lfloor \frac{n}{4} \right\rfloor\right\} \cup \{(u_1, v_1), (u_2, v_1), (u_3, v_1), (u_4, v_1), (u_1, v_n), (u_4, v_n)\}$ where $|D_2| = 4\left(\frac{n-1}{4}\right) + 6 = n + 5$. Clearly, D_2 is a dominating set, since for every vertex in $V\backslash D_2$, there exists at least one adjacent vertex in D_2. Also, D_2 is a restrained dominating set as $\langle V(P_4 \times P_n) - D_2 \rangle$ does not contains isolated vertices. Hence, $\gamma_r(P_4 \times P_n) \leq |D_2| = n + 5$. Since, there exists no restrained dominating sets of $P_4 \times P_n$ with cardinality less than $n + 5$, $\gamma_r(P_4 \times P_n) \geq n + 5$. Thus, $\gamma_r(P_4 \times P_n) = n + 5$. Since $\langle D_2 \rangle$ contains path and isloated vertices, $\chi(\langle D_2 \rangle) = 2$. This implies that, D_2 is a chromatic restrained dominaating set of $P_4 \times P_n$. Therefore, $\gamma_r^c(P_4 \times P_n) = |D_2| = n + 5, n \equiv 1(mod\ 4)$.

Case (iii): $n \equiv 2(mod\ 4)$

Let $D_3 = \{(u_2, v_{4j}), (u_3, v_{4j}), (u_2, v_{4j+1}), (u_3, v_{4j+1})/1 \leq j \leq \frac{n-2}{4}\} \cup \{(u_1, v_1),\ (u_2, v_1), (u_3, v_1), (u_4, v_1), (u_1, v_n), (u_4, v_n)\}$ and D_3 is of cardinality $4\left(\frac{n-2}{4}\right) + 6 = n + 4$. Clearly, D_3 is a restrained dominating set since every vertex in $V(P_4 \times P_n)\backslash D_3$ is adjacent to at least one vertex of D_3 and is adjacent to at least one vertex of $V(P_4 \times P_n)\backslash D_3$. Then, $\gamma_r(P_4 \times P_n) \leq |D_3| = n + 4$. Suppose there exists a restrained dominating set S such that $|S| < n + 4$. Since $\gamma(P_4 \times P_n) = n + 2$ [1], $n + 2 \leq |S| < n + 4$. But there does not exists a restrained dominating set with cardinality $n + 2$ or $n + 3$. Thus, $\gamma_r(P_4 \times P_n) = n + 4$. Since

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$(u_2, v_4)(u_3, v_5) \in E(P_4 \times P_n), \chi(\langle D_3 \rangle) = 2 = \chi(P_4 \times P_n)$. Then, D_3 is a chromatic restrained dominating set and $\gamma_r^c(P_4 \times P_n) = |D_3| = n + 4$.

Case(iv): $n \equiv 3(mod\ 4)$

Let $D_4 = \{(u_2, v_{4j}), (u_3, v_{4j}), (u_2, v_{4j+1}), (u_3, v_{4j+1})/1 \le j \le \frac{n-3}{4}\} \cup \{(u_1, v_1), (u_2, v_1), (u_3, v_1), (u_4, v_1), (u_1, v_{n-1}), (u_4, v_{n-1}), (u_1, v_n), (u_4, v_n)\}$ and $|D_4| = 4\left(\frac{n-3}{4}\right) + 8 = n + 5$. Then, D_4 is a restrained dominating set of $P_4 \times P_n$, since $\langle V(P_4 \times P_n) - D_4 \rangle$ contains no isolated vertex. Thus, $\gamma_r(P_4 \times P_n) \le |D_4| = n + 5$. But, there does not exists any restrained dominating set with cardinality less than $n + 5$. Thus, $\gamma_r(P_4 \times P_n) = n + 5$. Also, $\chi(\langle D_4 \rangle) = 2 = \chi(P_4 \times P_n)$. Therefore, D_4 is a chromatic restrained dominating set of $P_4 \times P_n$ and $\gamma_r^c(P_4 \times P_n) = |D_4| = n + 5$, where $n \equiv 3(mod\ 4)$.

Theorem 2.11. For $m, n \ge 2$, $\gamma_r^c(K_{1,m} \times K_{1,n}) = mn + 3$.

Proof. Let $V(K_{1,m}) = \{u_0, u_1, u_2, \dots, u_m\}$ and $V(K_{1,n}) = \{v_0, v_1, v_2, \dots, v_m\}$ where u_0 and v_0 are the full degree vertices of $K_{1,m}$ and $K_{1,n}$. Then $V(K_{1,m} \times K_{1,n}) = \{(u_i, v_j), 0 \le i \le m, 0 \le j \le n\}$ and $\left|V(K_{1,m} \times V(K_{1,n})\right| = (m + 1)(n + 1)$. Now, the vertex set of $K_{1,m} \times K_{1,n}$ can be partitioned into three disjoint subsets V_1, V_2, V_3 where $V_1 = \{(u_0, v_0), (u_i, v_j)/1 \le i \le m, 1 \le j \le n\}$, $V_2 = \{(u_0, v_j)/1 \le j \le n\}$ and $V_3 = \{(u_i, v_0)/1 \le i \le m\}$. Furthermore, vertices of V_1 forms a star graph $K_{1,mn}$ and the vertices of V_2 together with V_3 forms a complete bipartite graph. Also, $V_1 \cap (V_2 \cup V_3) = \emptyset$. Then $\chi(K_{1,m} \times K_{1,n}) = 2$. Let $D = V_1 \cup \{(u_0, v_1), (u_1, v_0)\}$ where $(u_0, v_1) \in V_2$ and $(u_1, v_0) \in V_3$. Clearly, D is a restrained dominating set of $K_{1,m} \times K_{1,n}$. Then, $\gamma_r(K_{1,m} \times K_{1,n}) \le |D| = mn + 3$. Since, any restrained dominating set of $K_{1,m} \times K_{1,n}$ must contain all the vertices of V_1, a vertex from V_2 and a vertex from V_3, $\gamma_r(K_{1,m} \times K_{1,n}) \ge mn + 3$. Therefore, $\gamma_r(K_{1,m} \times K_{1,n}) = mn + 3$. Also, $\chi(\langle D \rangle) = 2 = \chi(K_{1,m} \times K_{1,n})$. Therefore, D is a chromatic restrained dominating set of $K_{1,m} \times K_{1,n}$ and so, $\gamma_r^c(K_{1,m} \times K_{1,n}) = mn + 3$.

Observation 2.12. For any $n > 3$, $\gamma_r^c(K_{1,m} \times P_n) = \begin{cases} 2m + n - 1 & if\ \ n \equiv 0,1(mod\ 4) \\ 2m + n - 2 & if\ \ n \equiv 2,3(mod\ 4) \end{cases}$.

Observation 2.13. *(i)* $\gamma_r^c(K_{1,m} \times P_2) = 2(m + 1)$ *(ii)* $\gamma_r^c(K_{1,m} \times P_3) = 2m + 3$.

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3. Conclusion

In this paper, we have determined the chromatic restrained domination number for the direct product of certain standard graphs. An encouraging direction for future research is to analyse the bounds on the direct product of graphs and characterise the extremal graphs that represents the upper and the lower bound of the chromatic restrained domination number in direct product of graphs.

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COMPRESSED GAMMA GRAPH AND EXTENDED GAMMA GRAPH OF A ZERO-DIVISOR GRAPH

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Abstract

In this paper, we find some properties of compressed gamma graph of zero-divisor graph and extended gamma graph of zero-divisor graph. Let $\mathbb{Z}_n$ be a finite commutative ring. The compressed gamma graph is a graph with vertex set as the collection of all gamma sets of the compressed zero-divisor graph $\Gamma_E(\mathbb{Z}_n)$ and two distinct vertices r and s are adjacent if and only if $|r \cap s| = \gamma\big(\Gamma_E(\mathbb{Z}_n)\big) - 1$. This graph is denoted by $\gamma.\Gamma_E(\mathbb{Z}_n)$. The extended gamma graph of zero-divisor graph is a graph with vertex set as the collection of all gamma sets of extended zero-divisor graph $E\Gamma(\mathbb{Z}_n)$ and two (not necessarily distinct) vertices r and s are adjacent if and only if $|r \cap s| = \gamma\big(E\Gamma(\mathbb{Z}_n)\big) - 1$. This graph is denoted by $\gamma.E(\Gamma(\mathbb{Z}_n))$.

Keywords: Zero-divisor graph, gamma graph, compressed gamma graph of zero-divisor graph, extended gamma graph of zero-divisor graph.

2020 Mathematics Subject Classification (AMS): 05C25, 05C69, 05C10

1. Introduction

Over the past 20 years, there has been a growing interest in studying algebraic structures that utilize graph characteristics, leading to a number of fascinating discoveries and questions. Let R be a commutative ring with identity and $Z(R)^*$ be the set of all non-zero zero-divisors of R. D.F. Anderson and P.S. Livingston [6], associate a graph called zero-divisor graph $\Gamma(R)$ to R with vertex set $Z(R)^*$ and for two distinct $x, y \in Z(R)^*$, the vertices x and y are adjacent if and only if $xy = 0$ in R. [11] A set $D \subseteq V$ of vertices in a graph $G = (V, E)$ is called a dominating set if for every vertex $u \in V - D$, there exists a vertex $v \in D$ such that v is adjacent to u. A dominating set D is minimal if no proper subset of D is a dominating set. The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a minimal dominating set of G. A dominating set D in a graph G with cardinality γ is called $\gamma -$

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set of G. There are so many domination parameters in the literature and one can refer [10] for more details. The relationship between ring-theoretic properties of R and graph-theoretic properties of $\Gamma(R)$ has been studied extensively.

For any elements r and s of R, define $r \sim s$ if and only if $ann_R(r) = ann_R(s)$. Then $\sim$ is an equivalence relation on R; for any $r \in R$, let $[r]_R = \{s \in | \ r \sim s\}$. For example, it is clear that $[0]_R = \{0\}, [1]_R = R/Z(R)$, and $[r]_R \subseteq Z(R)/\{0\}$ for every $r \in R([0]_R \cup [1]_R)$. Furthermore, the operation on the equivalence classes given by $[r]_R[s]_R = [rs]_R$ is well-defined (i.e., $\sim$ is a congruence relation on R) and thus makes the set $R_E = \{[r]_R | r \in R\}$ into a commutative monoid. Moreover, R_E is a commutative Boolean monoid if R is a reduced ring. The monoid R_E has been studied in [[1], [9], [2], [3], [4]].

[5]The relation on R given by $r \sim s$ if and only if $ann_R(r) = ann_R(s)$ is an equivalence relation. The compressed zero-divisor graph $\Gamma_E(R)$ of R is the (undirected) graph with vertices the equivalence classes induced by $\sim$ other than [0] and [1], and distinct vertices r and s are adjacent if and only if $rs = 0$. Let R_E be the set of equivalence classes for $\sim$ on R. Then R_E is a commutative monoid with multiplication $[r][s] = [rs]$.

The concept of a zero-divisor graph of a commutative ring R was introduced by I. Beck in [7]. The compressed zero-divisor graph $\Gamma_E(R)$ (using different notation) was first defined by S.B. Mulay in [13], where it was noted in passing that several graph-theoretic properties of $\Gamma(R)$ remain valid for $\Gamma_E(R)$. The compressed zero-divisor graph $\Gamma_E(R)$ has been explicitly studied in [[2], [8], [14]].

[12]Let R be a finite commutative ring with $1 \neq 0$. Then, the extended zero-divisor graph $E\Gamma(R)$ is defined as the graph with vertex set R where two (not necessarily distinct) vertices $x, y \in R$ are adjacent if and only if $xy = 0$. In this paper, we find some properties of compressed gamma graph of zero-divisor graph and extended gamma graph of zero-diviosr graph. Let $\mathbb{Z}_n$ be a finite commutative ring. The compressed gamma graph of zero-divisor graph is a graph with vertex set as the collection of all gamma sets of the compressed zero-divisor graph $\Gamma_E(\mathbb{Z}_n)$ and two distinct vertices r and s are adjacent if and only if $|r \cap s| = \gamma\big(\Gamma_E(\mathbb{Z}_n)\big) - 1$. This graph is denoted by $\gamma.\Gamma_E(\mathbb{Z}_n)$. The extended gamma graph of zero-divisor graph is a graph with vertex set as the collection of all gamma sets of extended zero-divisor graph $E\Gamma(\mathbb{Z}_n)$ and two (not necessarily distinct) vertices r and s are adjacent if and only if $|r \cap s| = \gamma\big(E\Gamma(\mathbb{Z}_n)\big) - 1$. This graph is denoted by $\gamma.E\Gamma(\mathbb{Z}_n)$.

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2. Compressed gamma graph of a Zero-divisor graph

Definition 2.1. The Compressed gamma graph of a zero-divisor graph is a graph with vertex set as the collection of all gamma sets of the compressed zero-divisor graph $\Gamma_E(\mathbb{Z}_n)$ and two distinct vertices r and s are adjacent if and only if $|r \cap s| = \gamma\big(\Gamma_E(\mathbb{Z}_n)\big) - 1$. This graph is denoted by $\gamma.\Gamma_E(\mathbb{Z}_n)$.

Example: For n=12

$V(\Gamma_{12})$ = {2, 3, 4, 6, 8, 9, 10}

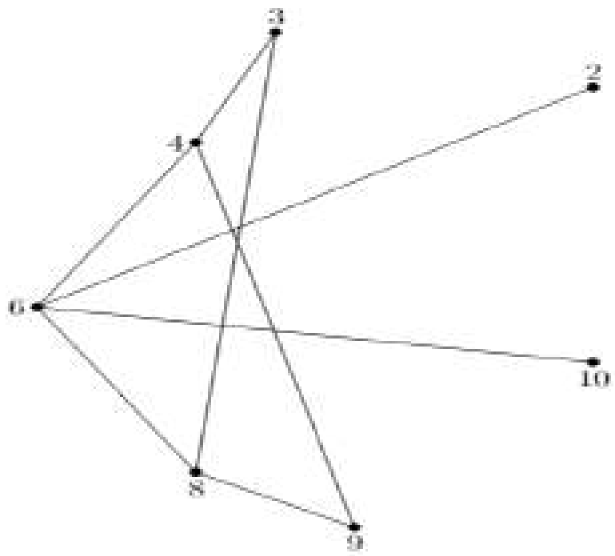

Figure 1: $\Gamma(\mathbb{Z}_{12})$

$V(\Gamma_E(\mathbb{Z}_{12}))$ = {[2], [3], [4], [6]}

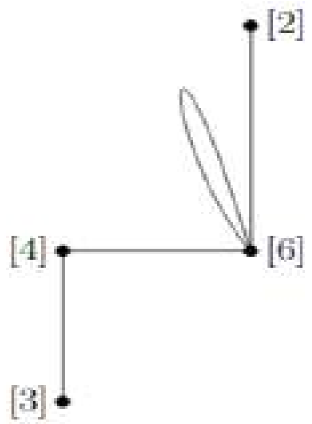

Figure 2: $\Gamma_E(\mathbb{Z}_{12})$

$V(\gamma.\Gamma_E(\mathbb{Z}_{12}))$. = {{[6], [4]}, {[6], [3]}, {[3], [2]}, {[2], [4]}}

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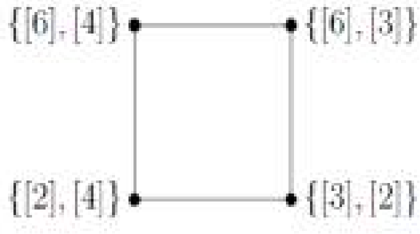

Figure 3: $\gamma.\Gamma_E(\mathbb{Z}_{12})$

Theorem 2.2. If $n = p_1^{k_1} p_2^{k_2},\ k_1, k_2 > 1$ where $p_1 < p_2,\ p_1, p_2$ are distinct primes, then the compressed gamma are graph of a zero-divisor graph $\gamma.\Gamma_E(\mathbb{Z}_n)$ is K_1.

Proof. If $n = p_1^{k_1} p_2^{k_2},\ k_1, k_2 > 1$ where $p_1 < p_2,\ p_1, p_2$ are distinct primes then, $\{\ [p_1^{k_1}.p_2^{k_2-1}], [p_1^{k_1-1}.p_2^{k_2}]\ \}$ be the only dominating set of $\Gamma_E(\mathbb{Z}_n)$.

Hence $\gamma.\Gamma_E(\mathbb{Z}_n) = K_1$

Theorem 2.3. If $n = pq, p^k q, pq^k$ where $k > 2,\ p < q,\ p$ and q are distinct primes, then the compressed gamma graph of a zero-divisor graph $\gamma.\Gamma_E(\mathbb{Z}_n)$ is K_2.

Proof.

Case(i):

If $n = pq$ where $p < q,\ p$ and q are distinct primes, then the only dominating set of $\Gamma_E(\mathbb{Z}_n)$ is $\{[p], [q]\}$.

Hence $\gamma.\Gamma_E(\mathbb{Z}_n) = K_2$.

Case(ii):

If $n = p^k q$ where $k > 2,\ p < q,\ p$ and q are distinct primes, then the only dominating set of $\Gamma_E(\mathbb{Z}_n)$ is $\{\{[p^{k-1}.q], [q]\},\ \{[p^{k-1}.q], [p^k]\}\}$

Hence $\gamma.\Gamma_E(\mathbb{Z}_n) = K_2$.

Case(iii):

If $n = pq^k$ where $k > 2,\ p < q,\ p$ and q are distinct primes, then the only dominating set of $\Gamma_E(\mathbb{Z}_n)$ is $\{\{[pq^{k-1}], [p]\},\ \{[pq^{k-1}], [q^k]\}\}$.

Hence $\gamma.\Gamma_E(\mathbb{Z}_n) = K_2$.

Theorem 2.4. If $n = p_1^{k_1} p_2^{k_2},\ pq,\ p^k q,\ pq^k$ where $k_1, k_2 > 1,\ k > 2,\ p_1 < p_2,\ p < q,$ p, q, p_1 and p_2 are distinct primes, then $\gamma.\Gamma_E(\mathbb{Z}_n)$ is planar.

Proof.

Case(i):

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If $n = p_1^{k_1} p_2^{k_2},\ k_1, k_2 > 1$ where $p_1 < p_2$ are distinct primes, then $\gamma.\Gamma_E(\mathbb{Z}_n)$ is K_1.

Hence $\gamma.\Gamma_E(\mathbb{Z}_n)$ is planar.

Case(ii):

If $n = pq,\ p^k q,\ pq^k$ where $k > 2,\ p < q,\ p$ and q are distinct primes, then $\gamma.\Gamma_E(\mathbb{Z}_n)$ is K_2.

Hence $\gamma.\Gamma_E(\mathbb{Z}_n)$ is planar.

Corollary 2.5. If $n = pq,\ p^k q,\ pq^k$ where $k > 2,\ p < q,\ p$ and q are distinct primes, then $diam(\gamma.\Gamma_E(\mathbb{Z}_n))$ is 1.

3. Extended gamma graph of a Zero-divisor graph

Definition 3.1. Let $\mathbb{Z}_n$ be a finite commutative ring with $1 \neq 0$. The extended gamma graph of a zero-divisor graph is a graph with vertex set as the collection of all gamma sets of extended zero-divisor graph $E\Gamma(\mathbb{Z}_n)$ and two(not necessarily distinct) vertices r, s are adjacent if and only if $|r \cap s| = \gamma\left(E\Gamma(\mathbb{Z}_n)\right) - 1$. This graph is denoted by $\gamma.E\Gamma(\mathbb{Z}_n)$.

Example: For n = 8

$V(\Gamma(\mathbb{Z}_8))$ = {2, 4, 6}

Figure 4: $\Gamma(\mathbb{Z}_8)$

$V(E\Gamma(\mathbb{Z}_8))$ = {0, 1, 2, 3, 4, 5, 6, 7 }

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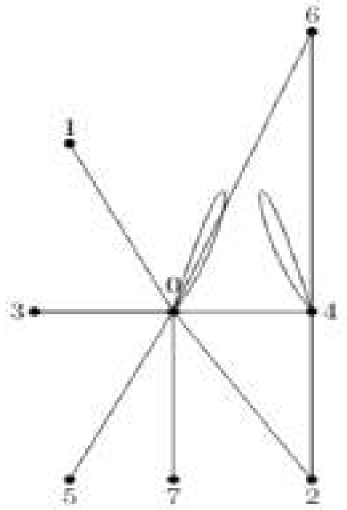

Figure 5: $E\Gamma(\mathbb{Z}_8)$

$V(\gamma.E\Gamma(\mathbb{Z}_8)) = \{\{0\}\}$

Figure 6: $\gamma.E\Gamma(\mathbb{Z}_8)$

Theorem 3.2. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ where $p_1 < p_2 < \cdots < p_r$, $k_1, k_2, \dots, k_r$ are integers, $p_1, p_2, \dots, p_r$ are distinct primes, then $\gamma.E\Gamma(\mathbb{Z}_n)$ is K_1.

Proof. In extended zero-divisor graph $\{0\}$ act as a universal vertex. Then $\{\{0\}\}$ is the only dominating set of $E\Gamma(\mathbb{Z}_n)$.

Hence $\gamma.E\Gamma(\mathbb{Z}_n) = K_1$.

Corollary 3.3. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ where $p_1 < p_2 < \cdots < p_r$, $k_1, k_2, \dots, k_r$ are integers, $p_1, p_2, \dots, p_r$ are distinct primes, then $\gamma.E\Gamma(\mathbb{Z}_n)$ is planar.

4. Conclusion

In this paper, we have discussed about some properties of compressed gamma graph and extended gamma graph of a Zero-divisor graph. It is planned to explore different graph properties in future work regarding to this concept.

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THE BASIC REPRODUCTION NUMBER OF ZIKA VIRUS BY VECTOR TRANSMISSION IN DISEASE FREE EQUILIBRIUM USING GRAPH THEORY

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Abstract

Zika virus is a mosquito-borne flavivirus that has emerged as a global health threat due to its potential for rapid spread and severe health complications. Understanding the dynamics of disease-free equilibrium (DFE) in the context of Zika virus transmission is crucial for devising effective control strategies. Graph theory, a mathematical framework for modeling relationships and networks, provides an innovative approach to study the transmission pathways of the virus. By representing the host-vector interactions and environmental factors as a directed graph, we examine the stability of the disease-free state through key graph parameters such as the basic reproduction number (R_0). Analytical results show that the DFE is stable when $R_0 < 1$, implying that the infection will die out in the long term. Conversely, if $R_0 \geq 1$, the disease may persist or become endemic.

Keywords: Zika virus, Mosquito biting rate, Basic reproduction number, Distance- weighted matrix, Vector transmission.

2020 Mathematics Subject Classification (AMS): 05C20, 05C22, 05C85, 05C90, 47A10, 00A71

1. Introduction

The Zika virus has emerged as a significant global health threat, primarily transmitted through the bites of infected Aedes aegypti mosquitoes. Additional modes of transmission, including sexual, perinatal and vertical transmission, have exacerbated its public health impact. Outbreaks have been reported worldwide, particularly in tropical and subtropical

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regions[7],[10]. The Zika virus was first identified from rhesus monkey in the Zika forest of Uganda in 1947 and from humans in Nigeria in 1954, but it was not spread in epidermic form among the human population until 2007. The first Zika outbreak among human occurred in Yap Island, Micronesia in 2007. Afterward, this disease highly spread among human in a different countries. Brazil is one of the most affected countries. The number of suspected cases in Brazil was estimated at 4,40,000 to 13,00,000 in 2015 [3],[13]. Anxiously, the increase of microcephaly incidence was unexpectedly observed in the outbreaks. Hence, the World Health Organization (WHO) decided to elevate the ZIKV epidemic status to the level of "a Public Health Emergency of International Concern (PHEIC)" on February 1, 2016.

Two species of mosquitoes, namely, Aedes aegypti and Aedes albopictus, were identified as the main vectors for ZIKV transmission [8],[6]. Aedes aegypti and Aedes albopictus seem to have different biological lifestyles, feeding preferences, and susceptibilities to ZIKV. Aedes aegypti extensively feeds on human blood whereas Aedes albopictus feeds on a more variety of host species. Both species are diurnal feeders providing high chance to expose and bite humans. Aedes aegypti basically breeds in manmade containers such as jars and old tires while Aedes albopictus may also extend the breeding sites to some other natural water holders, for examples, tree holes and coconut shells. The symptoms of Zika infection includes fever, headaches, rash, conjunctivitis and joint pain. Also the infection increases the chances of microcephaly, Guillain - Barre syndrome and other neurological disorder in new born babies from infected mothers. Zika remains a potential future epidemic threat, emphasizing the need for proactive surveillance, advanced research, and global collaboration to mitigate its impact.

Many mathematical models are constructed in Zika virus dissemination by various researchers in different countries. *Banuelos. S. et. al* presented a mathematical model to determine the effect of sexual transmission of the Zika Virus by using Wolbachia for vector control [2]. *Agusto F. B. et. al* analyzed a Zikv model that includes human vertical transmission, birth of babies with microcephaly and asymptomatic infected individuals [1]. *Suparit et. al* formulated a mathematical model for fitting the virus transmission in Bahia, Brazil during the 2015-2016 outbreaks and investigating the impact of vector control strategies [9]. In this study the graphs are generated from the models and then the basic reproduction number (R_0) in disease free equilibrium are calculated using graph theory [4] which helps to investigate epidemiology in Zika Virus transmission.

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In the field of epidemiology, understanding the transmission dynamics of infectious diseases is paramount for effective disease control and public health interventions. Central to this understanding is the concept of the basic reproduction number, denoted as R_0 [5]. The basic reproduction number serves as a key metric in assessing the transmissibility of infectious diseases within population. The **Basic Reproduction Number R_0,** is defined as the number of new infections produced by one infected individual in a completely susceptible population. It is a function of the baseline parameters. If $R_0 < 1$, each infected individual, on average, infects less than one other person. This indicates that the disease will likely die out in the population over time as the number of infected individual's decreases. If $R_0 = 1$, each infected individual, on average, infects exactly one other person. In this situation, the disease may persist in the population, but it will neither grow nor decline. This condition is often referred to as the disease being endemic. If $R_0 > 1$, each infected individual, on average, infects more than one other person. This indicates that the disease is likely to spread within the population, leading to an epidemic or pandemic if not controlled.

2. Graph Formulation

The human population follows an SEIR model with compartments for Susceptible (S_h), Exposed (E_h), Infectious (I_h), and Recovered (R_h) while the vector population follows an SEI model with compartments for Susceptible (S_v), Exposed (E_v) and Infectious (I_v). These seven compartments collectively form the vertices of a graph G=(V,E), where V represents the compartments and E represents edges denoting transitions between states. This graph-based representation captures the interactions and disease dynamics between human and vector populations. The graph, $G = (V, E)$ where

$$V(G)=\{S_h,E_h,I_h,R_h,S_v,E_v,I_v\}$$

$$E(G)=\{S_hE_h,\ E_hI_h,\ I_hR_h,\ I_vS_h,\ I_hS_v,\ S_vE_V,\ E_vI_v\}$$

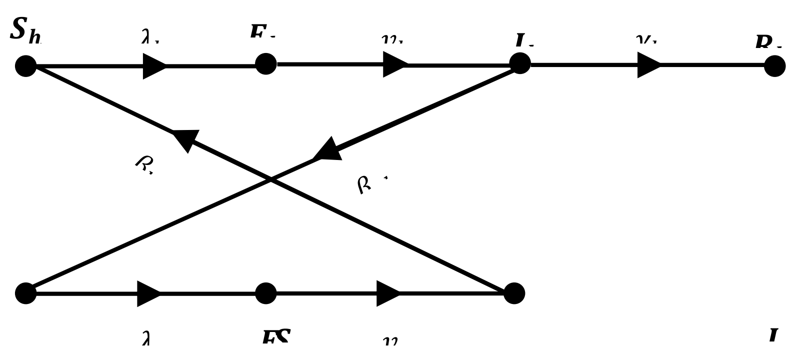

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Figure 1: Schematic of the Zika transmission graphical model. The arrow represents transitions between the compartments and also the interactions between humans and mosquitoes [11].

Parameter	**Description**
λ_h	The force from infection for humans
λ_v	The force from infection for mosquitoes
v_h	Human progression rate from exposed rate state to infectious state
v_v	Mosquito progression rate from exposed rate state to infectious state
β_{vh}	Probability of pathogen transmission from an infectious human to a susceptible mosquito
β_{hv}	Probability of pathogen transmission from an infectious mosquito to a susceptible human
γ_h	Human recovery rate
σ_h	Maximum number of bites a human can sustain
σ_v	Mosquito biting rate
N_h	Human population size in Bahia, Brazil
N_v	Mosquito population size

Table 1: Description of all parameters used in this model

The force from infection for humans (λ_h) and force from infection for mosquitoes (λ_v) are the rates at which infectious individuals infect others are calculated using the formula

$$\lambda_h = \frac{\sigma_v \sigma_h N_v}{\sigma_v N_v + \sigma_h N_v} \beta_{hv} \frac{I_v}{N_v} \cdots\cdots\cdots (1)$$

$$\lambda_v = \frac{\sigma_v \sigma_h N_v}{\sigma_v N_v + \sigma_h N_v} \beta_{vh} \frac{I_h}{N_h} \cdots\cdots\cdots (2)$$

3. Disease-Free Equilibrium

A disease-free equilibrium (DFE) is the state of an epidemic model when there is no infection, and the infected population is zero [12].

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Theorem 3.1. The basic reproduction number at disease-free equilibrium (DFE) is

$$R_0 = (\lambda_h + v_h + \beta_{vh})(\lambda_v + v_v + \beta_{hv}).$$

Proof. In the DFE, $E_h = 0, I_h = 0, R_h = 0, I_v = 0, E_v = 0$

The matrix $M(G)_{DFE}$ =

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \lambda_h + v_h + \beta_{vh} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_v + v_v + \beta_{hv} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The characteristic equation of the above matrix is

$$\lambda^2 - (\lambda_h + v_h + \beta_{vh})(\lambda_v + v_v + \beta_{hv}) = 0 \quad \text{----------(3)}$$

Spectral radius of the above characteristic equation is the basic reproduction number and is given by

$$R_0 = (\lambda_h + v_h + \beta_{vh})(\lambda_v + v_v + \beta_{hv})$$

Hence Proved

Theorem 3.2. If $\frac{1}{14} \le v_h \le \frac{1}{3}, \frac{1}{12} \le v_v \le \frac{1}{8}$ and $0.1 \le \beta_{hv}, \beta_{vh} \le 0.77$, then the basic reproduction number $R_0 < 1$.

Proof. Given: $\frac{1}{14} \le v_h \le \frac{1}{3}, \frac{1}{12} \le v_v \le \frac{1}{8}$, and $0.1 \le \beta_{hv}, \beta_{vh} \le 0.77$

To prove: $R_0 < 1$

The Basic Reproduction Number, $R_0 = (\lambda_h + v_h + \beta_{vh})(\lambda_v + v_v + \beta_{hv})$

The values of force from infections for humans (λ_h) and mosquitoes (λ_v) are 0.00000458 and 0.000007 respectively. [Using (1) and (2)]

Case (i): Let $v_h = \frac{1}{14}, \ v_v = \frac{1}{12}$ and $\beta_{hv} = \beta_{vh} = 0.1$

Therefore $R_0 = \left(0.00000458 + \frac{1}{14} + 0.1\right)\left(0.000007 + \frac{1}{12} + 0.1\right)$

$$= (0.00000458 + 0.0714 + 0.1)(0.000007 + 0.0833 + 0.1)$$

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$= (0.1714058)(0.183340)$

$= 0.0314 < 1$

Case (ii): Let $v_h = \frac{1}{3},\ v_v = \frac{1}{8}$ and $\beta_{hv} = \beta_{vh} = 0.1$

Therefore $R_0 = \left(0.00000458 + \frac{1}{3} + 0.1\right)\left(0.000007 + \frac{1}{8} + 0.1\right)$

$= (0.00000458 + 0.3333 + 0.1)(0.000007 + 0.125 + 0.1)$

$= (0.43330458)(0.225007)$

$= 0.0975 < 1$

Case (iii): Let $v_h = \frac{1}{14},\ v_v = \frac{1}{12}$ and $\beta_{hv} = \beta_{vh} = 0.77$

Therefore $R_0 = \left(0.00000458 + \frac{1}{14} + 0.77\right)\left(0.000007 + \frac{1}{12} + 0.77\right)$

$= (0.00000458 + 0.0714 + 0.77)(0.000007 + 0.0833 + 0.77)$

$= (0.8414058)(0.853307)$

$= 0.718 < 1$

Case (iv): Let $v_h = \frac{1}{3},\ v_v = \frac{1}{8}$ and $\beta_{hv} = \beta_{vh} = 0.77$

Therefore $R_0 = \left(0.00000458 + \frac{1}{3} + 0.77\right)\left(0.000007 + \frac{1}{8} + 0.77\right)$

$= (0.00000458 + 0.3333 + 0.77)(0.000007 + 0.125 + 0.77)$

$= (1.10330458)(0.895007)$

$= 0.9875 < 1$

Case (v): Let $v_h = \frac{1}{3},\ v_v = \frac{1}{8}, \beta_{hv} = 0.1\ and\ \beta_{vh} = 0.77$

Therefore $R_0 = \left(0.00000458 + \frac{1}{3} + 0.1\right)\left(0.000007 + \frac{1}{8} + 0.77\right)$

$= (0.00000458 + 0.3333 + 0.1)(0.000007 + 0.125 + 0.77)$

$= (0.43330458)(0.895007)$

$= 0.3878 < 1$

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Similarly, $R_0 < 1$ for all the given range of values in all possible ways.

Remark 3.3. The basic reproduction number $R_0 \geq 1$ for all $\frac{1}{14} \leq v_h \leq \frac{1}{3}, \frac{1}{12} \leq v_v \leq \frac{1}{8}$ and $\beta_{hv}, \beta_{vh} > 0.77$.

4. Conclusion

This study used graph theory to analyze the basic reproduction number (R_0) of the Zika virus and understand its spread through vector transmission. The basic reproduction number R_0 in disease-free equilibrium is calculated as $R_0 = (\lambda_h + v_h + \beta_{vh})(\lambda_v + v_v + \beta_{hv})$ and analyzed with the range of parameters. This shows that the DFE is stable when $R_0 < 1$, implying that the infection will die out in the long term. Conversely, if $R_0 \geq 1$, the disease may persist or become endemic. The results showed how interactions between humans and vectors impact the disease's ability to spread.

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UNFAITHFUL SET OF N-GROUP

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Abstract

In this paper we discuss the condition that is antonym to the faithful of an N-group in near-rings. This condition is named as unfaithful set of Γ. We construct and proved the properties for the unfaithful set of Γ. Then we define unfaithful of Γ. We then discuss the properties of unfaithful of Γ with annihilator, faithful, nilpotent of near-rings and its N-group.

Keywords: N-group, faithful, annihilator, nilpotent.

2020 Mathematics Subject Classification (AMS): 16Y30

1. Introduction

The concept of near-rings was introduced by Dickson [2] in 1905. Near-fields were the first near-ring founded. N-group is the analogue of the concept of a module in ring theory [3]. In this paper near-rings are the right near-rings and it is denoted by N and N-groups are the left near-modules and the set is denoted by Γ. In this paper the near-rings and N-groups that we use are from Gunter Pilz's "Near-Ring: The theory and its application" [3]. In this paper first we define the unfaithful set of Γ. The unfaithful set of Γ is the subset of N which satisfies equality condition. In this paper we learn properties of the unfaithful set of Γ. Based on the unfaithful set of Γ we define k-unfaithful of Γ. The condition $n^k\gamma = \gamma_1, k \geq 2 \; and \; \gamma, \gamma_1 \in \Gamma \; and \; n \in N$ in k-unfaithful is inspired from definition of $(\Delta_1 : \Delta_2)$ [3]. Here we prove results of k-unfaithful with annihilator, nilpotent, faithful of near-rings and its N-group.

2. Preliminaries

Definition 2.1.[3] A near-ring is a non-empty set N together with two binary operations " $+$ " and " $\cdot$ " such that

a) (N, $+$) is a group (not necessarily abelian)
b) (N, $\cdot$) is a semigroup

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c) $\forall\, n_1, n_2, n_3 \in$N such that $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ ("right distributive law").

Class of all near-rings will be denoted by $\mathcal{N}$.

Definition 2.2. [3] Let $(\Gamma, +)$ be a group with zero 0 and let $N \in \mathcal{N}$. Let $\mu: N \times \Gamma \longrightarrow \Gamma$. $(\Gamma, +)$ is called an N-group (near-module over N) if $\forall \gamma \in \Gamma\ \forall n, n' \in N$:

1. $(n + n')\gamma = n\gamma + n'\gamma$.
2. $(nn')\gamma = n(n'\gamma)$.

We write N^{Γ} for the N-group.

Definition 2.3. [3]

a) A subgroup M of a near-ring N with M.M⊆M is called a subnear-ring of N (M≤N).
b) A subgroup Δ of N^{Γ} with $N\Delta \subseteq \Delta$ is said to be an N-subgroup of Γ ($\Delta \leq_N \Gamma$).

Definition 2.4. [3] Let $N \in \mathcal{N}$. A normal subgroup I of $(N, +)$ is called ideal of N ($I \trianglelefteq N$) if

a) $IN \subseteq I$.
b) $for\ all\ n, n' \in N\ \ and\ for\ all\ i \in I\ such\ that\ n(n' + i) - nn' \in I$.

Normal subgroups R of $(N, +)$ with a) are called right ideals of N $(R \trianglelefteq_r N)$, while normal subgroups L of $(N, +)$ with b) are said to be left ideals N $(L \trianglelefteq_l N)$.

Definition 2.5. [3] Let Δ_1, Δ_2 be subsets of N^{Γ}. $(\Delta_1 : \Delta_2) = \{n \in N |\ n\Delta_2 \subseteq \Delta_1\}$. $(0:\Delta)$ is called the annihilator of Δ and it is denoted by $ann(\Delta)$.

Definition 2.6. [3] N^{Γ} is called faithful if $(0:\Gamma) = \{0\}$.

Definition 2.7. [4] An element $\gamma(\neq 0) \in \Gamma$ is a nilpotent element with index $k > 1$ if there exist a proper ideal I of N such that $I^k\gamma = 0\ and\ I^{k-1}\gamma \neq 0$.

Definition 2.8. [1] An N-group Γ is rigid if for all $\gamma \in \Gamma, n \in N$ and a positive integer k, $n^k\gamma = 0$ implies that $n\gamma = 0$.

Definition 2.9. [3] $n \in N$ is called nilpotent if there exist $k \in \mathbb{N}$ such that $n^k = 0$.

3. Main Results

Definition 3.1. Let Γ be an N-group of N. For all $\in\ \Gamma\ and\ 0 \neq n \in N$, the set $\{n \in N/\ \ n\gamma = \gamma_1,\ \ for\ any\ \gamma_1 \in \Gamma\} = N/\{0\}$ is called the unfaithful set of Γ.

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Remarks 3.2.

- The unfaithful set of Γ is contained in N.
- If $\Gamma = N$ then the unfaithful set of Γ contained in Γ.
- If $\Gamma = N$ then the unfaithful set of Γ is not an N-subgroup of Γ.
  For, additive identity element does not in the set.
- If $\Gamma = N$ and Γ is faithful, then unfaithful set of Γ contained in Γ.
- The unfaithful set of Γ does not form an additive group.
  For, additive identity element does not in the set.
- The unfaithful set of Γ is not a near-ring.
- The unfaithful set of Γ is a subset of N but neither a subgroup nor a sub near-ring of N.

Theorem 3.3. If Γ is an N-group of N then the non-zero annihilator of Γ is contained in the unfaithful set of Γ.

Proof. Let Γ be an N-group of N.

To prove, non-zero annihilator of Γ contained in unfaithful set of Γ.

Let take an element n(say) in the non-zero annihilator of Γ

since $ann(\Gamma) = (0{:}\Gamma) = \{n \in N/\, n\gamma = 0\}$

$\Rightarrow n\gamma = 0$

$\Rightarrow either\ n = 0\ or\ \gamma = 0$

Since $n \neq 0$,

γ must be zero

$\Rightarrow 0 = \gamma \in \Gamma$

We have $n\gamma = 0$

$\Rightarrow n\gamma = \gamma$

$\Rightarrow n \in \{n \in N/\, n\gamma = \gamma\}$

therefore, n belongs to the unfaithful set of Γ

Hence non-zero annihilator of Γ contained in unfaithful set of Γ.

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Theorem 3.4. Every element of the unfaithful set of Γ need not be an element of annihilator of Γ.

Proof. Let n be an arbitrary element in the unfaithful set of Γ.

$$for\ all\ \gamma \in \Gamma\ and\ n \neq 0, n\gamma = \gamma_1\ for\ any\ \gamma_1 \in \Gamma$$

Suppose $\gamma_1 = 0$

Now, $n\gamma = 0$

$\Rightarrow n \in ann(\Gamma)$

Suppose $\gamma_1 \neq 0$

Now, $n\gamma = \gamma_1 \quad for\ any\ non\ zero\ \gamma_1 \in \Gamma$

$\Rightarrow n \notin ann(\Gamma)$

Hence every element of the unfaithful set of Γ need not be an element of annihilator of Γ.

Definition 3.5. An N-group Γ of near-ring N is called k-unfaithful if $for\ all\ \gamma \in \Gamma$ $there\ exist\ n^k \neq 0$ where $k \geq 2$ such that $\{n \in N/\ \ n^k\gamma = \gamma_1, for\ any\ \gamma_1 \in \Gamma\} = N/\{0\}$.

Remark 3.6.

- If $k < 2$, then the definition is similar to $(\Delta_1 : \Delta_2)$.
- If $\gamma = n_1$, where n_1 is the identity element of N under multiplication, then $\gamma_1 = n^k$.

Remark 3.7. If $\Gamma = N$ is k-unfaithful then the unfaithful set of Γ contained in k-unfaithful.

Theorem 3.8. If the N-group Γ of N is k-unfaithful then N does not have a nilpotent element.

Proof. Let Γ be an k-unfaithful N-group of near-ring N

Suppose N has a nilpotent element

Let $n_1 \in N$ be the nilpotent element

$\Rightarrow n_1^k = 0\ \ for\ k \in Z^+$

which is a contradiction.

Hence N does not have nilpotent elements.

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Theorem 3.9. If N has a nilpotent element then every N-group Γ of N is not k-unfaithful.

Proof. Let Γ be an N-group of N

Given N has a nilpotent element

Let $n \in N$ be a nilpotent element

$therefore\, n^k = 0 \;\; for\, k \in Z^+$

By the definition of k-unfaithful, $\{n \in N/ \;\; n^k\gamma = \gamma_1, for\ any\ \gamma_1 \in \Gamma\} \neq N/\{0\}$

Hence Γ is not k-faithful.

Theorem 3.10. If an element $0 \neq \gamma \in \Gamma$ is nilpotent. Then Γ is not k-unfaithful.

Proof. Let $0 \neq \gamma$ be a nilpotent element of Γ

Then there exists a proper ideal I of N such that $I^k\gamma = 0\ and\ I^{k-1}\gamma \neq 0$. [4]

If N has nilpotent element n(say)

then $n^k = 0$

By definition of k-unfaithful, $\{n \in N/ \;\; n^k\gamma = \gamma_1, for\ any\ \gamma_1 \in \Gamma\} \neq N/\{0\}$

Γ need not be k-unfaithful.

Remark 3.11. If Γ does not have nilpotent element then Γ need not k-unfaithful.

For, suppose near-ring N of Γ has a nilpotent element.

Theorem 3.12. If Γ is a faithful module of N where N has no nilpotent element then Γ is k-unfaithful.

Proof. Let Γ be a faithful N-group of N where N has no nilpotent element

$\Rightarrow (0{:}\,\Gamma) = \{0\}$

$\Rightarrow \{n\ /n\gamma = 0\} = \{0\}$

$\Rightarrow n = 0$

$\Rightarrow 0 \neq n \notin (0{:}\,\Gamma)$

$therefore\ n\gamma \neq 0$

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$\Rightarrow n\gamma = \gamma_1, \quad for\ any\ 0 \neq \gamma_1 \in \Gamma$

Since N has no nilpotent element, $n^k \neq 0$

$therefore\ n^k\gamma = \gamma_1,\ for\ any\ 0 \neq \gamma_1 \in \Gamma$

Γ is k-unfaithful.

Theorem 3.13. If Γ is k-unfaithful then it need not be faithful.

Proof. Let Γ be k-faithful

$for\ all\ \gamma \in \Gamma\ there\ exist\ n^k \neq 0$ where $k \geq 2$ such that

$$\{n \in N/\ \ n^k\gamma = \gamma_1, for\ any\ \gamma_1 \in \Gamma\} = N/\{0\}$$

$\Rightarrow n^k\gamma = \gamma_1, \quad for\ any\ \gamma_1 \in \Gamma$

Suppose $\gamma_1 = 0$

$\Rightarrow n^k\gamma = 0$

$\Rightarrow n^k = 0\ or\ \gamma = 0$

$n^k \neq 0$ since Γ is k-unfaithful

$therefore\ \ \gamma = 0\ and\ n^k \neq 0\ \Rightarrow n \neq 0$

$\Rightarrow n\gamma = 0, where\ \gamma = 0\ and\ n \neq 0$

Suppose $\gamma_1 = 0\ and\ \gamma \neq 0$

Now, $n^k\gamma = 0$

$n^k = 0$

Which is contradiction

Hence Γ need not be faithful.

Theorem 3.14. If Γ is an N-group of N then every element of $ann(\Gamma)$ does not belongs to k-unfaithful of Γ.

Proof. Suppose every element of annihilator of Γ belongs to k-unfaithful of Γ.

Let $n \in ann(\Gamma)$

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$n\gamma = 0 \quad for\ all\ \gamma \in \Gamma$

$n = 0\ \ or\ \gamma = 0$

Since every element of annihilator of Γ belongs to k-unfaithful of Γ,

$n^k \neq 0 \Rightarrow n \neq 0\ , k \geq 2$

$therefore\ \gamma = 0$

Hence $n\gamma = 0$, In particular $\gamma = 0$

Which is a contradiction

Every element of annihilator of Γ does not belongs to k-unfaithful of Γ.

Theorem 3.15. If Γ is rigid then it need not be k-unfaithful.

Proof. Let Γ be a rigid

$for\ all\ \gamma \in \Gamma, n \in N\ and\ positive\ integer\ k, n^k\gamma = 0 \Rightarrow n\gamma = 0$ [1]

$therefore\ either\ n = 0\ or\ \gamma = 0$

$if\ n = 0\ and\ \gamma \neq 0\ \Rightarrow n^k = 0$

$if\ \gamma = 0\ and\ n \neq 0$

Case (i) $n^k \neq 0$

$n^k 0 = \gamma$

Case (ii) $n^k = 0$

$\{n \in N/\ \ n^k\gamma = \gamma_1, for\ any\ \gamma_1 \in \Gamma\} \neq N/\{0\}$

Hence Γ need not be k-unfaithful.

4. Conclusion

The unfaithful set of Γ and k-unfaithful of Γ are defined and their properties are constructed and results are proved.

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ON THE GENUS OF GAMMA GRAPH OF ZERO-DIVISOR GRAPH FROM $\mathbb{Z}_p \times \mathbb{Z}_q$

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Abstract

Let R be a commutative ring with non-zero identity. The gamma graph of γ-sets in the zero-divisor graph, $\Gamma(R)$ is the graph, $\gamma.(\Gamma(R))$ with vertex set D as the collection of all γ −sets of the zero-divisor graph, $\Gamma(R)$ and two distinct vertices D_1 and D_2 are adjacent if and only if $|D_1 \cap D_2| = \gamma(\Gamma(R)) - 1$, where $\gamma(\Gamma(R))$ denotes the cardinality of γ −set. In this paper, we investigate gamma graph of zero-divisor graph of $\mathbb{Z}_p \times \mathbb{Z}_q$, where p and q are distinct primes and classify the graphs which are planar and toroidal.

Keywords: graph embedding, gamma graph, zero-divisor graph, planar graph

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1. Introduction

In order to study the interplay between the algebraic structure of the given object and the graph theoretic properties of the graph to which it corresponds, many different graphs have been assigned to rings. Beck (1998) introduced a graph whose vertices are the elements of the ring R and two distinct vertices x and y are adjacent if and only if xy=0. During 1999, Anderson and Livingston slightly modified this idea, considering only the non-zero zero-divisors of ring as vertices of the graph with the same adjacency condition and they named the graph as zero-divisor graph, which is denoted by $\Gamma(R)$. A set $S \subseteq V$ of vertices in a graph G is called a dominating set, if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. A dominating set S is minimal, if no proper subset of S is a dominating set. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G. In a graph G, a dominating set of cardinality $\gamma(G)$ is called a γ −set. Let D be the collection of all γ −sets in G. The gamma graph of G, denoted by $\gamma.G$, is the graph with vertex set D and

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any two vertices D_1 and D_2 are adjacent if $|D_1 \cap D_2| = \gamma(G) - 1$. Let S_k denote the sphere with k handles, where k is a non-negative integer, that is, S_k is an oriented surface of genus k. The genus of a graph denoted by $g(G)$, is the smallest integer n such that the graph can be embedded in S_n. Intuitively, G is embedded in a surface if it can be drawn in the surface so that its edges intersect only at their common vertices. A genus 0 graph is called a planar graph and a genus 1 graph is called a toroidal graph. If H is a subgraph of a graph G, then $g(H) \leq g(G)$. Throughout this paper, G denotes the zero-divisor graph of R and $\gamma.G$ denotes the corresponding gamma graph.

2. Preliminaries

Lemma 2.1. [4] $g(K_n) = \left\lceil \frac{(n-3)(n-4)}{12} \right\rceil$ if $n \geq 3$. In particular, $g(K_n) = 1$ if $n = 5, 6, 7$.

Lemma 2.2. [4] $g(K_{m,n}) = \left\lceil \frac{(m-2)(n-2)}{4} \right\rceil$ if $m, n \geq 2$. In particular, $g(K_{4,4}) = g(K_{3,n}) = 1$ if $n = 3, 4, 5, 6$. Also $g(K_{5,4}) = g(K_{6,4}) = g(K_{m,3}) = 2$ if $m = 7, 8, 9, 10$.

Lemma 2.3. [7] If G is a finite connected graph with n vertices and m edges; then,

$n - m + f = 2 - 2g$, where the graph is embedded upon a surface $S_k$with genus k and f is the number of faces created when G is embedded on S_k.

Lemma 2.4. [7] If G is a triangle-free graph with n vertices and m edges, then $g(G) \geq \left\lceil \frac{m}{4} - \frac{n}{2} + 1 \right\rceil$.

Lemma 2.5. [6] Let G be a connected graph with $n \geq 3$ vertices, q edges and genus g. Then $(G) \geq \left\lceil \frac{q}{6} - \frac{n}{2} + 1 \right\rceil$.

3. Genus of $\boldsymbol{\gamma}.(\boldsymbol{\Gamma}(\mathbb{Z}_p \times \mathbb{Z}_q))$

Theorem 3.1. If $R = \mathbb{Z}_p \times \mathbb{Z}_q$, where p and q are distinct prime numbers and $p, q > 3$, then $\gamma.G$ is a regular graph with $(p-1)(q-1)$ vertices of degree $p + q - 4$.

Proof. Consider $V(G) = \{(1,0), (2,0), \dots, (p-1,0), (0,1), (0,2), \dots, (0, q-1)\}$. Let $X = \{(1,0), (2,0), \dots, (p-1,0)\} \subseteq V(G)$ and $Y = \{(0,1), (0,2), \dots, (0, q-1)\} \subseteq V(G)$. Clearly, X and Y forms a bipartition of $V(G)$ and hence $G \cong K_{p-1,q-1}$

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Now, $N((i,0)) = \{(0,j)|j = 1,2,\dots,q-1\}$, for every $i = 1,2,\dots,p-1$. Thus $V(\gamma.G) = \{\gamma_{ij}|\gamma_{ij} = \{(i,0),(0,j)\}$, where $i = 1,2,\dots,p-1$ and $j = 1,2,\dots,q-1\}$ and $N(\gamma_{ij}) = \{\gamma_{im}, \gamma_{nj}|m = 1,2,\dots,j-1,j+1,\dots,q-1;\ n = 1,2,\dots,i-1,i+1,\dots,p-1\}$

Hence $|V(\gamma.G)| = (p-1)(q-1)$ and $d(\gamma_{ij}) = q-2+p-2 = p+q-4$, for every i and j.

Theorem 3.2. Let $= \mathbb{Z}_p \times \mathbb{Z}_q$, where p and q are distinct prime numbers. Then $g(G) = 0$ if and only if R is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_q, \mathbb{Z}_3 \times \mathbb{Z}_q$. Also $g(\gamma.G) = 0$ if and only if R is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_q$.

Proof. Assume $g(G) = 0$. Let $R_i = \mathbb{Z}p_i \times \mathbb{Z}q_i$, where p_i and q_i are distinct prime numbers and G_i be the corresponding zero-divisor graphs, where $i = 1,2$

Claim: If $p_1 < p_2$ and $q_1 < q_2$ then G_1 is a subgraph of G_2

Now, $V(G_1) = \{(1,0),(2,0),\dots,(p_1-1,0),(0,1),(0,2),\dots,(0,q_1-1)\}$ and $V(G_2) = \{(1,0),(2,0),\dots,(p_1,0),(p_1+1,0),\dots,(p_2-1,0),(0,1),(0,2),\dots,(0,q_1),(0,q_1+1),\dots,(0,q_2-1)\}$. Clearly, $V(G_1) \subseteq V(G_2)$. Let $X_i = \{(0,1),(0,2),\dots,(0,q_i-1)\}$ and $Y_i = \{(1,0),(2,0),\dots,(p_i-1,0)\}$. Note that, In the graph G_i every vertex in X_i is adjacent to every vertex in Y_i and no two vertices in X_i are adjacent and hence for Y_i. Thus $G_i \cong Kp_i-1, q_i-1$. Since $p_1 < p_2$, $p_1 - 1 < p_2 - 1$ and hence $q_1 - 1 < q_2 - 1$. Thus G_1 is a subgraph of G_2.

Claim: $p \leq 3$ or $q \leq 3$

Suppose not, then $p > 3$ and $q > 3$. Then zero-divisor graph G' of $\mathbb{Z}_5 \times \mathbb{Z}_7$ must be a subgraph of G. Hence by lemma 2.4, $g(G) \geq g(G') \geq \left\lceil \frac{35}{4} - \frac{12}{2} + 1 \right\rceil > 0$, which is a contradiction. Hence either $p \leq 3$ or $q \leq 3$.

Thus R is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_q$ or $\mathbb{Z}_3 \times \mathbb{Z}_q$.

Conversely, Suppose R is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_q$ or $\mathbb{Z}_3 \times \mathbb{Z}_q$ then G is isomorphic to $K_{1,q-1}$ or $K_{2,q-1}$and it is clear that both graphs are planar. Thus $g(G) = 0$

Now, Assume $g(\gamma.G) = 0$. Let $G_1$and G_2be the zero-divisor graphs of $\mathbb{Z}_3 \times \mathbb{Z}_5$ and $\mathbb{Z}_3 \times \mathbb{Z}_q$ respectively.

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Claim: $\gamma. G_1$ is a subgraph of $\gamma. G_2$

$V(\gamma. G_1) = \{ \{(1,0),(2,0)\},\ \gamma_{ij} | \gamma_{ij} = \{(i,0),(0,j)\},$ where $i = 1,2$ and $j = 1,2,3,4\}$ and $(\gamma. G_2) = \{ \{(1,0),(2,0)\}, \gamma_{ij} | \gamma_{ij} = \{(i,0),(0,j)\},$ where $i = 1,2$ and $j = 1,2,3,\dots,q-1\}$. Clearly, $V(\gamma. G_1) \subseteq V(\gamma. G_2)$ and the graph induced by $V(\gamma. G_1)$ is a subgraph of the graph induced by $V(\gamma. G_2)$. Thus $\gamma. G_1$ is a subgraph of $\gamma. G_2$.

Let $G_1{}'$ and $G_2{}'$ be the zero-divisor graphs of $\mathbb{Z}_5 \times \mathbb{Z}_7$ and $\mathbb{Z}_p \times \mathbb{Z}_q, p \geq 5$ and $q \geq 7$

Claim: $\gamma. G_1{}'$ is a subgraph of $\gamma. G_2{}'$

$V(\gamma. G_1{}') = \{ \gamma_{ij} | \gamma_{ij} = \{(i,0),(0,j)\},$ where $i = 1,2,3,4$ and $j = 1,2,3,4,5,6\}$ and $(\gamma. G_2{}') = \{ \gamma_{ij} | \gamma_{ij} = \{(i,0),(0,j)\},$ where $i = 1,2,\dots,p-1$ and $j = 1,2,\dots,q-1\}$. Since $V(\gamma. G_1{}') \subseteq V(\gamma. G_2{}')$, $\gamma. G_1{}'$ is a subgraph of $\gamma. G_2'$.

Claim: Either p = 2 or q = 2

Suppose not, then p > 2 and q > 2. Suppose $p = 3$ and $q = 5$, $R = \mathbb{Z}_3 \times \mathbb{Z}_5$ by lemma 2.5, $g(\gamma. G_2) \geq g(\gamma. G_1) \geq \left\lceil \frac{24}{6} - \frac{9}{2} + 1 \right\rceil > 0$, which is a contradiction to $g(\gamma. G) = 0$. Suppose $p = 5$ and $= 7$, $R = \mathbb{Z}_5 \times \mathbb{Z}_7,$ by lemma 2.5, $g(\gamma. G_2{}') \geq g(\gamma. G_1{}') \geq \left\lceil \frac{96}{6} - \frac{24}{2} + 1 \right\rceil > 0$, which is a contradiction to $g(\gamma. G) = 0$. Hence either p = 2 or q = 2.

Thus $R \cong \mathbb{Z}_2 \times \mathbb{Z}_q$

Conversely, Suppose $R \cong \mathbb{Z}_2 \times \mathbb{Z}_q$

$\gamma. G$:

Figure 1

Theorem 3.3. If $= \mathbb{Z}_p \times \mathbb{Z}_q, p < q$, where p and q are distinct prime numbers then there is no $\gamma. G$ with $g(\gamma. G) = 1$

Proof. By theorem 3.2, R cannot be $\mathbb{Z}_2 \times \mathbb{Z}_q\ (q > 2)$. If $R = \mathbb{Z}_3 \times \mathbb{Z}_7,$ then by lemma 2.5, $g(\gamma. G) \geq \left\lceil \frac{48}{6} - \frac{13}{2} + 1 \right\rceil > 1$. From the proof of theorem 3.2, R cannot be $\mathbb{Z}_3 \times \mathbb{Z}_q\ (q \geq 7)$

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and $\mathbb{Z}_5 \times \mathbb{Z}_q$ $(q > 5)$. Hence it is enough to check whether $g(\gamma.G)$ of $R = \mathbb{Z}_3 \times \mathbb{Z}_5$ is 1 or not.

Suppose $R = \mathbb{Z}_3 \times \mathbb{Z}_5$

$\gamma.G$:

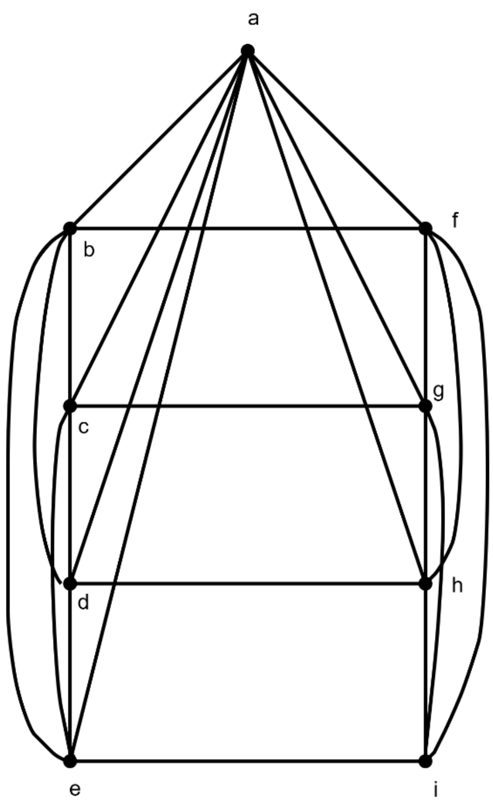

Figure 2

Consider the faces of $\gamma.G$ in S_1

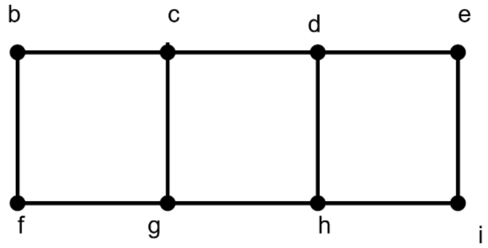

Figure 3

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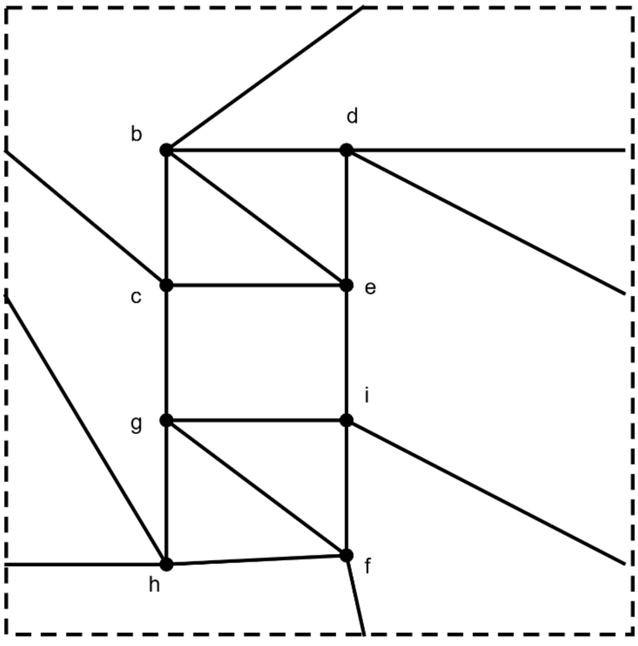

Figure 4

Clearly, vertex u cannot be inserted in any faces so that $N(a) = \{b, c, d, e, f, g, h, i\}$ without crossing while embedding it in S_1. Therefore, $g(\gamma.G) \neq 1$.

Hence the proof.

Theorem 3.4. If $R = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \ldots \times \mathbb{Z}_2$ (k times), $k \geq 4$, then $\gamma.G$ is $K_{1,k}$ and so $g(\gamma.G) = 0$

Proof. Consider the following vertices of G,

$v_i = (1,1,1,\ldots,0[i\ th],1,\ldots,1)$, $b_i = (0,0,\ldots,0,1[i\ th],0,..,0)$,

$u_{ij} = (1,1,\ldots,1,0[i\ th],1,\ldots,0[j\ th],1,\ldots,1\}$

$w_{ij} = \{(0,0,\ldots,0,1[i\ th],0,..,0,1[j\ th],\ldots,0)$, where $i,j = 1,2,\ldots,k$

Clearly, $N(v_i) = \{b_i\}$, where $i = 1,2,..,k$

$N(b_1) = \{(0, a_{12}, a_{13}, \ldots, a_{1k}) | a_{1j} \in \mathbb{Z}_2, j = 2,3,\ldots,k$ and a_{1j} cannot be zero simultaneously$\}$, $N(b_2) = \{(a_{21,}0, a_{23}, \ldots, a_{2k}) | a_{2j} \in \mathbb{Z}_2, j = 1,3,\ldots,k$ and a_{2j} cannot be zero simultaneously$\}, \ldots, N(b_k) = \{(a_{k1,}a_{k2}, \ldots, a_{k(k-1)}, 0) | a_{kj} \in \mathbb{Z}_2, j = 1,2,\ldots,k-1$ and a_{kj} cannot be zero simultaneously$\}$, $N(u_{ij}) = \{w_{ij}, b_i, b_j\}$. Thus $\{b_1, b_2, \ldots, b_k\}$ is a γ –set.

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Claim: Both $v_i, v_j (i \neq j)$ cannot be in any γ −set

Suppose not, then there exists a γ −set (say) γ' such that $v_i, v_j \in \gamma'$, where $i \neq j$. Without Loss of Generality, Let $\gamma' = \{v_1, v_2, b_3 \ldots, b_k\}$. Now, $N(u_{12}) = \{w_{12}, b_1, b_2\}$ and it is clear that γ' is not even a dominating set, which is a contradiction. Hence our claim.

Thus the γ −sets are $\gamma_0 = \{b_1, b_2, \ldots, b_k\}$, $\gamma_1 = \{v_1, b_2, \ldots, b_k\}$, $\gamma_2 = \{b_1, v_2, \ldots, b_k\}$, …, $\gamma_k = \{b_1, b_2, \ldots, b_{k-1}, v_k\}$. Clearly, $N(\gamma_0) = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$ and $N(\gamma_i) = \{\gamma_0\}$, where $i = 1,2,..,k$

Hence $\gamma.G$ is nothing but $K_{1,k}$, which is planar. Thus $g(\gamma.G) = 0$

4. Conclusion

Through this paper, we have analysed gamma graph of zero-divisor graph of the ring $\mathbb{Z}_p \times \mathbb{Z}_q$. Also, we have dealt with its embedding nature and found for those rings the gamma graph of zero-divisor graph is planar and toroidal.

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TOTAL OUTER CONNECTED DOMINATING SETS AND TOTAL OUTER CONNECTED DOMINATION POLYNOMIALS OF PATH P_n

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Abstract

Let $G = (V, E)$ be a simple graph. A set $D \subseteq V(G)$ is a total outer-connected dominating set of G if D is total dominating, and the induced subgraph $G[V(G) - D]$ is a connected graph. Let P_n be the path and $\widetilde{D}_{tc}(P_n, i)$ denote the family of all total outer- connected dominating sets of P_n with cardinality i. Let $\tilde{d}_{tc}(P_n, i) = |\widetilde{D}_{tc}(P_n, i)|$. In this paper, we obtain recursive formula for $\tilde{d}_{tc}(P_n, i)$. Using this recursive formula, we construct the polynomial, $\widetilde{D}_{tc}(P_n, x) = \sum_{i=1}^{n} \tilde{d}_{tc}(P_n, i)x^i$ which we call total outer- connected domination polynomial of P_n and obtain some properties of this polynomial.

Keywords: Domination, Total outer- connected domination, Total outer- connected domination number, Total outer- connected dominating set, Total outer- connected domination polynomial.

2020 Mathematics Subject Classification (AMS): 05C69

1. Introduction

By a graph $G = (V, E)$, we mean a finite, undirected graph with neither loops nor multiple edges. The order $|V|$ and the size $|E|$ of G are denoted by n and m respectively. For any vertex $v \in V(G)$, the open neighbourhood of v is the set $N_G(v) = \{u \in V(G)/uv \in E(G)$ and the closed neighbourhood of v is the set $N_G[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \cup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$.

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A dominating set of G is a set $D \subseteq V(G)$ such that $N_G[V] \cap D \neq \emptyset$, for all $v \in V(G)$. The domination number of G is the minimum cardinality of a dominating set of G and it is denoted by $\gamma(G)$. Similarly, a total dominating set of G is a set $D \subseteq V(G)$ such that for each $v \in V(G)$, $N_G(v) \cap D \neq \emptyset$. The total domination number $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set of G.

The concept total outer connected dominating set is introduced by J. Cyman. A path is a connected graph in which end vertices have degree 1 and the remaining vertices have degree 2 and is denoted by P_n [1].

Definition 1.1. Let G be a simple connected graph. A set $D \subseteq V(G)$ is a total outer connected dominating set of G if D is total dominating, and the induced subgraph $G[V(G) - D]$ is a connected graph. The total outer connected domination number of G, denoted by $\tilde{\gamma}_{tc}(G)$, is the minimum cardinality of a total outer connected dominating set of G.

Definition 1.2. Let G be a simple connected graph. Let $\widetilde{D}_{tc}(G,i)$ denote the family of all total outer connected dominating set of G with cardinality i and let $\tilde{d}_{tc}(G,i) = \left|\widetilde{D}_{tc}(G,i)\right|$. Then the total outer connected domination polynomial $\widetilde{D}_{tc}(G,x)$ of G is defined as $\widetilde{D}_{tc}(G,i) = \sum_{i=\tilde{\gamma}_{tc}(G)}^{|V(G)|} \tilde{d}_{tc}(G,i)\, x^i$, where $\tilde{\gamma}_{tc}(G)$ is the total outer connected domination number of G.

In the next section we study total outer connected dominating sets and total outer connected domination polynomial of P_n, which is needed for the study of total outer connected domination polynomial of complete bipartite graph P_n.

2. Total outer connected dominating sets and total outer connected domination polynomial of P_n

Lemma 2.1. For every $n \in N$, $\tilde{\gamma}_{tc}(P_n) = \begin{cases} n, & if \quad n = 1,2 \\ n-1, & if \quad n = 3,4,5 \\ n-2, & if \quad n \geq 6 \end{cases}$

n/i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1														
2	0	1													
3	0	2	1												
4	0	0	2	1											

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5	0	0	0	3	1										
6	0	0	0	1	4	1									
7	0	0	0	0	2	5	1								
8	0	0	0	0	0	3	6	1							
9	0	0	0	0	0	0	4	7	1						
10	0	0	0	0	0	0	0	5	8	1					
11	0	0	0	0	0	0	0	0	6	9	1				
12	0	0	0	0	0	0	0	0	0	7	10	1			
13	0	0	0	0	0	0	0	0	0	0	8	11	1		
14	0	0	0	0	0	0	0	0	0	0	0	9	12	1	
15	0	0	0	0	0	0	0	0	0	0	0	0	10	13	1

Table 1: $d(P_n, 15)$

3. Total outer connected dominaton polynomial of $\boldsymbol{P_n}$.

Definition 3.1. Let $\widetilde{D}_{tc}(P_n, i)$ be the family of dominating sets of Path P_n with cardinality i and let $d(P_n, i) = |\widetilde{D}_{tc}(P_n, i)|$. Then the domination polynomial $\widetilde{D}_{tc}(P_n, x)$ of P_n is defined as $\widetilde{D}_{tc}(P_n, x) = \sum_{i=\widetilde{\gamma}_{tc(P_n)}}^{n} d(P_n, i)x^i$

Lemma 3.2.

(i) For every $n > 4$, $\widetilde{D}_{tc}(P_n, x) = x^n + (n-2)x^{n-1} + (n-5)x^{n-2}$

(ii) For every $n \geq 6$, $\widetilde{D}_{tc}(P_n, x) = x[\widetilde{D}_{tc}(P_{n-1}, x)] + x^{n-3}[\widetilde{D}_{tc}(P_2, x) + \widetilde{D}_{tc}(P_3, x)]$, where $\widetilde{D}_{tc}(P_2, x) = x^2, \widetilde{D}_{tc}(P_3, x) = x^3 + 2x^2$

Theorem 3.2. The following properties hold for coefficients of $\widetilde{D}_{tc}(P_n, x)$:

(i) For every $n > 4$, $\tilde{d}_{tc}(P_n, n-1) = (P_{n-1}, n-2) + (P_{n-1}, n-1)$

(ii) For every $n \geq 6$, $\tilde{d}_{tc}(P_n, n-2) = n-5$

(iii) For every $n \in N$, $\tilde{d}_{tc}(P_n, n) = 1$

(iv) For every $n \geq 4$, $\tilde{d}_{tc}(P_n, n-1) = n-2$

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(v) For every $n \geq 6$, $\tilde{d}_{tc}(P_n, n-2) + \tilde{d}_{tc}(P_n, n-1) + \tilde{d}_{tc}(P_n, n) = [\tilde{d}_{tc}(P_{n-1}, n-3) + \tilde{d}_{tc}(P_{n-1}, n-2) + \tilde{d}_{tc}(P_{n-1}, n-1)] + 2$

(vi) For every $n \geq 6$, $\tilde{d}_{tc}(P_{n+1}, n-1) + \tilde{d}_{tc}(P_n, n-1) + \tilde{d}_{tc}(P_{n-2}, n-1) = [\tilde{d}_{tc}(P_n, n-2) + \tilde{d}_{tc}(P_{n-1}, n-2) + \tilde{d}_{tc}(P_{n-2}, n-2)] + 2$

Proof. The proof follows from the table 2.1

4. Conclusion

This paper discusses and analyses the total outer connected dominating sets of path and total outer connected domination polynomial of path. Using recursive formula, we constructed the polynomial $\widetilde{D}_{tc}(P_n, x) = \sum_{i=\widetilde{\gamma}_{tc(P_n)}}^{n} d(P_n, i)x^i$, which we call total outer connected domination polynomial of P_n and obtain some properties of the polynomial.

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RESTRICTED ZUMKELLER LABELING AND RESTRICTED EDGE ZUMKELLER LABELING OF GRAPHS

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Abstract

A bijective function $f: V(G) \to \{1,2,\ldots, | V(G) |\}$ is considered a restricted Zumkeller labeling of the graph G if the induced function $f^*: E(G) \to N$, defined as $f^*(xy) = f(x)f(y)$ for all $xy \in E(G)$, yields Zumkeller numbers. Similarly, a bijective function $f: E(G) \to \{1,2,\ldots, | E(G) |\}$ is termed a restricted edge Zumkeller labeling of the graph G if the induced function $f^*: V(G) \to N$, defined as $f^*(v) = \prod_{u \in N(v)} f(uv)$ for all $v \in V(G)$ (where $N(v)$ represents the neighborhood of v), assigns Zumkeller numbers to all vertices in $V(G)$.

Keywords: Graph labeling; Zumkeller Numbers; Zumkeller labeling; Edge Zumkeller labelling

2020 Mathematics Subject Classification (AMS): 05C78

1. Introduction

Graph labeling represents an engaging and evolving domain within graph theory, involving the assignment of values, typically integers, to edges or vertices, adhering to specific mathematical criteria. Originating in the mid-sixties, Alex Rosa [1] formally introduced this concept. Gallian's comprehensive work [4] continuously gathers and revises previous labeling schemes across various established graph families.

A positive integer n is termed perfect if it equals the sum of all its proper positive divisors, denoted by $\sigma(n) = 2n$, where $\sigma(n)$ represents the sum of positive divisors. This property has fascinated mathematicians for centuries due to its elusive nature and unique properties. In 2003, the Encyclopaedia of Integer Sequences [8] delved into the concept of Zumkeller numbers,

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offering a broader perspective on perfect numbers. Initiated by R. H. Zumkeller, Zumkeller numbers represent a fascinating extension of perfect numbers. These are positive integers wherein the sum of their positive factors can be elegantly partitioned into two distinct sets of equal sums, adding a layer of complexity and intrigue to the study of number theory. The formal introduction of Zumkeller numbers was attributed to Clark et al. [3], sparking further exploration and analysis in subsequent studies, as documented in [7,9]. In 2013, Balamurugan et al. introduced the notion of Zumkeller labeling [2], a concept deeply rooted in graph theory. Zumkeller labeling is defined as an injective function $f: V(G) \rightarrow N$, where $V(G)$ represents the vertices of a graph G, such that the induced function $f^*: E(G) \rightarrow N$, defined by $f^*(xy) = f(x)f(y)$, yields a Zumkeller number for all $xy \in E(G)$, $x, y \in V(G)$. Furthermore, the concept of edge-Zumkeller labeling was introduced by Linta K Wilson and Bebincy V M[6], defining it as an injective function $f: E(G) \rightarrow N$, where $E(G)$ represents the edges of a graph G. In this labeling scheme, the induced function $f^*: V(G) \rightarrow N$, defined by $f^*(v) = \prod_{u \in N(u)} f\,(uv)$assigns a Zumkeller number for all $v \in V(G)$ (where N(v) represent neighborhood of v). This elegant connection between number theory and graph theory adds a new dimension to the study of both fields.

In 2019, Joshua and Wong [5] pioneered the concept of restricted super totient labeling of graphs. Here, a super totient labeling of G is deemed "restricted" if the range of f is confined to the set {1,2,...,|V(G)|}. Drawing inspiration from restricted super totient labeling, we further developed the concept of restricted Zumkeller labeling of graphs, building upon the foundation of Zumkeller labeling. Additionally, we introduced the notion of restricted edge Zumkeller labeling of graphs, leveraging the framework of edge Zumkeller labeling.

2. Preliminaries

Definition 2.1. A positive integer n is said to be a Zumkeller number if the positive divisors of n can be partitioned into two disjoint subset of equal sum.

Properties of Zumkeller Numbers

- If the prime factorization of even Zumkeller number n is $2^k p_1^{k_1} p_2^{k_2} \dots p_m^{k_m}$. Then atleast one of k_i must be an odd number.
- If n is a Zumkeller number and p is a prime with $(n, p) = 1$, then np^l is Zumkeller for any positive integer l.

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- For any prime $p \neq 2$ and positive integer k with $p \leq 2^{k+1} - 1$, the number $2^k p$ is a Zumkeller number.
- Let $n = 2^\alpha p^\beta$ be a positive integer. Then n is a Zumkeller number if and only if $p \leq 2^{k+1} - 1$ and β is an odd number.

3. Restricted zumkeller labelling of graphs

Definition 3.1. Let $G = (V(G), E(G))$ be a simple connected graph. An bijective function $\mathrm{f}: \mathrm{V(G)} \to \{1,2,\dots,|\mathrm{V(G)}|\}$ is said to be restricted Zumkeller labeling of the graph G, if the induced function $\mathrm{f}^*: \mathrm{E(G)} \to \mathrm{N}$ defined as $\mathrm{f}^*(\mathrm{xy}) = \mathrm{f(x)f(y)}$ Zumkeller number for all $\mathrm{xy} \in \mathrm{E(G)}$, $\mathrm{x,y} \in \mathrm{V(G)}$. A graph that admits restricted Zumkeller labeling is called a restricted Zumkeller graph, denoted as RZG.

Example 3.2.

Figure 1. P_6 is a restricted Zumkeller Graph

Theorem 3.3. For $n \geq 3$, every spanning subgraph of a restricted Zumkeller graph is a restricted Zumkeller graph.

Proof. Let G be a restricted Zumkeller graph. As we remove edges from G to get H, the restriction of f to H is a restricted Zumkeller labeling for H. That is, H is a restricted Zumkeller graph.

Theorem 3.4. Let *H* be a spanning subgraph of a simple connected graph G. If *H* is not *RZG* then *G* is not RZG.

Proof. Suppose *G* be a simple connected graph and *H* is a spanning sub graph of *G*. *H* is not *RZG*, then there exist at least one edge labled with non Zumkeller number. That edge also in *G*. Thus, *G* is also not *RZG*.

Remark 3.5. Let *H* be induced sub graph of simple connected graph *G*. If *H* is not *RZG* then *G* is need not be *RZG*.

Example 3.6. For illustration consider the graph $G = C_4 \odot K_1$ is RZG by given labeling. But $H = C_4$ is not *RZG*.

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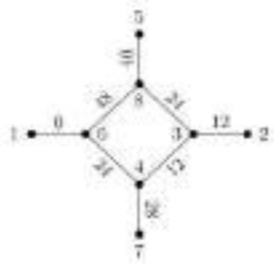

Figure 2. $C_4 \odot K_1$ is a restricted Zumkeller Graph

Theorem 3.7. Let *n* be a positive integer. Then, the complete graph K_n is a restricted Zumkeller graph if and only if *n=1*.

Proof. In the case *n=1* is trivial. Further, the case $n \geq 2$ follows from the fact that the vertices labeled as 1 and 2 must be adjacent, but the induced edge label 2 is not a Zumkeller number.

Theorem 3.8. Let G be a connected graph. Then G is restricted Zumkeller graph if $|V(G)| \geq 6$.

Proof. Since G is connected, the degree of each vertex of G is ≥ 1. Let v_0 be the vertex labeled as 1. Then 2, 3, 4 and 5 are can not be labeled for adjacent vertices of v_0 because 2, 3, 4 and 5 are not Zumkeller numbers. That is the first possibility of adjacent vertex of v_0 be 6. By the definition of restricted Zumkeller labeling it easily follows that $|V(G)| \geq 6$, when G is connected.

Theorem3.9. The complete bipartite graph K_{mn} is not restricted Zumkeller Graph for any positive integers *m* and *n*.

Proof. Consider Complete bipartite graph K_{mn} with $m + n \geq 6$. We divide the numbers 1,2,3..., $m + n$ in to 2 distinct sets A and B with size *m* and *n*. If $1 \in A$, then the set B contains only Zumkeller numbers. Suppose $2p^l \in B$ where 1 is an odd number. The number *p* belongs to *A* or *B*. If $p \in A$, then we get edge label as $2p^{l+1}$ which is not a Zumkeller number. Also if $p \in B$ then we get an edge label as *p*, which is not a Zumkeller number. Both case we get contradition. Thus, the complete bipartite graph K_{mn} is not restricted Zumkeller Graph.

Corollary 3.10. For any $n \geq 1$, star graph K_{1n} can not admits restricted Zumkeller labeling.

Theorem 3.11. For any positive integer n, the wheel graph W_n is not *RZG*.

Proof. The star graph $K_{1,n-1}$ is the spanning sub graph of wheel graph W_n. Using Theorem 3.4. and corollary 3.10. we get the result.

Theorem 3.12. For any positive integer n, the fan graph F_n is not *RZG*.

Proof. The star graph $K_{1,n-1}$ is the spanning sub graph of fan graph F_n. Using Theorem 3.4. and corollary 3.10. we get the result.

Theorem 3.13. For any positive integer *n*, the friendship graph Fr_n is not *RZG*.

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Proof. The wheel graph W_n is the spanning sub graph of friendship graph F_n. Using Theorem 3.4. and Theorem 3.12 we get the result.

Theorem 3.14. For any $n \geq 2$, bistar graph $B_{n,n}$ admits restricted Zumkeller labeling.

Proof. The vertex set of $B_{n,n}$ is $V(B_{n,n}) = \{v_i, u_i: 0 \leq i \leq n\}$ and the edge set of $B_{n,n}$ is $E(B_{n,n}) = \{ v_0u_0, v_0v_i, u_0u_i: 1 \leq i \leq n\}$.

Define the bijective function $f: V(G) \rightarrow \{ 1,2, \ldots, |V(G)|\}$as follows. Fix $f(v_0) = 6,\ f(u_0)$ be the largest number of the form 2^k where k is a positive integer, f(v_i)=1 or the number of the form 2^k (where k is positive integer) or p^{2j} (where j is positive integer and p is prime number $n \geq 3$) and its multiples and $f(u_i) = 3^{2j-1}$ (where j is positive integer) and its multiples. Leftover vertices take remaining numbers randomly. Now we calculate the induced edge labels.

$f^*(v_ou_o) = f(v_o).f(u_o) = 6.2\text{^}k = 2\text{^}\{k+1\}$.3 is a Zumkeller number.

$f^*(v_ov_i) = f(v_o).f(v_i) = \{6.1 = 6 \qquad 6.2^k = 2^{k+1}.3 \qquad m.p^{2j}.6 = m.2.3.p^{2j}$ (where *m* is positive integer.)

$f^*(u_ou_i) = f(v_o).f(u_i) = 2^k.n.3^{2j+1}$ (where *n* is positive integer.)

Now we consider remaining vertices. For the case of v_i, Clearly 3 is not a divisor of these labels, $f^*(v_ov_i) = f(v_o).f(v_i) = 2.3.f(v_i)$, which are Zumkeller numbers. In the case of u_i, there is an odd prime ($n \geq 3$) have odd power is a factor then $f^*(u_ou_i) = 2^k.f(u_i)$, which are Zumkeller numbers. Clearly all edges labeled as Zumkeller numbers. Thus, bistar graph $B_{n,n}$ admits restricted Zumkeller labeling.

Example 3.15.

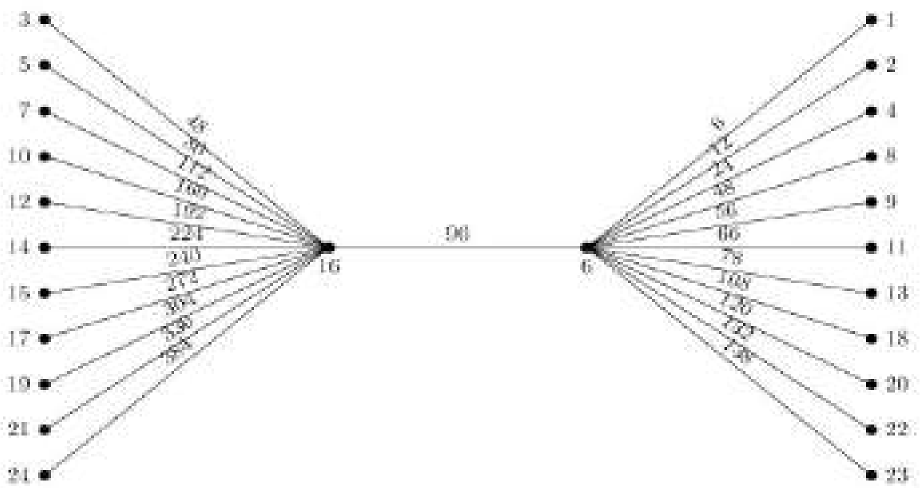

Figure 3. $B_{11,11}$ is a restricted Zumkeller Graph

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Theorem 3.16. For any $n \geq 5$, jellyfish graph $J_{n,n}$ admits restricted Zumkeller labeling.

Proof. The vertex set of $J_{n,n}$ is $V(J_{n,n}) = \{u, v, , w, x\} \cup \{v_i, u_i: 1 \leq i \leq n\}$ and the edge set of $J_{n,n}$ is $E(J_{n,n}) = \{vv_i, uu_i: 1 \leq i \leq n\} \cup \{vw, wu, ux, xv, wx\}$

Define the bijective function $f: V(G) \rightarrow \{1,2, \ldots, |V(G)|\}$ as follows. Fix $f(v) = 6,\ f(w) = 10, f(x) = 14$ and $f(u)$ be the largest number of the form 2^k where k is a positive integer. The remaining vertices are labeled same as in $B_{n+1,n+1}$.

Now we calculate the induced edge labels.

$f^*(vw) = f(v).f(w) = 6.10 = 2^2.3.5$, which is Zumkeller number.

$f^*(wu) = f(w).f(u) =$ 10.2^{k} $= 2^{k+1}.5$, which is Zumkeller number.

$f^*(ux) = f(u).f(x) =$ 2^{k}.14 $= 2^{k+1}.7$, which is Zumkeller number.

$f^*(vx) = f(v).f(x) = 6.14 = 2^2.3.7$, which is Zumkeller number.

$f^*(wx) = f(w).f(x) = 10.14 = 2^2.5.7$, which is Zumkeller number.

Using same argument in Theorem 3.14., we can show that all edge labels are Zumkeller numbers. Thus, the graph $J_{n,n}$ admits restricted Zumkeller labeling.

Example 3.17.

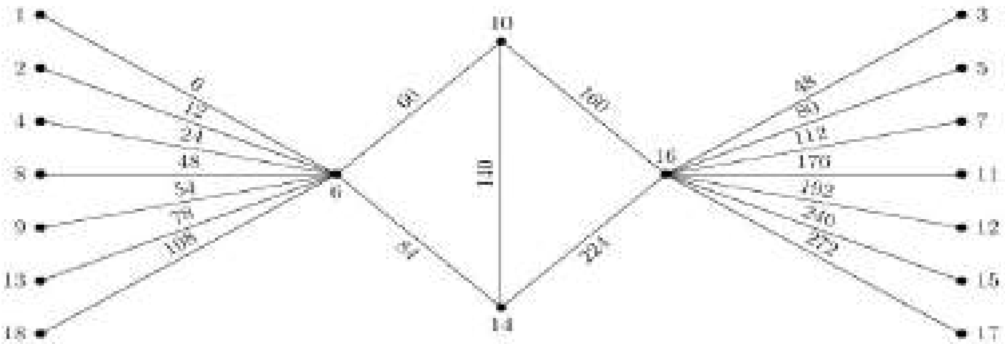

Figure 4. $J_{7,7}$ is a restricted Zumkeller Graph

Theorem 3.18. For any $n \geq 1$, the corona $P_3 \odot \underline{K_n}$ admits restricted Zumkeller labeling.

Proof: The vertex set of $P_3 \odot \underline{K_n}$ is $V\left(P_3 \odot \underline{K_n}\right) = \{v_i, u_i, w_i: 0 \leq i \leq n\}$ and the edge set of $P_3 \odot \underline{K_n}$ is $E(P_3 \odot \underline{K_n}) = \{u_0v_0, v_0w_0, u_0u_i, v_0v_i, w_0w_i: 1 \leq i \leq n\}$.

Define the bijective function $f: V(G) \rightarrow \{1,2, \ldots, |V(G)|\}$ as follows. Fix f(u_0)=3, f(v_0)= the largest number of the form 2^k (where k is a positive integer), $f(w_0) = 6, f(u_i)$ =the number of the form 2^j (where j is positive integer), $f(v_i) = 3^{2j-1}$(where j is positive integer) and its multiples and $f(w_i) = 1$ or p^{2j} (where j is positive integer and p is prime number $n \geq 3$) and its multiples . Next we label u_i with leftover even numbers.

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Also, left over odd numbers of the form $m.3^{2j}-2$ or $n.3^{2j}-1$ labeled as v_i and $m.3^{2j}-2$ or $n.3^{2j}-1$ labeled as w_i.

Now we calculate the induced edge labels.

$$f^*(u_0v_0)=f(u_0).f(v_0)=3.2^k=2^k.3$$

$$f^*(v_0w_0)=f(v_0).f(w_0)=2^k.6=2^{k+1}.3$$

$$f^*(u_0u_i)=f(u_0).f(u_i)=3.2^j=2^j.3$$

$$f^*(v_0v_i)=f(v_0).f(v_i)=2^k.m.3^{2j-1}$$

$$f^*(w_0w_i)=f(w_0.f(w_i)=\{6.1=6 \qquad\qquad 6.n.3^{2j}=2.n.3^{2j+1}$$

Next we go through remaining even numbers that is $f(u_i)=2.m$, then $f^*(u_0u_i)=f(u_0).f(u_i)=3.2.m$, which are Zumkeller numbers. Finally we go through remaining odd numbers. First we consider $f(v_i)=m.3^{2j}-1$ or $n.3^{2j}-2$, the edge labels are $f^*(v_0v_i)=f(v_0).f(v_i)=2^k.m.3^{2j}-1$ or $2^k.n.3^{2j}-2$, is Zumkeller numbers because m and n contain at least one odd prime other than 3. Next we consider $f(w_i)=m.3^{2j}-2$ or $n.3^{2j}-1$ the edge entries are $f^*(w_0w_i)=6.m.3^{2j-1}-1$ or $6.m.3^{2j-1}-2$, is Zumkeller numbers because 3 is not a factor of m.

Here we show that all edge labels are Zumkeller number. Thus, the graph the corona $P_3 \odot \underline{K_n}$ admits restricted Zumkeller labeling.

Example 3.19.

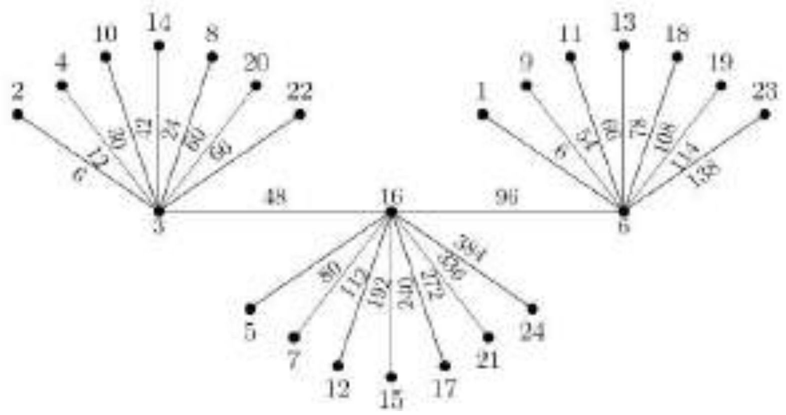

Figure 5. $P_3 \odot \underline{K_7}$ is a restricted Zumkeller Graph

4. Restricted edge Zumkeller labeling of the graphs

Definition 4.1. Let $G=(V(G),E(G))$ be a graph. A bijective function $f:E(G)\to\{1,2,\ldots,|E(G)|\}$ is said to be restricted edge Zumkeller labeling of the graph G if the induced function $f^*:V(G)\to N$ defined as $f^*(v)=\prod_{u\in N(u)} f\,(uv)$assigns a Zumkeller

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number for all $v \in V(G)$ (where *N(v)* represent neighborhood of *v*). A graph that admits restricted edge Zumkeller labeling is called a restricted edge Zumkeller graph.

Example 4.2.

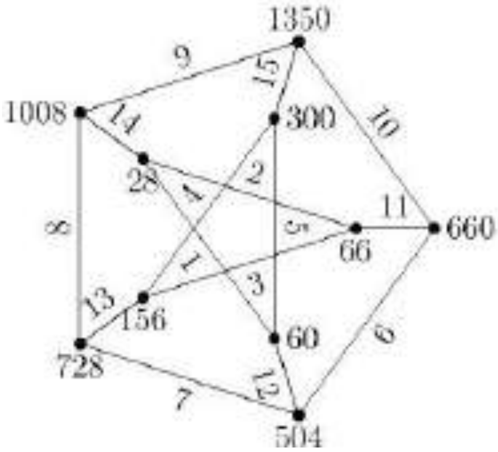

Figure 6. Peterson graph is a restricted edge Zumkeller Graph

Theorem 4.3. Any graph *G* with isolated vertex is not a restricted edge Zumkeller graph.

Proof. It easly follows from the definition of restricted edge Zumkeller graph.

Remark 4.4. Let *G* be restricted edge Zumkeller graph. Then sub graph of *G* need not be restricted edge Zumkeller graph.

Example 4.5. For this graph removing the chord edge we get cycle C_7 is not restricted edge Zumkeller graph

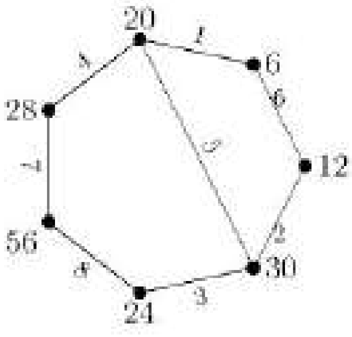

Figure 7

Remark 4.6. Let *H* be sub graph of simple connected graph *G*. If *H* is *REZG* then *G* is need not be *REZG*.

Theorem 4.7. Let *G* be a simple graph with *p* pendent vertices. Then *G* is *REZG* if cardinality of Zumkeller number $\leq |E(G)|$ is atleast *p*.

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Proof. By the definition of restricted edge Zumkeller labeling of graph all pendent vertices labeled by Zumkeller numbers and the function is bijective this is only possible when cardinality of Zumkeller number $\leq |E(G)|$ is atleast *p*.

Corollary 4.8. For any $n \geq 1$, star graph $K_{1,n}$ can not admits restricted edge Zumkeller labeling.

Proof. For star graph $K_{1,n}$ has *n* pendent vertices and *n* edges. Using theorem 4.7 we get the result.

Corollary4.9. For any $n \geq 3$, helm graph H_n can not admits restricted edge Zumkeller labeling.

Proof. It is easily from theorem 4.7.

Theorem 4.10. Let *G* be a simple graph. Then *G* is restricted edge Zumkeller graph if $|E(G)| \geq 6$.

Proof. Suppose $|E(G)| \geq 5$. If *G* contain pendent vertices, then *G* is not restricted Zumkeller graph. That is every vertices have degree ≥ 2. The only possible labels of incident edges in vertices are 1.2.3, 1.3.4, 1.4.5, 2.3, 4.3 and 4.5. If we draw with these as vertices we get parallel edge in graph. Thus, *G* is restricted edge Zumkeller graph if $|E(G)| \geq 6$.

Theorem 4.11. The path P_n cannot be $REZG$ *if* $n \geq 35$.

Proof. The graph P_n be a path has vertex set $V(P_n) = \{v_i: 1 \leq i \leq n\}$ and edge set $E(P_n) = \{v_i v_{i+1}: 1 \leq i \leq n-1\}$. We know that path graph has two pendent vertices namely v_1 and v_n. Thus, P_n is not $REZG$ if $n < 12$. For any *n* there is $\lceil \frac{n}{2} \rceil$ odd numbers and $\lfloor \frac{n}{2} \rfloor$ even numbers. Both end vertices are Zumkeller numbers. Also 945=3^3.5.7 is the least odd Zumkeller number. Then the possibility of 2 odd numbers put adjacent in restricted edge Zumkeller labeling is 27.35. Thus, the path P_n is $REZG$ if $n \geq 35$.

Theorem 4.12. *For* $n \geq 3$, the wheel graph W_n is $REZG$.

Proof. The graph W_n has vertex set $V(W_n) = \{v_i: 0 \leq i \leq n\}$ and edge set $E(W_n) = \{v_0 v_i, v_i v_{i+1}: 1 \leq i \leq n\}$.

Define an bijective function $f: E(G) \to \{1, 2, \ldots, |E(G)|\}$ as follows

$$f(v_0 v_i) = 2i \qquad 0 \leq i \leq n$$

$$f(v_1 v_2) = 3$$

$$f(v_2 v_3) = 1$$

$$f(v_i v_{i+1}) = 2i + 1 \qquad 3 \leq i \leq n$$

Then by the definition of *f* we obtain the induced function f^* as follows:

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$$f^*(v_0) = \prod 2i = 2^n.n!$$

$$f^*(v_1) = 2.3.1 = 6$$

$$f^*(v_2) = 1.4.5 = 20\$$$

$$f^*(v_i) = (2i-1).2i.(2i+1) = 2i(4i^2-1)$$

Using properties of Zumkeller numbers we get all vertex labels are Zumkeller numbers.

Thus, the wheel graph W_n admits restricted edge Zumkeller labeling.

Example 4.13.

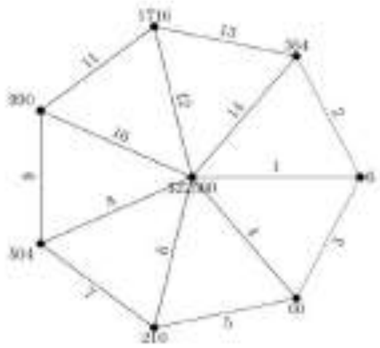

Figure 8. W_7 is a restricted edge Zumkeller Graph

5. Conclusion

This study introduces and explores the concept of restricted Zumkeller labeling for both vertices and edges of a graph. The restricted vertex Zumkeller labeling assigns a bijective function to the vertices, ensuring that the induced edge labeling satisfies the Zumkeller number conditions. Similarly, the restricted edge Zumkeller labeling defines a bijective function for edges that leads to Zumkeller numbers for all vertices in the graph when considering their neighborhoods. These new labeling approaches contribute to the broader understanding of graph labeling theory and may offer potential applications in network design and optimization.

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OCTAGONAL PRIME GRACEFUL LABELING OF SOME SPECIAL GRAPHS

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Abstract

Let G be a graph with p vertices and q edges. Define a bijection $f: V\ (G) \rightarrow \{1, 8, \dots \dots,$ $p(3p-2)\}$ by $f\ (v_i) = i(3i-2)$ for every i from 1 to p and define a $1-1$ mapping $f^{\ *}_{opgl}: E(G) \rightarrow$ set of natural number N such that $f^{\ *}(uv) = |f\ (u) - f\ (v)|$ for all edges $(uv) \in E(G)$. The induced function f is said to be octagonal prime graceful labeling if the $gcin$ of each vertex of degree atleast 2 is one.

Keywords: Graceful labeling, Prime graceful labeling, Octagonal graceful labeling, Octagonal prime graceful labeling,

2020 Mathematics Subject Classification (AMS): 05C78

1. Introduction

Numbers of the form $O_n = n(3n-2)$ for all $n \geq 1$ are called octagonal numbers. Octagonal graceful labeling on some graphs is studied by S. Mahendran and it is defined as follows: Let $f : V\ (G) \rightarrow \{0, 1, 2, \dots, M_q\}$ where M_q is the q^{th} octagonal number be an injective function. Define the function $f^{\ *} : E(G) \rightarrow \{1, 8, \dots, M_q\}$ such that $f^{\ *}(uv) = |f\ (u) - f\ (v)|$ for all edges $uv \in E(G)$. If $f^{\ *}(E(G))$ is a sequence of distinct consecutive octagonal numbers $\{M_1, M_2, \dots, M_q\}$, then the function f is said to be octagonal graceful labeling [7] and the graph which admits such a labeling is called a octagonal graceful graph. In this paper we discussed the octagonal prime graceful labeling of some graphs with illustrations.

2. Preliminaries

Definition 2.1. [2] The Jelly fish graph $J(m, n)$ is obtained from a $4-$cycle v_1, v_2, v_3, v_4 by

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joining v_1 and v_3 with an edge and appending m pendent edges to v_2 and n pendent edge to v_4.

Definition 2.2. [8] A double fan graph $F_{2,n}$ is defined as the graph join $\overline{K_2} + P_n$ where $\overline{K_2}$ is the empty graph on two vertices and P_n be a path of length n.

Definition 2.3. The joint sum of two graphs G_1 and G_2 is the graph obtained by joining a vertex of G_1 with a vertex of G_2 by an edge.

Definition 2.4. Let the graph G_1 and G_1 have disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their union $G = G_1 \cup G_1$ is a graph with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$. Clearly, $G_1 \cup G_1$ has $p_1 + p_2$ vertices and $q_1 + q_2$ edges.

3. Main results

3.1. Octagonal Prime Graceful Labeling of Some Special Graphs

Theorem 3.1.1. The Jelly fish graph $J(m,n)$ [2] is an octagonal prime graceful for $m,n \geq 1$.

Proof. Let $J(m,n)$ be the Jelly fish graph with $m+n+4$ vertices and $m+n+5$ edges. Without loss of generality let us assume $n \geq m$.

Let $V(G)$ and $E(G)$ be the vertex and edge set respectively.

Then $V(G) = V_1 \cup V_2 \cup V_3$

$$= \{x, y, u, v\} \cup \{u_i : 1 \leq i \leq m\} \cup \{v_j : 1 \leq j \leq n\} \text{ and}$$

$$E(G) = E_1 \cup E_2 \cup E_3$$

$$= \{xu, uy, yv, vx, xy\} \cup \{uu_i : 1 \leq i \leq m\} \cup \{v_j : 1 \leq j \leq n\}$$

Define $f : V(G) \rightarrow \{1, 8, \ldots, p(3p - 2)\}$ by

$f(u) = O_1, f(v) = O_2, f(x) = O_3, f(y) = O_4$

$f(u_i - m) = i(3i - 2), for\ 5 \leq i \leq 2m$

$f(v_i - 2m) = i(3i - 2), for\ 2m + 1 \leq i \leq 3n$

The edge labels are given by

$f^*_{opgl}(xv) = f(x) - f(v)$

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$f^*_{opgl}(xu) = f(x) - f(u)$

$f^*_{opgl}(yu) = f(y) - f(u)$

$f^*_{opgl}(yv) = f(y) - f(v)$

$f^*_{opgl}(u_i - 3u) = 3i^2 + 4i, for\ 4 \leq i \leq 2m - 1$

$f^*_{opgl}(v_i v) = f(v_i) - f(v), for\ 1 \leq i \leq n$

Clearly f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(G)$ such that

$f^*_{opgl}(uv) = |f(u) - f(v)|$.

Also the $gcin$ of each vertex of degree greater than one is 1. Therefore f admits

octagonal prime graceful labeling.

Hence the Jelly fish $J(m, n)$ is an octagonal prime graceful graph.

Example 3.1.2. Octagonal prime graceful labeling of graph Jelly fish $J(4, 4)$ is shown below.

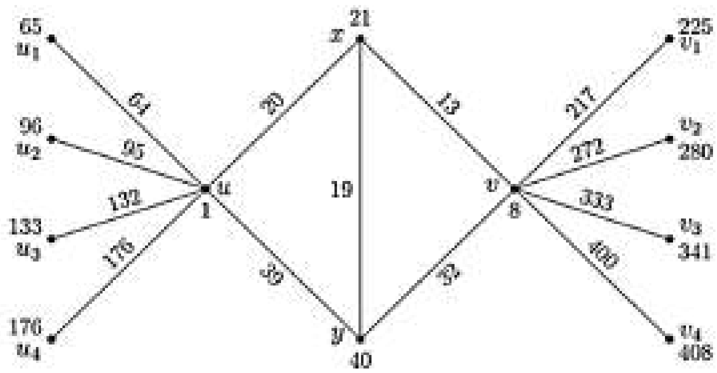

Figure 1: Octagonal prime graceful labeling of Jelly fish $J(4, 4)$

Theorem 3.1.3. The double fan graph $F_{2,n}$ [8] is an octagonal prime graceful graph.

Proof. Let $F_{2,n}$ be a double fan graph.

Then $F_{2,n}$ has $p = n + 2$ and $q = 3n - 1$ number of vertices and edges respectively.

$(ie)\ p = |V(F_{2,n})| = n + 2$ and $q = |E(F_{2,n})| = 3n - 1$

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Let $V\ (F_{2,n}) = \{v, u, v_i : 1 \ \leq \ i \ \leq \ n\}$ and

$E(F_{2,n}) = \{uv_i, vv_i : 1 \ \leq \ i \ \leq \ n\} \cup \{v_i v_{i+1} : 1 \ \leq \ i \ \leq \ n \ - \ 1\}$

Define a function $f : V\ (F_{2,n}) \rightarrow \{1, 8, \ldots, p(3p \ - \ 2)\}$ by

$f\ (v_i) = 3i^2 - 2i, for\ 1 \ \leq \ i \ \leq \ n$

$f\ (u) = O_{n+1}\ and\ f\ (v) = O_{n+2}$

The edges of $F_{2,n}$ are labeled in such a way that

$f\ ^*_{opgl}(v_i v_{i+1}) = 6i + 1, for\ 1 \ \leq \ i \ \leq \ n \ - \ 1$

$f\ ^*_{opgl}(uv_i) = f\ (u) - O_i, for\ 1 \ \leq \ i \ \leq \ n$

$f\ ^*_{opgl}(vv_i) = f\ (v) - O_i, for\ 1 \ \leq \ i \ \leq \ n$

Clearly $f\ ^*_{opgl}$ is an injection and f induces the function $f\ ^*_{opgl}$ on $E(F_{2,n})$ such that

$f\ ^*_{opgl}(uv) = |f\ (u) - f\ (v)|$.

Also the $gcin$ of u = gcd of edges incident on u

$\Rightarrow\ gcd\ \{uv_i / 1 \ \leq \ i \ \leq \ n\} = 1$

gcin of v = gcd of edges incident on v

$\Rightarrow\ gcd\ \{vv_i / 1 \ \leq \ i \ \leq \ n\} = 1$

gcin of v_i = gcd of edges incident on v_i

$= 1$

Hence the $gcin$ of each vertex of degree atleast 2 is one.

Therefore f admits octagonal prime graceful labeling.

Hence the double fan $F_{2,n}$ is an octagonal prime graceful graph.

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Example 3.1.4. Octagonal prime graceful labeling of graph $F_{2,7}$ is shown below.

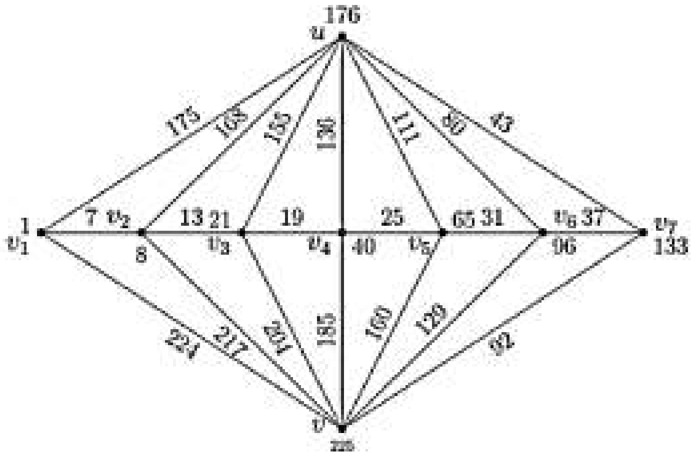

Figure 2: Octagonal prime graceful labeling of $F_{2,7}$

Theorem 3.1.5. The graph $(P_2 \cup mK_1) + N_2$ [2] is an octagonal prime graceful graph for $m \leq 4$.

Proof. Let $G = (P_2 \cup mK_1) + N_2$

Then G has $m + 4$ vertices and $2m + 5$ edges respectively.

Let x, y, u, v and $v_1, v_2, \dots, v_m$ be the vertices of G.

Let $V(G) = V_1 \cup V_2 \cup V_3$

$= \{x, y\} \cup \{u, v\} \cup \{v_i : 1 \leq i \leq m\}$ and

$E(G) = E_1 \cup E_2 \cup E_3$

$= \{x, y\} \cup \{xu, xv, yu, yv\} \cup \{uv_i : 1 \leq i \leq m\} \cup \{vv_i : 1 \leq i \leq m\}$

Define a function $f : V(G) \to \{1, 8, \dots, p(3p - 2)\}$ by

$f(u) = O_1, f(v) = O_2, f(x) = O_3, f(y) = O_4$

$f(v_i) = O_{i+m}\ for\ 1 \leq i \leq m$

The edge labels are given by

$f^*_{opgl}(xu) = f(x) - f(u)$

$f^*_{opgl}(xv) = f(x) - f(v)$

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$f^{*}_{opgl}(yu) = f(y) - f(u)$

$f^{*}_{opgl}(yv) = f(y) - f(v)$

$f^{*}_{opgl}(v_i u) = f(v_i) - f(u)$

$f^{*}_{opgl}(v_i v) = f(v_i) - f(v)$

Clearly f^{*}_{opgl} is an injection and f induces the function f^{*}_{opgl} on $E(G)$ such that

$f^{*}_{opgl}(uv) = |f(u) - f(v)|$.

Also the $gcin$ of each vertex of degree atleast 2 is one.

Therefore f admits octagonal prime graceful labeling.

Hence the graph $(P_2 \cup mK_1) + N_2$ is an octagonal prime graceful graph for $m \le 4$.

Example 3.1.6. The octagonal prime graceful labeling of $(P_2 \cup 4K_1) + N_2$ is shown below.

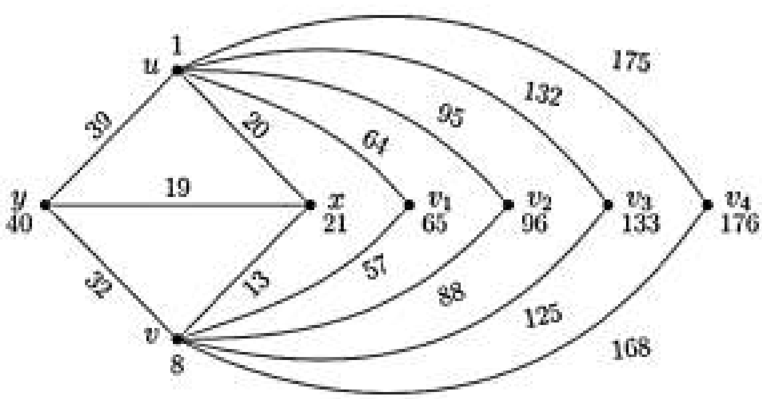

Figure 3: Octagonal prime graceful labeling of $(P_2 \cup 4K_1) + N_2$

Remark 3.1.7. The graph $(P_2 \cup mK_1) + N_2$ [2] is octagonal graceful but does not admits an octagonal prime graceful labeling for $m \ge 5$.

Proof. The graph $(P_2 \cup mK_1) + N_2$ is octagonal graceful but not octagonal prime graceful graph for $m \ge 5$.

This is shown below by a figure $(P_2 \cup 5K_1) + N_2$

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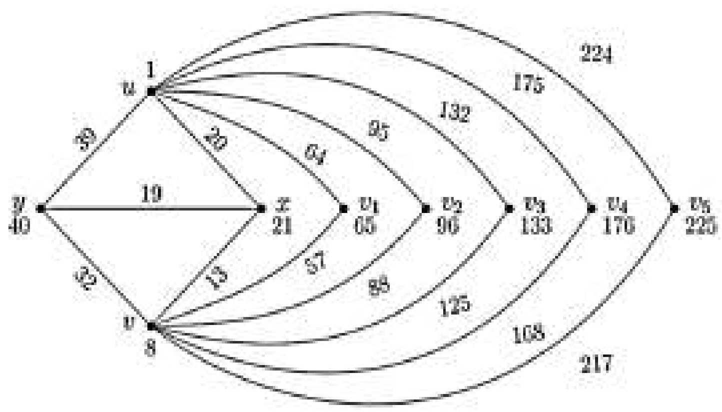

Figure 4: $(P_2 \cup 5K_1) + N_2$

Here gcin of vertex $v_5 = gcd\,\{224, 217\} \neq 1$

4. Application

Graph theory finds its application in various fields such as coding theory, radar, astronomy, security designs, missile guidance, communication networks, X-ray crystallography, and database management [3]. Nowadays, it is widely used in the medical field also. The application of graph theory has not yet found its way into dental application which could help the dentist to plan the treatment easily. Suitable labeling is applied on a graph to represent the given sample in a simple way. When graph theory is used to depict the dental arch, it, in turn, gives a visual idea which would be easier to analyze than the standard formulas. The geometrical representation of graph structure provides a powerful aid for visualizing and understanding dental arch form. The octagonal prime graceful labeling of graph serves as models whether the patient has spacing or crowding. The variations can be used to predict if arch expansion is needed as a part of the treatment for correcting the malocclusion [5]. Determination of the need for arch expansion in orthodontics using graph labeling and graceful labeling of dental arch and the application of different types of graph labeling in dental arch structure have been studied by P. Lalitha, M. Gayathri, L. Tamilselvi, A. V. Arun[4].

5. Conclusion

In this paper we discussed the octagonal prime graceful labeling of some special graphs and the octagonal prime graceful labeling of join of two graphs. Also we have discussed the

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application of octagonal prime graceful labeling in the field of dentistry. A possible direction of future research is to investigate the octagonal prime graceful labeling of other graphs.

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SECURE CONNECTED DOMINATING SETS AND SECURE CONNECTED DOMINATION POLYNOMIALS OF COMPLETE BIPARTITE GRAPH $K_{2,n}$

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Abstract

Let $G = (V, E)$ be a simple graph. A connected dominating set S of $V(G)$ is a secure connected dominating set of G if for each $u \in V(G)\backslash S$, there exists $v \in S$ such that $uv \in E(G)$ and the set $(S\backslash\{v\}) \cup \{u\}$ is a connected dominating set of G. The minimum cardinality of a secure connected dominating set of G, denoted by $\gamma_{sc}(G)$, is called the secure connected domination number of G. Let $K_{2,n}$ be the complete bipartite graph and let $D_{sc}(K_{2,n}, i)$ denote the family of all secure connected dominating sets of $K_{2,n}$ with cardinality i. Let $d_{sc}(K_{2,n}, i) = |D_{sc}(K_{2,n}, i)|$. In this paper, we obtain recursive formula for $d_{sc}(K_{2,n}, i)$. Using this recursive formula, we construct the polynomial, $D_{sc}(K_{2,n}, x) = \sum_{i=\gamma_{sc}(K_{2,n})}^{n+2} d_{sc}(K_{2,n}, i)x^i$ which we call secure connected domination polynomial of $K_{2,n}$ and obtain some properties of this polynomial.

Keywords: Domination, Connected Domination, Secure Connected Domination Number, Secure Connected Dominating Set, Secure Connected Domination Polynomial.

2020 Mathematical Subject Classification (AMS): 05C69

1. Introduction

Let $G = (V, E)$ be a graph with no self loops and no parallel edges. The order and size of the graph is denoted by $|V(G)|$ and $|E(G)|$ respectively. For any vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V: uv \in E\}$ and the closed neighborhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \bigcup_{v\in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ is a dominating set of G, if $N[S] = V$, or equivalently, every vertex in $V - S$ is adjacent to atleast one vertex in S. The dominating set S is said to be a connected dominating set if the subgraph

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$\langle S \rangle$ induced by S is connected in G. A connected dominating set S of $V(G)$ is a secure connected dominating set of G if for each $u \in V(G) \setminus S$, there exists $v \in S$ such that $uv \in E(G)$ and the set $(S \setminus \{v\}) \cup \{u\}$ is a connected dominating set of G. The minimum cardinality of a secure connected dominating set of G, denoted by $\gamma_{sc}(G)$, is called the secure connected domination number of G. The study of secure connected domination in graphs was initiated by Amerkhan G. Cabaro, Sergio S. Canoy, Jr. and Imelda S. Aniversario[1]. Let $K_{2,n}$ be the complete bipartite graph with $n+2$ vertices. Let $D_{sc}(K_{2,n}, i)$ denote the family of all secure connected dominating sets of $K_{2,n}$ with cardinality i and let $d_{sc}(K_{2,n}, i)= \left| D_{sc}\left(K_{2,n}, i\right) \right|$. The polynomial, $D_{sc}(K_{2,n}, x) = \sum_{i=\gamma_{sc}(K_{2,n})}^{n+2} d_{sc}(K_{2,n}, i)x^i$ which we call secure connected domination polynomial of $K_{2,n}$.

2. Secure connected dominating sets of $K_{2,n}$

Theorem 2.1. $\gamma_{sc}\left(K_{2,n}\right) = 3, n \in \mathbb{N}$.

Proof. Let $K_{2,n}$, $n \geq 1$ be the complete bipartite graph with $n+2$ vertices and $2n$ edges.

By the definition of secure connected dominating sets, every secure connected dominating of $K_{2,n}$ must contain atleast three vertices, that is., the minimum cardinality is 3.

Therefore, $\gamma_{sc}\left(K_{2,n}\right) = 3, n \in \mathbb{N}$.

Theorem 2.2. For all $n \in \mathbb{Z}^+$, $D_{sc}\left(K_{2,n}, i\right) = \emptyset$ if and only if $i > n+2$ or $i < 3$.

Proof. Since the minimum cardinality of the secure connected dominating set of $K_{2,n}$ is 3, there cannot exists a set with cardinality less than this minimum cardinality.

Hence, $D_{sc}\left(K_{2,n}, i\right) = \emptyset$ if $i < 3$.

Also, since $K_{2,n}$ contains $n+2$ vertices, there cannot exist a secure connected dominating set with cardinality greater than the number of vertices of the graph.

Hence, $D_{sc}\left(K_{2,n}, i\right) = \emptyset$, if $i > n+2$.

Theorem 2.3. Let $K_{2,n}$ be the complete bipartite graph with $n+2$ vertices, then

$$d_{sc}\left(K_{2,n}, i\right) = \begin{cases} n, & if\ i = 3, n \geq 3 \\ \binom{n}{i-2}, & if\ 3 < i \leq n \\ \binom{n+2}{i}, & if\ i > n \end{cases}.$$

Proof. Let $K_{2,n}, n \geq 1$ be the complete bipartite graph with $n+2$ vertices. Let $v_1, v_2 \in V$ be the vertices with degree n and $v_3, v_4, \dots, v_{n+2}$ be the remaining vertices.

For the construction of secure connected dominating sets, the set must contain v_1 and v_2 if $3 \leq i \leq n$.

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If $v_1 \notin D_{sc}(K_{2,n}, i)$ or $v_2 \notin D_{sc}(K_{2,n}, i)$ or $\{v_1, v_2 \notin D_{sc}(K_{2,n}, i)$, then the resultant set will never be a secure connected dominating set.

Now, for $i = 3$, $K_{2,n}$ contains 'n' number of subsets which includes v_1 and v_2.

Therefore, $d_{sc}(K_{2,n}, i) = n$ if $i = 3$.

For $3 < i \leq n$, $K_{2,n}$ contains $\binom{n}{i-2}$ number of subsets which includes v_1 and v_2.

Therefore, $d_{sc}(K_{2,n}, i) = \binom{n}{i-2}$, if $3 < i \leq n$.

For $i > n$, $K_{2,n}$ contains $\binom{n+2}{i}$ number of sets, that are all secure connected dominating sets.

Therefore, $d_{sc}(K_{2,n}, i) = \binom{n+2}{i}$, if $i > n$.

Hence, the proof.

Remark 2.4.

(i) $d_{sc}(K_{2,n}, i) = d_{sc}(K_{2,n-1}, i) + 1$ for all $n > 2$ and $i = 3$.

(ii) $d_{sc}(K_{2,n}, i) = d_{sc}(K_{2,n-1}, i) + d_{sc}(K_{2,n-1}, i-1)$, for $3 < i < n$ and $i = n$.

(iii) $d_{sc}(K_{2,n}, i) = [d_{sc}(K_{2,n-1}, i) + d_{sc}(K_{2,n-1}, i-1)] - 2$, for $i = n$.

Proof.

(i) From the table, it is obviously

$$d_{sc}(K_{2,n}, i) = d_{sc}(K_{2,n-1}, i) + 1 \text{ for all } n > 2 \text{ and } i = 3.$$

(ii) From the table, for $3 < i < n$ and $i > n$

$$\binom{n-1}{i-1} + \binom{n-1}{i} = \binom{n}{i}$$

$$\Rightarrow d_{sc}(K_{2,n-1}, i-1) + d_{sc}(K_{2,n-1}, i) = d_{sc}(K_{2,n}, i).$$

(iii) Also from the table, for $i = n$

$$[\binom{n-1}{i-1} + \binom{n-1}{i}] - 2 = \binom{n}{i}$$

$$\Rightarrow [d_{sc}(K_{2,n-1}, i-1) + d_{sc}(K_{2,n-1}, i)] - 2 = d_{sc}(K_{2,n}, i).$$

i	1	2	3	4	5	6	7	8	9	10	11	12	13
2,n													
2,1	0	0	1										
2,2	0	0	4	1									
2,3	0	0	3	5	1								
2,4	0	0	4	6	6	1							
2,5	0	0	5	10	10	7	1						

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2,6	0	0	6	15	20	15	8	1					
2,7	0	0	7	21	35	35	21	9	1				
2,8	0	0	8	28	56	70	56	28	10	1			
2,9	0	0	9	36	84	126	126	84	36	11	1		
2,10	0	0	10	45	120	210	252	210	120	45	12	1	
2,11	0	0	11	55	165	330	462	462	330	165	55	13	1

Table 1 $d_{sc}(K_{2,n}, i)$, the number of secure connected dominating sets with cardinality i

3. Secure connected domination polynomial of $K_{2,n}$

Definition 3.1. Let $D_{sc}(K_{2,n}, i)$ denote the family of all secure connected dominating sets of $K_{2,n}$ with cardinality i and let $d_{sc}(K_{2,n}, i)= |D_{sc}(K_{2,n}, i)|$. Then the secure connected domination polynomial $D_{sc}(K_{2,n}, x)$ of $K_{2,n}$ is defined as, $D_{sc}(K_{2,n}, x) = \sum_{i=\gamma_{sc}(K_{2,n})}^{n+2} d_{sc}(K_{2,n}, i)$, where $\gamma_{sc}(K_{2,n})$ is the secure connected domination number of $K_{2,n}$.

Theorem 3.2. $D_{sc}(K_{2,n}, x) = [(1 + \mathrm{x})D_{sc}(K_{2,n-1}, x)] - 2x^n + x^3$, with the initial value $D_{sc}(K_{2,2}, x) = x^4 + 4x^3$.

Proof. We have, $D_{sc}(K_{2,n}, x) = \sum_{i=3}^{n+2} d_{sc}(K_{2,n}, i)x^i$

$$= d_{sc}(K_{2,n}, 3)x^3 + \sum_{i=4}^{n+2} d_{sc}(K_{2,n}, i)x^i$$

$$= nx^3 + \sum_{i=4}^{n-1,n+1,n+2} d_{sc}(K_{2,n}, i)x^i + \sum_{i=n} d_{sc}(K_{2,n}, i)x^i$$

$$= nx^3 + \sum_{i=4}^{n-1,n+1,n+2}[d_{sc}(K_{2,n-1}, i) + d_{sc}(K_{2,n-1}, i-1)]x^i + \sum_{i=n}\{[d_{sc}(K_{2,n-1}, i) + d_{sc}(K_{2,n-1}, i-1)] - 2\}x^i$$

$$= nx^3 + \sum_{i=4}^{n-1,n+1,n+2}[d_{sc}(K_{2,n-1}, i) + d_{sc}(K_{2,n-1}, i-1)]x^i + d_{sc}(K_{2,n-1}, n)x^n + d_{sc}(K_{2,n-1}, n-1)x^n - 2x^n$$

$$= nx^3 + \sum_{i=4}^{n+2}[d_{sc}(K_{2,n-1}, i) + d_{sc}(K_{2,n-1}, i-1)]x^i - 2x^n$$

$$= nx^3 + \sum_{i=4}^{n+2} d_{sc}(K_{2,n-1}, i)x^i + \sum_{i=4}^{n+2} d_{sc}(K_{2,n-1}, i-1)x^i - 2x^n$$

Consider,

$$\sum_{i=4}^{n+2} d_{sc}(K_{2,n-1}, i)x^i = \sum_{i=3}^{n+2} d_{sc}(K_{2,n-1}, i)x^i - d_{sc}(K_{2,n-1}, 3)x^3$$

$$= \sum_{i=3}^{n+2} d_{sc}(K_{2,n-1}, i)x^i - (n-1)x^3$$

$$= D_{sc}(K_{2,n-1}, x) - (n-1)x^3$$

Again consider,

$$\sum_{i=4}^{n+2} d_{sc}(K_{2,n-1}, i-1)x^i = x\sum_{i=4}^{n+2} d_{sc}(K_{2,n-1}, i-1)x^{i-1}$$

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$$= x\sum_{i=3}^{n+1} d_{sc}(K_{2,n-1}, i)x^i$$
$$= xD_{sc}(K_{2,n-1}, x)$$

Now,

$$D_{sc}(K_{2,n-1}, x) = nx^3 + D_{sc}(K_{2,n-1}, x) - (n-1)x^3 + xD_{sc}(K_{2,n-1}, x) - 2x^n$$
$$\Rightarrow D_{sc}(K_{2,n}, x) = [(1 + \mathrm{x})D_{sc}(K_{2,n-1}, x)] - 2x^n + x^3.$$

Remark 3.3. $D_{sc}(K_{2,n}, x) = [\sum_{i=3}^{n+2}\binom{n}{i-2}x^i + \binom{n-1}{i-3}x^i] + 2x^{n+1}$

Theorem 3.4. The coefficients of $D_{sc}(K_{2,n}, x)$ possess the following characteristics:

(i) $d_{sc}(K_{2,n}, 1) = d_{sc}(K_{2,n}, 2) = 0.$

(ii) $d_{sc}(K_{2,n}, n+2) = 1$, for every n.

(iii) $d_{sc}(K_{2,n}, n+1) = n+2$, for every $n \geq 2$.

(iv) $d_{sc}(K_{2,n}, n) = \frac{n(n-1)}{2}$, for every $n \geq 3$.

(v) $d_{sc}(K_{2,n}, n-1) = \frac{n(n^2-3n+2)}{6}$, for every $n \geq 4$.

(vi) $d_{sc}(K_{2,n}, n-2) = \frac{n(n^3-6n^2+11n-6)}{24}$, for every $n \geq 5$.

(vii) $d_{sc}(K_{2,n}, i) = d_{sc}(K_{2,n-i-1}, i)$, for every $4 \leq i \leq n$.

Proof.

(i) Since every secure connected dominating set must contain atleast 3 vertices, we have

$$d_{sc}(K_{2,n}, 1) = d_{sc}(K_{2,n}, 2) = 0$$

(ii) Since $d_{sc}(K_{2,n}, n+2) = [n+2]$, we have the result.

(iii) We have, $d_{sc}(K_{2,n}, n+1) = \{[n+2] - \{x\}/x \in [n+2]\}$

Therefore, $d_{sc}(K_{2,n}, n+1) = n+2$, for every $n \geq 2$.

(iv) To prove $d_{sc}(K_{2,n}, n) = \frac{n(n-1)}{2}$, for every $n \geq 3$, we apply induction on n.

When $n = 3$, $LHS = d_{sc}(K_{2,3}, 3) = 3$(from the table)

$$RHS = \frac{1}{2} \times 3 \times 2 = 3$$

Therefore, $LHS = RHS$

Now, suppose that the result is true for all numbers less than n and we prove it for n.

We have, $d_{sc}(K_{2,n}, n) = d_{sc}(K_{2,n-1}, n) + d_{sc}(K_{2,n-1}, n-1) - 2$

$$= n + 1 + \frac{1}{2} \times (n-1) \times (n-2) - 2 = \frac{n(n-1)}{2}$$

Hence, $d_{sc}(K_{2,n}, n) = \frac{n(n-1)}{2}$, for every $n \geq 3$.

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(v) To prove $d_{sc}\left(K_{2,n}, n-1\right) = \frac{n(n^2-3n+2)}{2}$, for every $n \geq 4$, we apply induction on n.

When $n = 4$, $LHS = d_{sc}\left(K_{2,4}, 3\right) = 4$(from the table)

$$RHS = \frac{1}{6} \times 4 \times 6 = 4$$

Therefore, $LHS = RHS$

Now, suppose that the result is true for all numbers less than n and we prove it for n.

We have, $d_{sc}\left(K_{2,n}, n-1\right) = d_{sc}\left(K_{2,n-1}, n-1\right) + d_{sc}\left(K_{2,n-1}, n-2\right)$

$$= \frac{1}{2} \times (n-1) \times (n-2) \times \frac{1}{6} \times (n-1) \times (n^2 - 5n + 6)$$

$$= \frac{n(n^2-3n+2)}{2}$$

Hence,$d_{sc}\left(K_{2,n}, n-1\right) = \frac{n(n^2-3n+2)}{2}$, for every $n \geq 4$.

*(vi)*To prove $d_{sc}\left(K_{2,n}, n-2\right) = \frac{n(n^3-6n^2+11n-6)}{24}$, for every $n \geq 5$, we apply induction on n.

When $n = 5$, $LHS = d_{sc}\left(K_{2,5}, 3\right) = 5$(from the table)

$$RHS = \frac{5}{24} \times 24 = 5$$

Therefore, $LHS = RHS$

Now, suppose that the result is true for all numbers less than n and we prove it for n.

We have, $d_{sc}\left(K_{2,n}, n-2\right) = d_{sc}\left(K_{2,n-1}, n-2\right) + d_{sc}\left(K_{2,n-1}, n-3\right)$

$$= \frac{1}{6} \times (n-1) \times (n^2 - 5n + 6)$$

$$\times \frac{1}{24} \times (n-1) \times ((n-1)^3 - 6(n-1)^2 + 11(n-1) - 6)$$

$$= \frac{n(n^2-3n+2)}{2}$$

Hence,$d_{sc}\left(K_{2,n}, n-2\right) = \frac{n(n^3-6n^2+11n-6)}{24}$, for every $n \geq 5$.

(vii) The result is obvious from table1.

4. Conclusion

In this paper, we have studied and discussed some properties of secure connected dominating sets and secure connected domination polynomials of complete bipartite graph$K_{2,n}$. We can further study this property for various types of graphs.

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SOME RESULTS OF GRAPHS USING ABC-INDEX

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Abstract

The atom-bond connectivity (ABC) index (*ABC*-index) of a nontrivial connected graph G, denoted by ABC(G), is defined as $ABC(G) = \sum_{v_iv_j \in E(G)} \sqrt{\frac{d_i+d_j-2}{d_i d_j}}$, where d_i is the degree of vertex v_i in G. In this paper we find the ABC- Index of Wheel graph (W_n), Helm graph (H_n), Centepede graph (P_n^*), Gear graph (G_n). Also we discuss some results on ABC - index of graphs.

Keywords: ABC- index, centipede, wheel, helm, gear graphs

2020 Mathematical Subject Classification: 05C20, 05C05

1. Introduction

Let G be a graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(G)$. The degree of each vertex v_i, denoted by $d_G(v_i)$ (or simply d_i), is the number of neighbors of v_i in G. The maximum and minimum vertex degree in G are denoted by Δ and δ, respectively. The number of vertices of the largest clique in a graph is called its clique number and is denoted by ω. The vertex connectivity of a graph G, denoted by v, is the smallest number of vertices whose removal disconnects G or reduces it to a single vertex. The index or spectral radius λ_1 of G is the largest eigenvalue of its adjacency matrix. The algebraic connectivity of G, denoted by a, is the second smallest eigenvalue of the Laplacian matrix of G. A k-partite graph is said to be complete if any two vertices are adjacent if and only if they belong to different partition classes. Our terminology and notation not defined here will conform to those in [1].

In 1998, Estrada et al. proposed a new index, which is latter known as the atom-bond connectivity (ABC) index [8]. The atom-bond connectivity index of a nontrivial graph G,

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denoted by $ABC(G)$, is defined as $ABC(G) = \sum_{v_iv_j \in E(G)} \sqrt{\frac{d_i+d_j-2}{d_id_j}}$, where d_i is the degree of vertex v_i in G. In [8], Estrada et al. used *ABC*-index for the purpose of modeling thermodynamic properties of organic chemical compounds. In 2008, Estrada published another paper, in which *ABC*-index is used as a tool to explain the stability of branched alkanes. This work has attracted the attention by several Mathematicians, resulting in a remarkable number of research papers on the mathematical properties of the *ABC*-index, see [2], [3], [4], [6], [7], [8], [9], [11], [12].

In this paper, we explore some results of atom-bond connectivity index of graphs.

2. ABC index of some known graphs

Definition 2.1. A wheel graph $W_n = K_1 + C_n$ is a graph formed by connecting a single universal vertex to all vertices of a cycle C_n.

Example 2.2.

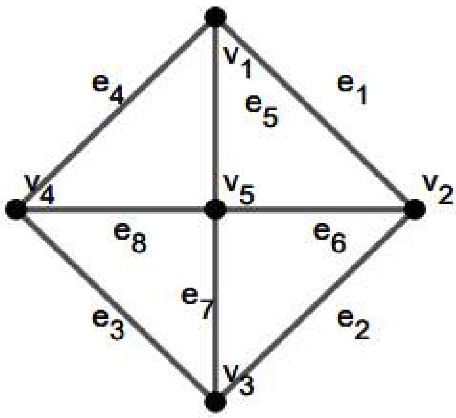

Figure 1: Wheel graph W_4

Theorem 2.3. For a wheel graph W_n, $ABC(W_n) = n\left[\frac{2}{3} + \sqrt{\frac{(n+1)}{3n}}\right]$, n ≥ 5

Proof. Let W_n be the wheel graph with $n+1$ vertices and $2n$ edges.

Let W_n, n edges having the sum of degrees of their vertices 6 and n edges having the sum of degrees of their vertices $n+3$

Let E_1 be the set of edges of W_n having the multiplication of degrees of their vertices 9 and sum of degrees of their vertices 6.

$$ABC(W_n) = \sum_{v_iv_j \epsilon E(G)} \sqrt{\frac{d_i+d_j-2}{d_id_j}}$$

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$$= \sum_{v_i v_j \in E(G)} \sqrt{\left(\frac{6-2}{9}\right)}$$

$$= n\sqrt{\left(\frac{4}{9}\right)}$$

$$= \frac{2n}{3}$$

Let E_2 be the set of edges of W_n having the multiplication of degrees of their vertices $3n$ and having the sum of degrees of their vertices $n+3$.

$$ABC(W_n) = \sum_{v_i v_j \epsilon E(G)} \sqrt{\frac{d_i+d_j-2}{d_i d_j}}$$

$$= \sum_{v_i v_j \epsilon E(G)} \sqrt{\frac{n+3-2}{3n}}$$

$$= \sum_{v_i v_j \epsilon E(G)} \sqrt{\frac{n+1}{3n}}$$

$$= n\sqrt{\frac{n+1}{3n}}$$

Therefore, $ABC(G) = \sum_{v_i v_j \in E(G)} \sqrt{\frac{d_i+d_j-2}{d_i d_j}}$

$$= \sum_{v_i v_j \epsilon E_1(G)} \sqrt{\frac{d_i+d_j-2}{d_i d_j}} + \sum_{v_i v_j \epsilon E_2(G)} \sqrt{\frac{d_i+d_j-2}{d_i d_j}}$$

$$= n\left(\frac{2}{3}\right) + n\sqrt{\frac{n+1}{3n}}$$

$$= n\left(\left(\frac{2}{3}\right) + \sqrt{\frac{n+1}{3n}}\right)$$

Defnition2.4. A Helm graph (H_n) is the graph obtained from a wheel graph W_n by adjoining a pendent edge at each node of the cycle.

Example 2.5.

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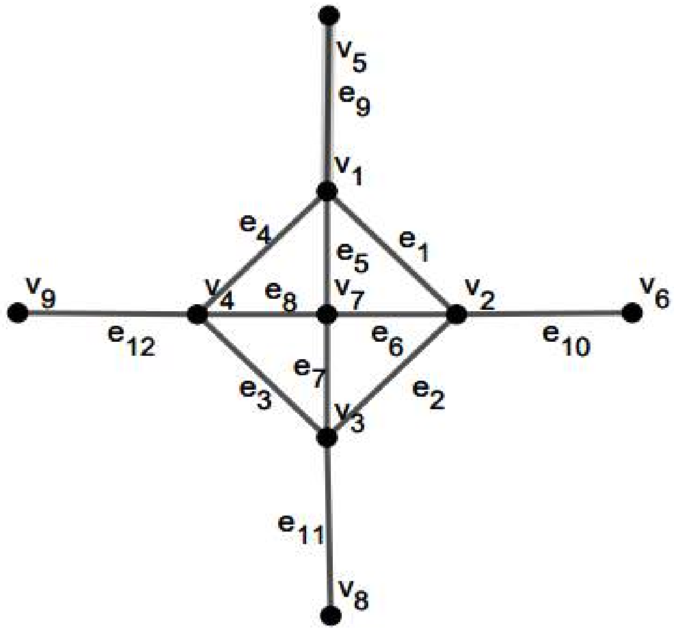

Figure 2: Helm graph H_4

Theorem 2.6. For the Helm graph H_n, $ABC(H_n) = n\sqrt{\frac{3}{4}} + \sqrt{\frac{3}{8}} + \sqrt{\frac{n+2}{4n}},$ n ≥ 5

Definition 2.7. A Centipede graph P_n^* is a graph on 2n vertices obtained by appending a single pendant edge to each vertex of a path P_n.

Example 2.8.

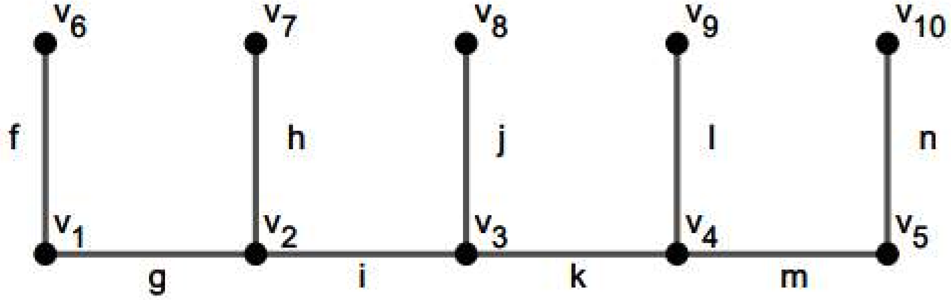

Figure 3: Centipede graph P_5^*

Theorem 2.9. For the Centipede graph P_n^*, $ABC(P_n^*) = 4\sqrt{\frac{1}{2}} + \sqrt{\frac{2}{3}}[(n-2)(n-3)], n \geq 3.$

Definition 2.10. A graph G_n is obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graph W_n. G_n has $2+n$ vertices and $3n$ edges.

Theorem 2.11. For the gear graph G_n, $ABC(G_n) = n\left(2\sqrt{\frac{3}{6}} + \sqrt{\frac{n+1}{3n}}\right), n \geq 3$

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3. Conclusion

In this paper, we determined ABC- index of some graphs. Further we can find ABC-index for new graph structures.

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EDGE COMBINATION CORDIAL LABELING OF GRAPHS

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Abstract

Let G be a graph with p vertices and q edges. An edge labeling $f: E(G) \to \left\{\binom{q}{i}, 0 \le i \le q\right\}$ is said to be an edge combination cordial labeling of G if it induces a vertex labeling $f^*: V \to \{0,1\}$ given by $f^*(v) = \begin{cases} 1 & if\ the\ labels\ of\ the\ edges\ incident\ to\ v\ are\ equal \\ 0 & otherwise \end{cases}$

such that $\left|v_{f^*}(0) - v_{f^*}(1)\right| \le 1$, where $v_{f^*}(0)$ is the number of edges labeled with 0 and $v_{f^*}(1)$ is the number of edges labeled with 1. A graph G is said to be an edge combination cordial graph if it admits edge combination cordial labeling. In this paper we prove the existence of this labeling of path, cycle, flower, $P_n \odot K_1$, $C_n \odot K_1$, ladder and jewel graph.

Keywords: cordial labeling, combination labeling, combination cordial labeling, edge combination cordial labeling.

2020 Mathematics Subject Classification (AMS): 05C78

1. Introduction

The graphs referred to here are assumed to be simple, finite, connected and undirected. We adopt Harary's [3] definitions for additional terminology. One of the prominent areas of research in graph theory is graph labeling. Graph labeling is an assignment of integers to the elements of a graph under certain conditions. Rosa [5] initially proposed graph labeling in 1967. Numerous types of graph labeling have been developed over the past fifty-five years. Gallian [2] elegantly categorized these labelings in his survey. Cordial labeling, one of the popular labelings was introduced by Cahit [1]. Suresh Manjanath Hegde et. al [6] proposed combinatorial labeling in 2005. Drawing from the idea of these two, B.J.Murali et. al [4] introduced the concept of combination cordial labeling. Building upon the previous concepts, we introduced a new notation namely edge combination cordial labeling as an edge counterpart of combination cordial labeling.

This paper focuses on exploring the existence of edge combination cordial labeling of path, cycle, flower, $P_n \odot K_1$, $C_n \odot K_1$, ladder and jewel graph.

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Definition 1.1. [6] A (p,q) graph $G=(V,E)$ is said to be combination graph if there exists a bijection $f:V(G)\rightarrow\{1,2,3,\dots\dots p\}$ such that the induced edge function $g_f:E(G)\rightarrow N$ defined as $g_f(uv)=\begin{cases} f(u)C_{f(v)} & if \quad f(u)>f(v) \\ f(u)C_{f(v)} & if \quad f(v)>f(u)\end{cases}$

is injective, where $f(u)C_{f(v)}$ is the number of combinations of $f(u)$ things taken $f(v)$ at a time. Such a labeling f is called combination labeling of G.

Definition 1.2. [1] Let f be a function from the vertices of G to $\{0,1\}$ and for each edge xy assign the label $|f(x)-f(y)|$. Call f a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1

Definition 1.3. [4] Let $G = (V, E)$ be a graph with n vertices. A function $f : V(G) \rightarrow \left\{\binom{n}{i}; 0\le i\le n\right\}$ of a graph G is said to be a combination cordial labeling if the induced edge function $f^* : E \rightarrow \{0, 1\}$ defined by $f^*(uv)=\begin{cases} 1 & \text{if } f(u)=f(v) \\ 0 & \text{if } f(u)\neq f(v)\end{cases}$ satisfies the condition $|e_{f^*}(0) - e_{f^*}(1)| \le 1$.

2. MAIN RESULTS

In this paper we introduce the concept of edge combination cordial labeling behavior of path, cycle , flower, $P_n\odot K_1$, $C_n\odot K_1$, ladder and jewel graph.

Theorem 2.1. The path graph P_n is edge combination cordial if $n\ge 3$.

Proof. Let P_n be a path graph with n vertices and n-1 edges.

$V(P_n)=\{v_i \,/\, 1\le i\le n\}$, $E(P_n)=\{v_iv_{i+1}/1\le i\le n-1\}$.

An edge labeling $f:E(P_n)\rightarrow\left\{\binom{n-1}{i}, 0\le i\le n-1\right\}$ for $n\ge 3$ is defined as follows:

$$f(v_1v_2) = \binom{n-1}{0}$$

$$f(v_{n-1}v_n) = \begin{cases} \binom{n-1}{0} & if\ \ n\ is\ even \\ \binom{n-1}{\frac{n-1}{2}} & if\ n\ is\ odd\,. \end{cases}$$

If n is even, then

$$f(v_iv_{i+1}) = f(v_{i+1}v_{i+2}) = \binom{n-1}{\frac{i}{2}} \quad where\ \ i=2,4,6\dots n-2.$$

If n is odd, then

$$f(v_iv_{i+1}) = f(v_{i+1}v_{i+2}) = \binom{n-1}{\frac{i}{2}} \quad where\ \ i=2,4,6\dots n-3.$$

Then the induced vertex labeling $f^*:V\rightarrow\{0,1\}$ is defined as follows:

Case (i) n is even.

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$$f^*(v_i) = \begin{cases} 1 & if \ i = 1,3,5, \dots . n-3, n-1 \\ 0 & if \ i = 2,4,6 \dots n-2, n \end{cases}$$

Thus we get $v_{f^*}(1) = \frac{n}{2}$, $v_{f^*}(0) = \frac{n}{2}$.

Case (ii) n is odd.

$$f^*(v_i) = \begin{cases} 1 & if \ i \ is \ odd \\ 0 & if \ i \ is \ even. \end{cases}$$

Thus we get $v_{f^*}(1) = \frac{n+1}{2}$, $v_{f^*}(0) = \frac{n-1}{2}$.

In both the cases $|v_{f^*}(0) - v_{f^*}(1)| \leq 1$.

Hence P_n is edge combination cordial if $n \geq 3$.

An example of edge combination cordial labeling of P_8 is given below:

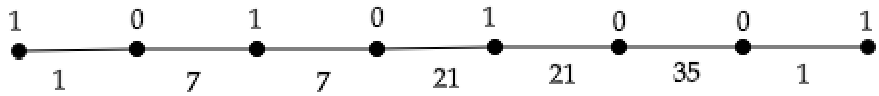

Figure 1

Theorem 2.2. The cycle graph C_n is edge combination cordial if $n \geq 3$.

Proof. Let C_n be a cycle graph with n vertices and n edges.

$V(C_n) = \{v_i \ / \ 1 \leq i \leq n\}$, $E(C_n) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\}$

An edge labeling $f: E(C_n) \rightarrow \left\{\binom{n}{i}, 0 \leq i \leq n\right\}$ for $n \geq 3$ is defined as follows :

$$f(v_1 v_2) = \binom{n}{0},$$

$$f(v_n v_1) = \begin{cases} \binom{n}{0} & if \ n \ is \ even \\ \binom{n}{\frac{n-1}{2}} & if \ n \ is \ odd . \end{cases}$$

If n is even, then

$$f(v_i v_{i+1}) = f(v_{i+1} v_{i+2}) = \binom{n}{\frac{i}{2}} \quad where \ i = 2,4,6 \dots n-2,$$

If n is odd, then

$$f(v_i v_{i+1}) = f(v_{i+1} v_{i+2}) = \binom{n}{\frac{i}{2}} \quad where \ i = 2,4,6 \dots n-1$$

Then the induced vertex labeling $f^*: V \rightarrow \{0,1\}$ is defined as follows:

Case (i): n is even.

$$f^*(v_i) = \begin{cases} 1 & if \ i \ is \ odd \\ 0 & if \ i \ is \ even . \end{cases}$$

Thus we get $v_{f^*}(1) = \frac{n}{2}$, $v_{f^*}(0) = \frac{n}{2}$.

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Case (ii): n is odd .

$$f^*(v_i) = \begin{cases} 1 & if\ i\ is\ odd,\ \ i \neq 1 \\ 0 & if\ i\ is\ even, i = 1 \end{cases}$$

Thus we get $v_{f^*}(1) = \frac{n-1}{2}$, $v_{f^*}(0) = \frac{n+1}{2}$.

In both the cases $\left|v_{f^*}(0) - v_{f^*}(1)\right| \leq 1$.

Hence C_n is edge combination cordial if $n \geq 3$.

An example of edge combination cordial labeling of C_5 is given below:

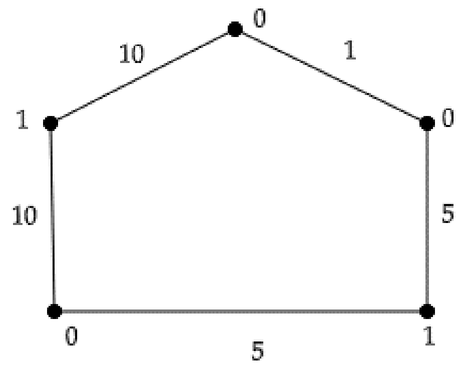

Figure 2

Theorem 2.3. The flower graph Fl_n is edge combination cordial if n≥ 3.

Proof. Let Fl_n be a flower graph with $2n + 1$ vertices and $4n$ edges.

$V(Fl_n) = \{v_i \,/\, 0 \leq i \leq n\} \cup \{w_i \,/\, 1 \leq i \leq n\}$, ,

$E(Fl_n) = \{v_0 v_i / 1 \leq i \leq n\} \cup \{v_0 w_i / 1 \leq i \leq n\} \cup \{v_i w_i / 1 \leq i \leq n\} \cup$
$\{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.

An edge labeling $f: E(Fl_n) \rightarrow \left\{\binom{4n}{i}, 0 \leq i \leq 4n\right\}$ for n≥ 3 defined as follows:

$$f(v_i v_{i+1}) = f(v_n v_1) = \binom{4n}{1}, \ 1 \leq i \leq n-1,$$

$$f(v_0 v_i) = f(v_i w_i) = f(v_0 w_i) = \binom{4n}{4n}, \ 1 \leq i \leq n.$$

Then the induced vertex labeling $f^*: V \rightarrow \{0,1\}$ is defined by,

$f^*(v_i) = 0, \ \ 0 \leq i \leq n, \ f^*(w_i) = 1, 1 \leq i \leq n$.

Thus we get, $v_{f^*}(1) = n$, $v_{f^*}(0) = n + 1$, Therefore $\left|v_{f^*}(0) - v_{f^*}(1)\right| \leq 1$.

Hence Fl_n is edge combination cordial if n≥ 3.

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An example of edge combination cordial labelling of Fl_5 is given below:

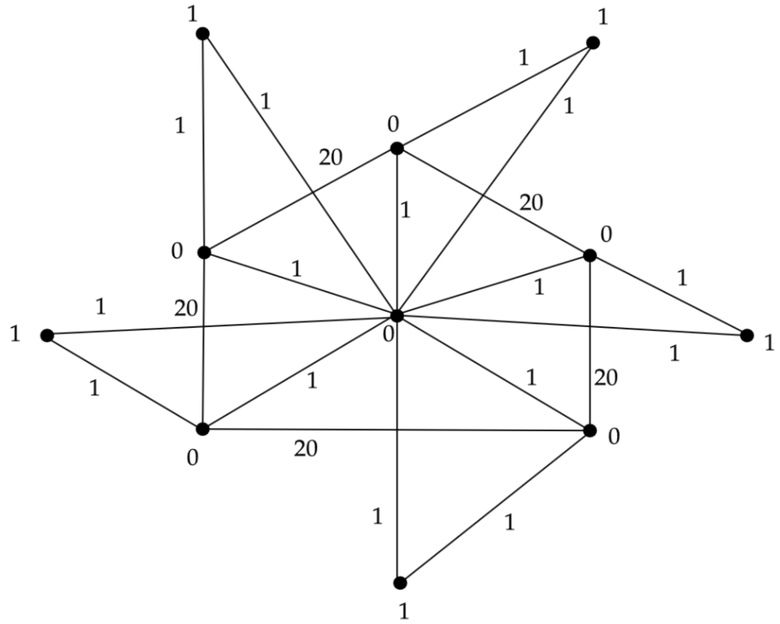

Figure 3

Theorem 3.4. The comb graph $P_n \odot K_1$ is edge combination cordial if n≥ 2.

Proof. Let $P_n \odot K_1$ be a comb graph with $2n$ vertices and $2n-1$ edges

$V(P_n \odot K_1) = \{u_i \,/\, 1 \leq i \leq n\} \cup \{v_i \,/\, 1 \leq i \leq n\}$,

$E(P_n \odot K_1) = \{u_i u_{i+1}/1 \leq i \leq n-1\} \cup \{u_i v_i/1 \leq i \leq n\}$.

An edge labeling $f: E(P_n \odot K_1) \rightarrow \left\{\binom{2n-1}{i}, 0 \leq i \leq 2n-1\right\}$ for n≥ 2 defined as follows:

$$f(u_i v_i) = \binom{2n-1}{0},\ 1 \leq i \leq n,$$

$$f(u_i u_{i+1}) = \binom{2n-1}{1},\ 1 \leq i \leq n-1.$$

Then the induced vertex labeling $f^*: V \rightarrow \{0,1\}$ is defined by,

$f^*(u_i) = 0,\ 1 \leq i \leq n,\ \ f^*(v_i) = 1,\ 1 \leq i \leq n.$

Thus we get $v_{f^*}(1) = n,\ \ v_{f^*}(0) = n$, Therefore $|v_{f^*}(0) - v_{f^*}(1)| \leq 1$.

Hence the comb graph $P_n \odot K_1$ is edge combination cordial if n≥ 2.

An example of edge combination cordial labeling of $P_6 \odot K_1$ is given below:

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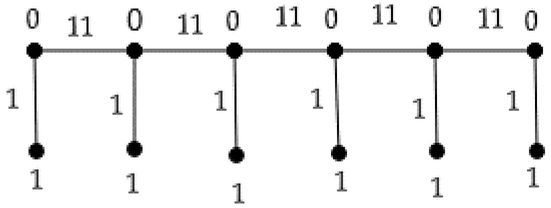

Figure 4

Theorem 3.5. The crown graph $C_n \odot K_1$ is edge combination cordial if $n \geq 3$.

Proof. Let $C_n \odot K_1$ be a graph with $2n$ vertices and $2n$ edges

$V(C_n \odot K_1) = \{u_i \,/\, 1 \leq i \leq n\} \cup \{v_i \,/\, 1 \leq i \leq n\}$,

$E(C_n \odot K_1) = \{u_i u_{i+1} 1 \leq i \leq n-1\} \cup \{u_i v_i 1 \leq i \leq n\} \cup \{u_n v_1 1 \leq i \leq n\}$

Ann edge labeling $f: E(C_n \odot K_1) \rightarrow \left\{\binom{2n-1}{i} \; 0 \leq i \leq 2n\right\}$ for $n \geq 3$ is defined as follows:

$$f(u_i v_i) = \binom{2n}{0},\; 1 \leq i \leq n,$$

$$f(u_i u_{i+1}) = \binom{2n}{1},\; 1 \leq i \leq n-1,$$

$$f(u_n u_1) = \binom{2n}{1}.$$

The induced vertex labeling $f^*: V \rightarrow \{0,1\}$ defined by,

$f^*(u_i) = 0, 1 \leq i \leq n,\; f^*(v_i) = 1, 1 \leq i \leq n$

Thus we get $v_{f^*}(1) = n,\; v_{f^*}(0) = n,$

Therefore $|v_{f^*}(0) - v_{f^*}(1)| \leq 1$

Hence $C_n \odot K_1$ is edge combination cordial graph if $n \geq 3$.

An example of edge combination cordial labeling of $C_5 \odot K_1$ is given below:

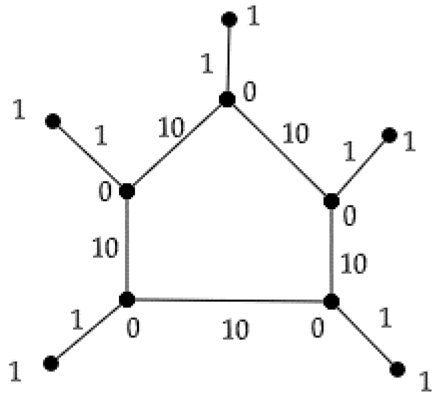

Figure 5

Theorem 3.6. The ladder graph $P_n X P_2$ is edge combination cordial if $n \geq 2$.

Proof. Let $P_n X P_2$ be a ladder graph with $2n$ vertices $3n-2$ edges.

$V(P_n X P_2) = \{u_i \,/\, 1 \leq i \leq n\} \cup \{v_i \,/\, 1 \leq i \leq n\}$,

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$E(P_nXP_2) = \{v_i\, v_{i+1} / \, 1 \leq i \leq n-1\} \cup \{u_iv_i \, / \, 1 \leq i \leq n\} \cup \{u_iu_{i+1} \, / \, 1 \leq i \leq n-1\}$.

An edge labeling $f: E(P_nXP_2) \to \left\{\binom{3n-2}{i},\ 0 \leq i \leq 3n-2\right\}$ for $n \geq 2$ is defined as follows:

$f(v_iv_{i+1}) = \binom{3n-2}{0}$, $1 \leq i \leq n-1$,

$f(u_iu_{i+1}) = \binom{3n-2}{1}$, $1 \leq i \leq n-1$,

$f(u_iv_i) \quad = \binom{3n-2}{0}$, $\quad 1 \leq i \leq n$.

Then the induced vertex labeling $f^*: V \to \{0,1\}$ is defined by,

$f^*(v_i) = 1$, $1 \leq i \leq n$, $f^*(u_i) = 0$, $1 \leq i \leq n$.

Thus we get $v_{f^*}(1) = n$, $v_{f^*}(0) = n$. Therefore $\left|v_{f^*}(0) - v_{f^*}(1)\right| \leq 1$.

Hence P_nXP_2 is edge combination cordial graph if $n \geq 2$.

An example of edge combination cordial labeling of P_5XP_2 is given below:

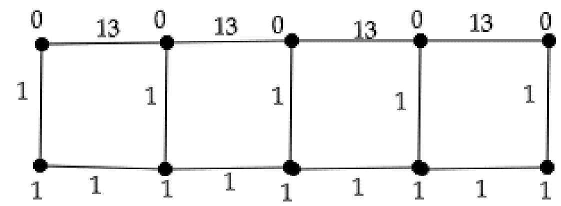

Figure 6

Theorem 3.7. The Jewel graph $(P_2 \cup mK_1) + N_2$ is edge combination cordial for $m \geq 1$.

Proof. Let $(P_2 \cup mK_1) + N_2$ be a graph with $m + 4$ vertices and $2m + 5$ edges

$V[\,(P_2 \cup mK_1) + N_2] = \{u, v, u_i / \, 1 \leq i \leq m\} \cup \{\, x, y\}$,

$E[\,(P_2 \cup mK_1) + N_2] = \{ux, uv, vx, uy, vy\} \cup \{\, u_ix \cup u_iy/1 \leq i \leq m\}$,

An edge labeling f: $E[\,(P_2 \cup mK_1) + N_2] \to \left\{\binom{2m+5}{i}, 0 \leq i \leq 2m+5\right\}$ is defined as follows:

Case (i): $1 \leq m \leq 3$

$f(uv) = f(vx) = f(vy) = \binom{2m+5}{0}$

$f(ux) = f(uy) = \binom{2m+5}{1}$, $f(u_ix) = f(u_iy) = \binom{2m+5}{0}$, $1 \leq i \leq 3$

Then the induced vertex labeling $f^*: V \to \{0,1\}$ is defined as follows:

$f^*(u) = f^*(x) = f^*(y) = 0$,

$f^*(v) = 1$, $\quad f^*(u_i) = 1$, $\quad 1 \leq i \leq 3$

Thus we get $v_{f^*}(1) = 3$, $v_{f^*}(0) = m + 1$

Case (ii): $m \geq 4$

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$f(uv) = \binom{2m+5}{0}$

$f(ux) = f(uy) = f(vx) = f(vy) = \binom{2m+5}{1}$

$f(u_i y) = \binom{2m+5}{0}$, $1 \le i \le m$

If m is odd, then

$$f(u_i x) = \begin{cases} \binom{2m+5}{0} & for \;\; 1 \le i \le \frac{m+5}{2}, \\ \binom{2m+5}{1} & for \;\; \frac{m+7}{2} \le i \le m \end{cases},$$

If m is even, then

$$f(u_i x) = \begin{cases} \binom{2m+5}{0} & for \;\; 1 \le i \le \frac{m+4}{2}, \\ \binom{2m+5}{1} & for \;\; \frac{m+6}{2} \le i \le m \end{cases}$$

Then the induced vertex labeling $f^*: V \to \{0,1\}$ is defined as follows:

$f^*(u) = f^*(v) = f^*(x) = f^*(y) = 0$

Subcase (i): m is odd

$f^*(u_i) = 1$ for $1 \le i \le \frac{m+5}{2}$, $f^*(u_i) = 0$ for $\frac{m+7}{2} \le i \le m$.

Thus we get $v_{f^*}(1) = \frac{m+5}{2}$, $v_{f^*}(0) = \frac{m+3}{2}$.

Subcase (ii): m is even

$f^*(u_i) = 1$ for $1 \le i \le \frac{m+4}{2}$, $f^*(u_i) = 0$ for $\frac{m+6}{2} \le i \le m$

Thus we get $v_{f^*}(1) = \frac{m+4}{2}$, $v_{f^*}(0) = \frac{m+4}{2}$,

In both the cases $|v_{f^*}(0) - v_{f^*}(1)| \le 1$.

Hence $(P_2 \cup mK_1) + N_2$ is edge combination cordial graph.

An example of edge combination cordial labeling of $(P_2 \cup mK_1) + N_2$ is given below:

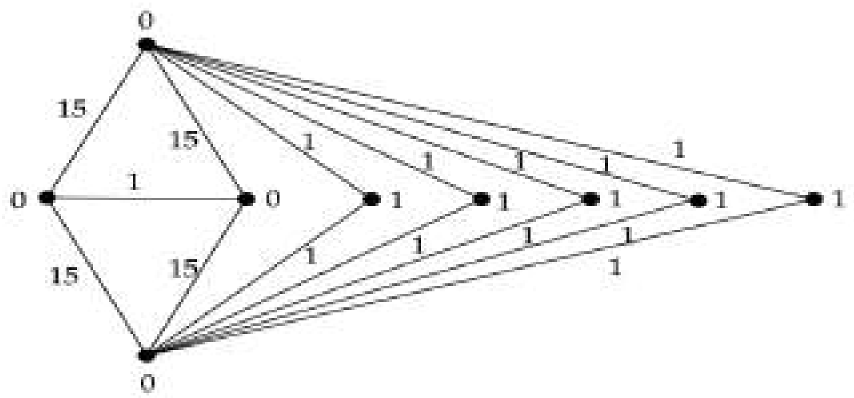

Figure 7

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4. Conclusion

We explore the concept of edge combination cordial labeling, a novel graph labeling technique. Through a comprehensive analysis, we demonstrated the existence of edge combination cordial labeling of path, cycle, flower, $P_n \odot K_1$, $C_n \odot K_1$, ladder and jewel graph.

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INTUITIONISTIC FUZZY IDEAL IN NEAR-RING

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Abstract

This paper investigate the concept of Intuitionistic fuzzy ideal and prime ideal in Near-ring. Also some definitions of Intuitionistic fuzzy ideal and prime ideal in a Near-ring. The purpose of this paper is to improve the concept of Intuitionistic fuzzy ideals of a Near-ring given a new characterization using the Intuitionistic fuzzy points. Moreover, some results and properties of Intuitionistic fuzzy prime ideal are discussed.

Keywords: Fuzzy ideal near-ring, Intuitionistic fuzzy ideal near-ring, Intuitionistic fuzzy points, Intuitionistic fuzzy prime ideal.

2020 Mathematical Subject Classification (AMS): 03E72

1. Introduction

Intuitionistic Fuzzy Ideal in Near-Ring is defined by Zhan Jianming & Ma Xueling[8] and fuzzy ideals of rings were introduced by Liu.W[6].The notion of fuzzy ideals and its properties were applied to various areas:semigroups [4,5]. In this paper we consider Intuitionistic Fuzzy Ideal in Near-Ring and investigate the related theorems and Properties.

2. Intuitionistic Fuzzy Ideal of Near-Ring

Definition 2.1. Let $\underline{R}$ be the subset of all **intuitionistic fuzzy points** of near-ring R and Let $\underline{A}$ denote the set of all intuitionistic fuzzy points contained in $A = <\mu_A, \lambda_A>$.

That is $\underline{A} = \{x_{(\alpha,\beta)} \in \underline{R} / \mu_A \geq \alpha \, and \, \lambda_A \leq \beta\}$

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Theorem 2.2. $A=<\mu_A,\lambda_A>$ is an intuitionistic fuzzy ideal of near-ring R if and only if:

i) $\forall x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in <\mu_A,\lambda_A>$, $x_{(\alpha,\beta)} - y_{(\alpha',\beta')} \in <\mu_A,\lambda_A>$

ii) $\forall x_{(\alpha,\beta)\in \underline{R}}, \forall y_{(\alpha',\beta')} \in <\mu_A,\lambda_A> \Rightarrow x_{(\alpha,\beta)} y_{(\alpha',\beta')} \in <\mu_A,\lambda_A>$

Proof. Assume $A=<\mu_A,\lambda_A>$ is an intuitionistic fuzzy ideal of near-ring R.

Since Near-ring R satisfies the following conditions

i) $\mu_A(y+x-y) \geq \mu_A(x)$
ii) $\lambda_A(y+x-y) \leq \lambda_A(x)$ ...... (1)

iii) $\mu_A(xy) \geq \mu_A(y)$
iv) $\lambda_A(xy) \leq \lambda_A(y)$ …… (2)

v) $\mu_A((x+z)y - xy) \geq \mu_A(z)$
vi) $\lambda_A((x+z)y - xy) \leq \lambda_A(z)$ …… (3) $\forall$ $x, y, z \in$ R

Now, Let $x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in <\mu_A,\lambda_A>$

From (1), we get $\mu_A(y+x-y) \geq \mu_A(x)$ and $\mu_A(x) \geq \alpha$

$\lambda_A(y+x-y) \leq \lambda_A(x)$ and $\lambda_A(x) \leq \beta$

$\Rightarrow \mu_A(y+x-y) \geq \mu_A(x) \geq \alpha$ and $\lambda_A(y+x-y) \leq \lambda_A(x) \leq \beta$

$\Rightarrow y+x-y_{(\alpha',\beta')} \in <\mu_A,\lambda_A>$

$\Rightarrow x_{(\alpha,\beta)} \in <\mu_A,\lambda_A>$

Similarly we can prove $x+y-x \in <\mu_A,\lambda_A>$

$\Rightarrow y_{(\alpha',\beta')} \in <\mu_A,\lambda_A>$

By assumption (1),

$x-y \in <\mu_A,\lambda_A>$

From (2),

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$\mu_A(xy) \geq \mu_A(y)$ and $\mu_A(y) \geq \alpha'$ and $\lambda_A(xy) \leq \lambda_A(y)$ and $\lambda_A(y) \leq \beta'$

$\Rightarrow \mu_A(xy) \geq \mu_A(y) \geq \alpha'$

and $\lambda_A(xy) \leq \lambda_A(y) \leq \beta'$

$\Rightarrow xy_{(\alpha',\beta')} \in <\mu_A, \lambda_A>$

Conversly, Assume

1. $\forall x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in <\mu_A, \lambda_A>$

$\Rightarrow \quad x - y \in <\mu_A, \lambda_A>$ …… (4)

2. $\forall x \in \underline{R}, y \in <\mu_A, \lambda_A> \Rightarrow \quad xy \in <\mu_A, \lambda_A>$ …… (5)

To prove that $A = <\mu_A, \lambda_A>$ is an Intuitionistic fuzzy ideal of near-ring R.

i.e. It is enough to prove that (1), (2), and (3)

Let x, $y \in$ R, we have $y+x$, $y \in$R

We have $y + x_{(\mu_A(x), \lambda_A(x))} \in <\mu_A, \lambda_A>$ and $y_{(\mu_A(x),\, \lambda_A(x))} \in <\mu_A, \lambda_A>$

Then By (4), we have $x + y_{(\mu_A(x), \lambda_A(x))} - y_{(\mu_A(x), \lambda_A(x))} \in <\mu_A, \lambda_A>$

Hence, $\mu_A(x + y - y) \geq \mu_A(x)$

and $\lambda_A(x + y - y) \leq \lambda_A(x)$

Hence (1) Proved

Now we show that $\mu_A(xy) \geq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(x)$

Let $x_{(\alpha,\beta)} \in \underline{R}$ and $y_{(\alpha',\beta')} \in <\mu_A, \lambda_A>$

By (5), $xy_{(u_{A(x)}, \lambda_{A(x)})} \in <\mu_A, \lambda_A>$

Hence $\mu_A(xy) \geq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(x)$

Hence (2) proved.

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Now, we show that

$$\mu_A((x+z)y-xy) \geq \mu_A(z) \text{ and } \lambda_A((x+z)y-xy) \leq \lambda_A(z)$$

we have, $x+z \in$ R and $y \in <\mu_A, \lambda_A>$

$$(x+z)y \in <\mu_A, \lambda_A \text{ [By (5)]}$$

Now, we have $(x+z)y \in <\mu_A, \lambda_A>$ and $xy \in <\mu_A, \lambda_A>$

From (4), $((x+z)y-xy)_{(\mu_A(z),\lambda_A(z))} \in <\mu_A, \lambda_A>$

Hence $\mu_A((x+z)y-xy) \geq \mu_A(z)$ and $\lambda_A((x+z)y-xy) \leq \lambda_A(z)$

Theorem 2.3. An Intuitionistic fuzzy ideal $<\mu_A, \lambda_A>$ of near-ring R is an intuitionistic fuzzy prime ideal iff for any two intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in \underline{R}$, $x_{(\alpha,\beta)} \bullet y_{(\alpha',\beta')} \in <\mu_A, \lambda_A>$. Implies either $x_{(\alpha,\beta)} \in <\mu_A, \lambda_A>$ or $y_{(\alpha',\beta')} \in <\mu_A, \lambda_A>$.

Theorem 2.4. A subset $<\mu_A, \lambda_A>$ of near-ring R is said to be an intuitionistic fuzzy prime ideal iff

$$\left.\begin{array}{ll} \text{i)} & \mu_A(y+x-y) \geq \mu_A(x) \\ \text{ii)} & \lambda_A(y+x-y) \leq \lambda_A(x) \end{array}\right\} \quad \ldots\ldots (6)$$

$$\left.\begin{array}{ll} \text{iii)} & \mu_A(xy) \geq \mu_A(y) \\ \text{iv)} & \lambda_A(xy) \leq \lambda_A(y) \end{array}\right\} \quad \ldots\ldots (7)$$

$$\left.\begin{array}{ll} \text{v)} & \mu_A((x+z)y-xy) \geq \mu_A(z) \\ \text{vi)} & \lambda_A((x+z)y-xy) \leq \lambda_A(z) \end{array}\right\} \quad \ldots\ldots (8)$$

Proof. Let $<\mu_A, \lambda_A>$ be an intuitionistic fuzzy prime ideal.

Suppose $\mu_A(y+x-y) < \mu_A(x)$ and $\lambda_A(y+x-y) > \lambda_A(x)$

$$\Rightarrow y+x-y \notin <\mu_A, \lambda_A>$$

$$\Rightarrow x \notin <\mu_A, \lambda_A>$$

Similarly we can prove $y \notin <\mu_A, \lambda_A>$

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$\Rightarrow \Leftarrow$

$\therefore$ Our assumption is wrong.

$\therefore \mu_A(y+x-y) \geq \mu_A(x)$ and $\lambda_A(y+x-y) \leq \lambda_A(y+x-y)$

Now, suppose $\mu_A(xy) < \mu_A(x)$ and $\lambda_A(xy) > \lambda_A(x)$

$\Rightarrow xy \notin < \mu_A, \lambda_A >$

Which is contradiction.

Hence $\mu_A(xy) = \mu_A(x)$ and $\lambda_A(xy) = \lambda_A(x)$

Now, suppose $\mu_A((x+z)y - xy) < \mu_A(z)$ and $\lambda_A((x+z)y - xy) > \lambda_A(z)$

$\Rightarrow (x+z)y - xy \notin < \mu_A, \lambda_A >$

$zy \notin < \mu_A, \lambda_A >$

Which is absurd

Then $\mu_A((x+z)y - xy) \geq \mu_A(z)$ and $\lambda_A((x+z)y - xy) \leq \lambda_A(z)$

Conversely, Assume (6), (7) and (8)

To we prove that $\forall x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in \underline{R}$

$x_{(\alpha,\beta)} \bullet y_{(\alpha',\beta')} \in < u_A, \lambda_A >$

Implies either $x_{(\alpha,\beta)} \in < \mu_A, \lambda_A >$ or $y_{(\alpha',\beta')} \in < \mu_A, \lambda_A >$

Suppose $x_{(\alpha,\beta)} \notin < \mu_A, \lambda_A >$ and $y_{(\alpha',\beta')} \notin < \mu_A, \lambda_A >$

$\Rightarrow \mu_A(x) < \alpha$ and $\lambda_A(x) > \beta$ and $\mu_A(y) < \alpha'$ and $\lambda_A(y) > \beta'$

Let $\alpha = \alpha' = \mu_A(xy)$ and $\beta = \beta' = \lambda_A(xy)$

$\Rightarrow \mu_A(xy) > \mu_A(y)$ and $\lambda_A(xy) < \lambda_A(y)$

Which is contradiction to (7),

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Hence $<\mu_A,\lambda_A>$ is an intuitionistic fuzzy prime ideal.

Definition 2.5. Let A and B be two IFSs of X. Then the **disjunctive sum** is $A+B=<\mu_{A+B}(x),\lambda_{A+B}(x)>$ and the **disjunctive Difference** is $A-B=<\mu_{A-B}(x),\lambda_{A-B}(x)>$

Where $\mu_{A+B}(x)=$ max $\lfloor\mu_{A\cap B^c}(x),\mu_{A^c\cap B}(x)\rfloor$ and $\lambda_{A+B}(x)=$ min$\lfloor\lambda_{A\cap B^c}(x),\lambda_{A^c\cap B}(x)\rfloor$ and $\mu_{A-B}(x)$ = min $\{\mu_A(x),\mu_{B^c}(x)\}$, $\lambda_{A-B}(x)=$ max $\{\mu_A(x),\mu_{B^c}(x)\}$

Theorem 2.6. If $A=<\mu_A,\lambda_A>$ is an IFI of R then $A^c=<\mu_{A^c},\lambda_{A^c}>$ is also an IFI of R.

Proof. Let $x, y, z \in$ R.

Given $A=<\mu_A,\lambda_A>$ is an IFI of R.

∴ A satisfies the following axioms.

i) $\mu_A(y+x-y)\geq\mu_A(x)$ and $\lambda_A(y+x-y)\leq\lambda_A(x)$ …… (1)

ii) $\mu_A(xy)\geq\mu_A(y)$ and $\lambda_A(xy)\leq\lambda_A(y)$ …… (2)

iii) $\mu_A((x+z)y-xy)\geq\mu_A(z)$ and $\lambda_A((x+z)y-xy)\leq\lambda_A(z)$ …… (3)

Now, $\mu_{A^c}(y+x-y)=1-\mu_A(y+x-y)\geq 1-\mu_A(x)$ [By 1] $=\mu_{A^c}(x)$

Similarly we can prove $\lambda_{A^c}(y+x-y)\leq\lambda_{A^c}(x)$

Now, $\mu_{A^c}(xy)=1-\mu_A(xy)\geq 1-\mu_A(y)$ [By 2] $=\mu_{A^c}(y)$

Similarly we can prove $\lambda_{A^c}(xy)\leq\lambda_{A^c}(y)$

Now, $\mu_{A^c}[(x+z)y-xy]=1-\mu_A[(x+z)y-xy]\geq 1-\mu_A(z)$ [By 3] $=\mu_{A^c}(z)$

Similarly, we can prove $\lambda_{A^c}[(x+z)y-xy]\leq\lambda_{A^c}(z)$

Hence $A^c = <\mu_A{}^c, \lambda_A{}^c>$ is an IFI of Near-ring R.

Theorem 2.7. If A and B are two IFIs of R then $A\cap B^c=<\mu_{A\cap B^c},\lambda_{A\cap B^c}>$ is also IFI of R.

Proof. Let $A=<\mu_A,\lambda_A>$ and $B=<\mu_B,\lambda_B>$

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By previous theorem, If $B = <\mu_B, \lambda_B>$ is an IFI of R then $B^c = <\mu_{B^c}, \lambda_{B^c}>$ is also IFI of R.

W.H.T $A \cap B^c = <\mu_{A\cap B^c}(x), \lambda_{A\cap B^c}(x)>$

Where, $\mu_{A\cap B^c} = \min\{\mu_A(x), \mu_{B^c}(x)\}$ and $\lambda_{A\cap B^c} = \max\{\lambda_A(x), \lambda_{B^c}(x)\}$

i) $\mu_{A\cap B^c}(y+x-y) = \min\{\mu_A(y+x-y), \mu_{B^c}(y+x-y)\} \geq \min\{\mu_A(x), \mu_{B^c}(x)\}$

$= \mu_{A\cap B^c}(x)$ [Since A and B^c are IFI]

Similarly, we can prove $\lambda_{A\cap B^c}(y+x-y) \leq \lambda_{A\cap B^c}(x)$

ii) $\mu_{A\cap B^c}(xy) = \min\{\mu_A(xy), \mu_{B^c}(xy)\} \geq \min\{\mu_A(y), \mu_{B^c}(y)\} = \mu_{A\cap B^c}(y)$

Similarly we can prove $\lambda_{A\cap B^c}(xy) \leq \lambda_{A\cap B^c}(y)$

iii) $\mu_{A\cap B^c}[(x+z)y - xy] = \min\{\mu_A[(x+z)y - xy], \mu_{B^c}[(x+z)y - xy]\}$

$\geq \min\{\mu_A(z), \mu_{B^c}(z)\} = \mu_{A\cap B^c}(z)$

Similarly we can prove $\lambda_{A\cap B^c}[(x+z)y - xy] \leq \lambda_{A\cap B^c}(z)$

Hence $A \cap B^c$ is an IFI of R.

Theorem 2.8. IF $A = <\mu_A, \lambda_A>$ and $B = <\mu_B, \lambda_B>$ be two IFI of nearing R then $A+B = <\mu_{A+B}, \lambda_{A+B}>$ is an IFI of R.

Proof. W.K.T $\mu_{A+B}(x) = \max[\mu_{A\cap B^c}(x), \mu_{A^c\cap B}(x)]$ and $\lambda_{A+B}(x) = \min[\mu_{A\cap B^c}(x), \mu_{A^c\cap B}(x)]$

i) $\mu_{A+B}(y+x-y) = \max\{\mu_{A\cap B^c}(y+x-y), \mu_{A^c\cap B}(y+x-y)\}$

$\geq \max\{\mu_{A\cap B^c}(x), \mu_{A^c\cap B}(x)\}$ [By theorem 2] $= \mu_{A+B}(x)$

Similarly, we can prove,

$$\lambda_{A+B}(y+x-y) \leq \lambda_{A+B}(x)$$

ii) $\mu_{A+B}(xy) = \max\{\mu_{A\cap B^c}(xy), \mu_{A^c\cap B}(xy)\}$

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$$\geq \max\{\mu_{A\cap B^c}(y), \mu_{A^c\cap B}(y)\} = \mu_{A+B}(y)$$

Similarly, we get $\lambda_{A+B}(xy) \leq \lambda_{A+B}(y)$

iii) $\mu_{A+B}[(x+z)y-xy] = \max\{\mu_{A\cap B^c}[(x+z)y-xy], \mu_{A^c\cap B}[(x+z)y-xy]\}$

$$\geq \max\{\mu_{A\cap B^c}(z), \mu_{A^c\cap B}(z)\} = \mu_{A+B}(z)$$

Similarly, we get $\lambda_{A+B}[(x+z)y-xy] \leq \lambda_{A+B}(z)$

Hence A+B is an IFI of Nearring R.

Theorem 2.9. If A and B are IFI of nearring R, Then A-B is also an IFI of nearing R.

Proof. W.K.T $\mu_{A-B}(x) = \min\{\mu_A(x), \mu_{B^c}(x)\}$ and

$$\lambda_{A-B}(x) = \max\{\lambda_A(x), \lambda_{B^c}(x)\}$$

The proof is similar to the theorem 2.

Properties 2.10. If A, B, C are IFI of Near-ring R and 0,1 are respectively fuzzy null and fuzzy universal subsets then we have.

1. $\left.\begin{array}{l} A\cap B = B\cap A \\ A\cup B = B\cup A \end{array}\right\} \rightarrow$ Commutativity

2. $A\cap A = A, = B\cup B = B \rightarrow$ Indempotent

3. $\begin{array}{ll} A\cup 0 = A; & A\cap 0 = 0 \\ A\cup 1 = 1; & A\cap 1 = A \end{array}$

4. $\begin{array}{l} (A\cup B)\cup C = A\cup(B\cup C) \\ (A\cap B)\cap C = A\cap(B\cap C) \end{array} \rightarrow$ associativity

5. $(A^c)^c = A \rightarrow$ Involution

6. $\left.\begin{array}{l} (A\cap B)^c = A^c\cup B^c \\ (A\cup B)^c = A^c\cap B^c \end{array}\right\} \rightarrow$ De'Morgan's law

We give the proofs of 4, 5 and 6. The rest can be proved in the same manners.

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Result 2.11. Intuitionistic Fuzzy ideal of a Nearring R does satisfy distributive law.

Preposition 2.12. If A and $\{Ai\}_{i\in f}$ are IFI of nearring R then

1.a) $A\cup\left[\bigcap_i Ai\right]=\bigcap_i[A\cup Ai]$ & b) $A\cap\left[\bigcup_i Ai\right]=\bigcup_i[A\cap Ai]$

2. a) $\left[\bigcup_i Ai\right]^c=\bigcap_i Ai^c$ & b) $\left[\bigcap_i Ai\right]^c=\bigcup_i Ai^c$.

3. Conclusion

In this paper, we consider a new kind of Intuitionistic Fuzzy Ideal of Near-Ring, which is a generalization of Fuzzy Ring and Fuzzy Near-Ring. Some related properties of Intuitionistic Fuzzy Ideal of Near-Ring are described.

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THE ATOM-BOND CONNECTIVITY INDEX OF A NANOROD GRAPH

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Abstract

Topological indices are numerical values derived from the structure of a chemical graph.The Nanorod graph G_{Nr} is a simple connected graph which is constructed with NaOH concentration as vertices and other reaction parameters such as pH, temperature, time and volume of solvent in a given ratio as edges. In this article, we discuss the degree based topological index namely atom-bond connectivity index of a Nanorod graph. Also we find the numerical value in step values, $k = 0.1, 0.09, 0.08, 0.07, 0.06, 0.05, 0.04, 0.03, 0.02$ and 0.01.

Keywords : Nanorod graph, atom-bond connectivity index

AMS Subject Classification : 05C90,05C92.

1 Introduction

For notation and graph theory terminology not given here we follow [1].Chemical graph theory is widely studied by researchers due to its extensive applications in daily life. A topological index is a numerical invariant used as a molecular descriptor. This topological descriptor, also referred to as a graph theoretic index is a numerical quantity that represents the molecular graph structure and its unique chemical and physical properties. There are different classes of topological indices such as distance based, counting based and degree based topological indices [2]. Degree based topological indices are extensively studied and have important applications in chemical graph theory. The authors Sonia et.al [3].S.Sobiya, S.Sujitha and M.K Angel Jebitha defined and generated the Nanorod graph [4] by using [3] and various graphical parameters are studied in the previous work [4].

The Nanorod graph G_{Nr} is a simple connected graph with vertex $V(G) = \{v_1, v_2, v_3, \cdots v_n\}$ and edge set $E(G)$. The verties of G_{Nr} correspond to NaOH concentration and

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an edge between two vertices corresponding to the UV spectrum (pH, temperature, time, volume of solvent in a given ratio) of these NaOH concentration.To construct the family of Nanorod graphs, various step values can be employed. In this paper, we utilize ten step values denoted by $k(k = 0.1, 0.09, 0.08, 0.07, 0.06, 0.05, 0.04, 0.03, 0.02, 0.01)$. The order of the Nanorod graph, represented as 'p'is determined by $p = \left\lfloor \frac{1.5}{k} + 1 \right\rfloor$,while the size is '$q$' and the reaction time is 't'[5]. The degree of vertex u, indicated by d_u, is the number of edges that are incident to v.

Extrada and Torres introduced the atom-bond connectivity index in 1998 [6].The atom-bond Connectivity index is defined as

$$ABC(G) = \sum_{uv\epsilon E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

2 The degree based topological indices of a Nanorod graph

Theorem 2.1 Let G_{Nr} be a Nanorod Graph. Then the ABC index is $ABC(G_{Nr})$ is

$$\begin{cases} (-168n + 14pn + 104p - 714)^{\frac{1}{2}}(-154n + 14pn + 104p - 610)^{-\frac{1}{2}} \\ if \ k = 0.1, n = t - 1 \ and \ k = 0.09, n = t \\ (-4n^2 - 477n + 42pn + 117p - 491)^{\frac{1}{2}}(-4n^2 - 435n + 42pn + 117p - 358)^{-\frac{1}{2}} \\ if \ k = 0.08, n = t - 1 \ and \ k = 0.07, n = t \\ (-7.8056n^6 + 50.9999n^5 - 309.1111n^4 - 1324.8333n^3 + 12960.3333n^2 - 25570.6666n + \\ 49.1667pn^3 - 389pn^2 + 628.8333pn - 166p + 11920)^{\frac{1}{2}} \\ (-7.8056n^6 + 65.6667n^5 - 313.2778n^4 - 1661.6667n^3 + 11922.1667n^2 - 17873n + 75pn^3 \\ -392pn^2 + 623pn - 253p + 13560)^{-\frac{1}{2}} \ if \ k = 0.06, n = t - 1, k = 0.05, n = t, k = 0.04, n = t + 1 \\ and \ k = 0.03, n = t + 2 \\ (-16445n^2 - 438851n + 7768pn - 6139p + 457298)^{\frac{1}{2}}(-16445n^2 - 429961n + 7768pn - 6139p + \\ 446995)^{-\frac{1}{2}} \ if \ k = 0.02, n = t - 1 \ and \ `k = 0.01, n = t \end{cases}$$

Proof. Let G_{Nr} be a Nanorod Graph

we know that the ABC index is

$$ABC(G_{Nr}) = \sum_{uv\epsilon E(G_{Nr})} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

Case (I) When $k = 0.1, n = t - 1$ and $k = 0.09, n = t$

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$$\sum_{uv\epsilon E(G_{Nr})}(d_u + d_v - 2)$$

$$= [(p-1)+(p-1)-2] + 2[(p-1)+(p-2)-2] + 2[(p-1)+(p-5)-2] + 2[(p-1)+(p-7)-2] + (-4n+12)[(p-1)+(p-10)-2] + (6n-4)[(p-1)+(p-13)-2] + 6[(p-1)+(p-11)-2] + 6[(p-1)+(p-12)-2] + [(p-2)+(p-7)-2] + (-2n+6)[(p-2)+(p-10)-2] + (3n-2)[(p-2)+(p-13)-2] + 3[(p-2)+(p-12)-2] + 3[(p-2)+(p-11)-2] + 3[(p-5)+(p-12)-2] + 3[(p-5)+(p-11)-2] + (-2n+5)[(p-5)+(p-10)-2] + 3[(p-7)+(p-11)-2] + (-2n+5)[(p-7)+(p-10)-2] + (-2n+5)[(p-10)+(p-10)-2] + (3n-3)[(p-5)+(p-13)-2] + (3n-3)[(p-7)+(p-12)-2] + (3n-3)[(p-10)+(p-11)-2] + (n-1)[(p-13)+(p-10)-2]$$

On Simplification, we get

$\sum_{uv\epsilon E(G_{Nr})}(d_u + d_v - 2) = -168n + 14pn + 104p - 714$ (a)

$\sum_{uv\epsilon E(G_{Nr})}(d_u + d_v) = -154n + 14pn + 104p - 610$ (b) [7]

From (a) and (b) $ABC(G_{Nr}) = \sqrt{\frac{-168n+14pn+104p-714}{-154n+14pn+104p-610}}$

Therfore,

$$ABC(G_{Nr}) = (-168n + 14pn + 104p - 714)^{\frac{1}{2}}(-154n + 14pn + 104p - 610)^{-\frac{1}{2}}$$

Case(II) When $k = 0.08, n = t - 1$ and $k = 0.07, n = t$ $\sum_{uv\epsilon E(G_{Nr})}(d_u + d_v - 2) = 3[(p-1)+(p-1)-2] + 3[(p-1)+(p-3)-2] + 3[(p-1)+(p-6)-2] + 3[(p-1)+(p-9)-2] + (-3n+9)[(p-1)+(p-11)-2] + 12[(p-1)+(p-14)-2] + (6n-3)[(p-1)+(p-15)-2] + (-6n+15)[(p-1)+(p-13)-2] + 9[(p-1)+(-n+8)-2] + (-n+3)[(p-3)+(p-11)-2] + 4[(p-3)+(p-14)-2] + (2n-1)[(p-3)+(p-15)-2] + (-2n+5)[(p-3)+(p-13)-2] + 3[(p-3)+(-n+8)-2] + 3[(p-6)+(p-14)-2] + (-2n+5)[(p-6)+(p-13)-2] + 3[(p-6)+7-2] + (2n-1)[(p-6)+(-2n+10)-2] + (-2n+5)[(p-9)+(p-13)-2] + 3[(p-9)+7-2] + (2n-1)[(p-9)+8-2] + 3[(p-11)+7-2] + (2n-1)[(p-11)+8-2] + (2n-1)[(p-14)+8-2] + (12n-12)[(p-1)+(p-17)-2] + (4n-4)[(p-3)+(p-17)-2] + (3n-3)[(p-6)+(p-17)-2] + (3n-3)[(p-9)+(p-16)-2] + (n-1)[(p-11)+(p-13)-2] + (n-1)[(p-14)+(p-13)-2] + (n-1)[(p-17)+$

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$(p-13)-2]$

Simplifying the above expression, we get

$\sum_{uv\epsilon E(G_{Nr})}(d_u+d_v-2)=-4n^2-477n+42pn+117p-491$ (c)

$\sum_{uv\epsilon E(G_{Nr})}(d_u+d_v)=-4n^2-435n+42pn+117p-358$ (d) [7]

From (c) and (d) $ABC(G_{Nr})=\sqrt{\frac{-4n^2-477n+42pn+117p-491}{-4n^2-435n+42pn+117p-358}}$

Therfore,

$$ABC(G_{Nr})=(-4n^2-477n+42pn+117p-491)^{\frac{1}{2}}(-4n^2-435n+42pn+117p-358)^{-\frac{1}{2}}$$

Case(III) When $k=0.06, n=t-1, k=0.05, n=t, k=0.04, n=t+1$ and $k=0.03, n=$

$\sum_{uv\epsilon E(G_{Nr})}(d_u+d_v-2)=[-\frac{1}{3}n^3+\frac{7}{2}n^2-\frac{49}{6}n+8][(p-1)+(p-1)-2]+[-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+5][(p-1)+(p-2)-2]+[-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+5][(p-1)+(p-5)-2]+[-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+5][(p-1)+(p-7)-2]+[-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+5][(p-1)+(p-10)-2]+[-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+5][(p-1)+(p-13)-2]+[-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+5][(p-1)+(p-15)-2]+[-\frac{5}{3}n^3+15n^2-\frac{127}{3}n+41][(p-1)+(p-18)-2]+[\frac{13}{3}n^3-\frac{69}{2}n^2+\frac{493}{6}n-49][(p-1)+(p-21)-2]+[-\frac{2}{3}n^3+\frac{5}{2}n^2-\frac{509}{6}n+59][(p-1)+(\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{11}{3}n+8)-2]+[-\frac{17}{3}n^3+\frac{5}{2}n^2+\frac{1}{6}n+7][(p-1)+(\frac{7}{2}n^2-\frac{23}{2}n+15)-2]+[-\frac{25}{6}n^3+\frac{57}{2}n^2-\frac{169}{3}n+41][(p-1)+(\frac{7}{6}n^3-5n^2+\frac{29}{6}n+8)-2]+[-\frac{1}{2}n^3+\frac{9}{2}n^2-10n+15][(p-1)+(\frac{13}{6}n^3-13n^2+\frac{137}{6}n-2)-2]+[(p-2)+(p-7)-2]+[(p-2)+(p-10)-2]+[(p-2)+(p-13)-2]+[(p-2)+(p-15)-2]+[-\frac{1}{2}n^3+\frac{9}{2}n^2-13n+13][(p-2)+(p-18)-2]+[\frac{3}{2}n^3-12n^2+\frac{57}{2}n-17][(p-2)+(p-21)-2]+3[(p-2)+(-\frac{5}{3}n^3+\frac{27}{2}n^2-\frac{167}{6}n+22)-2]+3[(p-2)+(\frac{7}{6}n^3-\frac{17}{2}n^2+\frac{58}{3}n-5)-2]+3[(p-2)+(\frac{7}{6}n^3-\frac{17}{2}n^2+\frac{58}{3}n-3)-2]+[\frac{1}{6}n^3-\frac{3}{2}n^2+\frac{13}{3}n][(p-2)+(-\frac{5}{6}n^3+8n^2-\frac{121}{6}n+23)-2]+[(p-5)+(p-15)-2]+[-\frac{1}{2}n^3+\frac{9}{2}n^2-13n+13][(p-5)+(p-18)-2]+[\frac{3}{2}n^3-12n^2+\frac{57}{2}n-17][(p-5)+(p-21)-2]+[\frac{1}{6}n^3-$

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$2n^2+\frac{35}{6}n-1][(p-5)+(\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{11}{3}n+8)-2]+[-\frac{4}{3}n^3+\frac{19}{2}n^2-\frac{121}{6}n+15][(p-5)+(\frac{7}{2}n^2-\frac{23}{2}n+15)-2]+[-\frac{5}{6}n^3+\frac{11}{2}n^2-\frac{32}{3}n+9][(p-5)+(\frac{7}{6}n^3-5n^2+\frac{29}{6}n+8)-2]+3[(p-5)+(\frac{13}{6}n^3-13n^2+\frac{137}{6}n-2)-2]+[\frac{1}{6}n^3-2n^2+\frac{35}{6}n-1][(p-7)+(\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{11}{3}n+8)-2]+[-\frac{5}{6}n^3+6n^2-\frac{79}{6}n+11][(p-7)+(\frac{7}{2}n^2-\frac{23}{2}n+15)-2]+[-\frac{5}{6}n^3+\frac{11}{2}n^2-\frac{32}{3}n+9][(p-7)+(\frac{4}{3}n^3-\frac{13}{2}n^2+\frac{55}{6}n+4)-2]+3[(p-7)+(\frac{7}{3}n^3-\frac{29}{2}n^2+\frac{163}{6}n-6)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n+2][(p-7)+(\frac{7}{6}n^3-7n^2+\frac{77}{6}n+3)-2]+3[(p-10)+(\frac{5}{3}n^3-\frac{19}{2}n^2+\frac{101}{6}n-2)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n+2][(p-10)+(n^3-\frac{11}{2}n^2+\frac{19}{2}n+3)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n+2][(p-10)+(\frac{1}{3}n^3-\frac{3}{2}n^2+\frac{13}{6}n+8)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n+2][(p-10)+(-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+12)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n+2][(p-13)+(\frac{1}{6}n^3-\frac{1}{2}n^2+\frac{1}{3}n+8)-2]+[-\frac{1}{6}n^3+n^2-\frac{11}{6}n+4][(p-13)-(\frac{1}{3}n^3+\frac{5}{2}n^2-\frac{31}{6}n+12)-2]+3[(p-13)+(-\frac{1}{3}n^3+\frac{5}{2}n^2-\frac{31}{6}n+13)-2]+3[(p-15)+(-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+11)-2]+3[(p-18)+(-\frac{1}{6}n^3+\frac{3}{2}n^2-\frac{10}{3}n+12)-2]+[\frac{11}{6}n^3-14n^2+\frac{229}{6}n-26][(p-1)+(4n^2-21n+37)-2]+[\frac{1}{3}n^3-2n^2+\frac{29}{3}n-8][(p-1)+(2n^2-11n+26)-2]+[-\frac{5}{2}n^2+\frac{29}{2}n-17][(p-2)+(8n^2-29n+32)-2]+[\frac{1}{6}n^3-\frac{3}{2}n^2+\frac{16}{3}n-4][(p-2)+(2n^2-11n+26)-2]+[\frac{2}{3}n^3-\frac{11}{2}n^2+\frac{89}{6}n-10][(p-5)+(4n^2-21n+37)-2]+[\frac{1}{6}n^3-\frac{3}{2}n^2+\frac{16}{3}n-4][(p-5)+(2n^2-11n+26)-2]+[\frac{2}{3}n^3-\frac{11}{2}n^2+\frac{89}{6}n-10][(p-7)+(\frac{3}{2}n^2-\frac{15}{2}n+20)-2]+[\frac{1}{6}n^3-\frac{3}{2}n^2+\frac{16}{3}n-4][(p-7)+(-n^2+6n+4)-2]+[\frac{1}{3}n^3-\frac{7}{2}n^2+\frac{67}{6}n-8][(p-10)+(-\frac{3}{2}n^2+\frac{17}{2}n)-2]+[-\frac{1}{2}n^2+\frac{7}{2}n-3][(p-10)+(-\frac{3}{2}n^2+\frac{17}{2}n+1)-2]+[-\frac{1}{2}n^3-\frac{9}{2}n^2+13n-9][(p-13)+(n+9)-2]+[-\frac{1}{2}n^2+\frac{7}{2}n-3][(p-13)+(n+10)-2]+[-\frac{1}{2}n^3-\frac{9}{2}n^2+13n-9][(p-15)+(n+9)-2]+[-\frac{1}{2}n^2+\frac{7}{2}n-3][(p-15)+(n+10)-2]+[-\frac{1}{2}n^3-\frac{9}{2}n^2+13n-9][(p-18)+(n+10)-2]+[-\frac{1}{2}n^3-\frac{9}{2}n^2+13n-9][(p-21)+(n+9)-2]+[-\frac{1}{2}n^2+\frac{7}{2}n-3][(p-21)+(n+10)-2]+[-\frac{1}{2}n^2+\frac{7}{2}n-3][(p-23)+(n+10)-2]+$

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$[-\frac{8}{3}n^3+22n^2-\frac{142}{3}n+28][(p-1)+(16-n)-2]+[-\frac{13}{3}n^3+32n^2-\frac{197}{3}n+38][(p-1)+(-4n+26)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-2)+(2n+7)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-2)+(2n+8)-2]+[-\frac{5}{6}n^3+\frac{13}{2}n^2-\frac{41}{3}n+8][(p-5)+(16-n)-2]+[-\frac{7}{6}n^3+\frac{17}{2}n^2-\frac{52}{3}n+10][(p-5)+(26-4n)-2]+[-\frac{7}{6}n^3+\frac{17}{2}n^2-\frac{52}{3}n+10][(p-7)+(26-4n)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-10)+(17-n)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-10)+15-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-13)+(n+11)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-13)+(n+12)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-15)+(n+11)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-15)+(n+12)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-18)+(n+11)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-18)+(n+12)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-21)+(n+11)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-21)+(n+12)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-23)+(n+11)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-23)+(n+12)-2]+[-n^3+\frac{15}{2}n^2-\frac{31}{2}n+9][(p-26)+(n+11)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-26)+(n+12)-2]+[-\frac{1}{2}n^3+4n^2-\frac{17}{2}n+5][(p-29)+(n+12)-2]+[\frac{5}{2}n^3-15n^2+\frac{55}{2}n-15][(p-1)+(p-40)-2]+[\frac{5}{2}n^3-15n^2+\frac{55}{2}n-15][(p-1)+(p-38)-2]+[\frac{5}{2}n^3-15n^2+\frac{55}{2}n-15][(p-1)+(p-36)-2]+[\frac{5}{2}n^3-15n^2+\frac{55}{2}n-15][(p-1)+(p-35)-2]+[\frac{5}{2}n^3-15n^2+\frac{55}{2}n-15][(p-1)+(p-33)-2]+[\frac{5}{2}n^3-15n^2+\frac{55}{2}n-15][(p-1)+(p-32)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n-1][(p-2)+(p-26)-2]+[\frac{1}{6}n^3-n^2+\frac{11}{6}n-1][(p-2)+(p-29)-2]+[\frac{2}{3}n^3-4n^2+\frac{22}{3}n-4][(p-2)+(p-39)-2]+[\frac{1}{3}n^3-2n^2+\frac{11}{3}n-2][(p-2)+(p-41)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-2)+(p-33)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-2)+(p-32)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-5)+(p-40)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-5)+(p-38)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-5)+(p-36)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-5)+(p-35)-2]+[\frac{1}{2}n^3-3n^2+\frac{11}{2}n-3][(p-5)+(p-33)-2]+[\frac{1}{2}n^3-$

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$3n^2 + \frac{11}{2}n - 3][(p-5) + (p-32) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-7) + (p-40) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-7) + (p-38) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-7) + (p-36) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-7) + (p-35) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-7) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-7) + (p-32) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-10) + (p-35) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-10) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-10) + (p-32) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-13) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-13) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-13) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-13) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-15) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-15) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-15) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-15) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-18) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-18) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-18) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-18) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-21) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-21) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-21) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-21) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-23) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-23) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-23) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-23) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-26) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-26) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-26) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-26) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-29) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-29) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-29) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-29) + (p-31) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-31) + (p-34) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-31) + (p-33) - 2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-31) + (p-32) - 2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-31) + (p-31) - 2] + [\frac{1}{2}n^3 -$

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$3n^2 + \frac{11}{2}n - 3][(p-34)+(p-33)-2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-34)+(p-32)-2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-34)+(p-31)-2] + [\frac{1}{2}n^3 - 3n^2 + \frac{11}{2}n - 3][(p-37)+(p-32)-2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-37)+(p-31)-2] + [\frac{1}{3}n^3 - 2n^2 + \frac{11}{3}n - 2][(p-39)+(p-31)-2]$

On Simplication, we get

$\sum_{uv\epsilon E(G_{Nr})} (d_u + d_v) = -7.8056n^6 + 65.6667n^5 - 313.2778n^4 - 1161.6667n^3 + 11922.1667n^2 - 17873n + 69pn^3 - 392pn^2 + 623pn - 144p + 12550$ (e) [7]

$\sum_{uv\epsilon E(G_{Nr})} (d_u + d_v - 2) = -7.8056n^6 + 50.9999n^5 - 309.1111n^4 - 1324.8333n^3 + 12960.3333n^2 - 25570.6666n + 49.1667pn^3 - 389pn^2 + 628.8333pn - 166p + 11920$ (f)

From (e) and (f)

$ABC(G_{Nr})$

$$= \sqrt{\frac{-7.8056n^6 + 50.9999n^5 - 309.1111n^4 - 1324.8333n^3 + 12960.3333n^2 - 25570.6666n + 49.1667p}{-7.8056n^6 + 65.6667n^5 - 313.2778n^4 - 1661.6667n^3 + 11922.1667n^2 - 17873n + 75pn^3}}$$

Therfore,

$$ABC(G_{Nr}) = (-7.8056n^6 + 50.9999n^5 - 309.1111n^4 - 1324.8333n^3 + 12960.3333n^2 - 25570.6666n + 49.1667pn^3 - 389pn^2 + 628.8333pn - 166p + 11920)^{\frac{1}{2}}(-7.8056n^6 + 65.6667n^5 - 313.2778n^4 - 1661.6667n^3 + 11922.1667n^2 - 17873n + 75pn^3 - 392pn^2 + 623pn - 253p + 13560)^{-\frac{1}{2}}$$

Case (IV) When $k = 0.02, n = t - 1$ and $k = 0.01, n = t$

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$$\sum_{uv\epsilon E(G_{Nr})} (d_u + d_v - 2)$$

$$= [57n - 36][(p-1) + (p-1) - 2] + [6n+1][(p-1) + (p-2) - 2] + [6n+1][(p-1) + (76n-6) - 2] + [6n+1][(p-1) + (76n-8) - 2] + [6n+1][(p-1) + (p-10) - 2] + [6n+1][(p-1) + (p-13) - 2] + [6n+1][(p-1) + (p-15) - 2] + [6n+1][(p-1) + (p-18) - 2] + [6n+1][(p-1) + (p-21) - 2] + [6n+1][(p-1) + (p-23) - 2] + [6n+1][(p-1) + (p-26) - 2] + [6n+1][(p-1) + (p-29) - 2] + [6n+1][(p-1) + (76n-32) - 2] + [6n+1][(p-1) + (74n-32) - 2] + [6n+1][(p-1) + (p-37) - 2] + [6n+1][(p-1) + (p-39) - 2] + [6n+1][(p-1) + (p-42) - 2] + [6n+1][(p-1) + (p-45) - 2] + [-15n+43][(p-1) + (74n-45) - 2] + [-15n+43][(p-1) + (p-50) - 2] + [-15n+43][(p-1) + (p-53) - 2] + [-15n+43][(p-1) + (p-55) - 2] + [-15n+43][(p-1) + (p-58) - 2] + [-15n+43][(p-1) + (p-61) - 2] + [-8n+29][(p-1) + (72n-56) - 2] + [-8n+29][(p-1) + (68n-51) - 2] + [-8n+29][(p-1) + (63n-44) - 2] + [-8n+29][(p-1) + (60n-40) - 2] + [-8n+29][(p-1) + (55n-33) - 2] + [-8n+29][(p-1) + (50n-26) - 2] + [-8n+29][(p-1) + (47n-22) - 2] + [-8n+29][(p-1) + (42n-15) - 2] + [-8n+29][(p-1) + (38n-10) - 2] + [6n+1][(p-1) + (34n-4) - 2] + [(p-2) + (76n-8) - 2] + [(p-2) + (p-10) - 2] + [(p-2) + (p-13) - 2] + [(p-2) + (p-15) - 2] + [(p-2) + (p-18) - 2] + [(p-2) + (p-21) - 2] + [(p-2) + (p-23) - 2] + [(p-2) + (p-26) - 2] + [(p-2) + (p-29) - 2] + [(p-2) + (76n-32) - 2] + [(p-2) + (74n-32) - 2] + [(p-2) + (p-37) - 2] + [(p-2) + (p-39) - 2] + [(p-2) + (p-42) - 2] + [(p-2) + (p-45) - 2] + [-3n+7][(p-2) + (74n-45) - 2] + [-3n+7][(p-2) + (p-50) - 2] + [-3n+7][(p-2) + (p-53) - 2] + [-3n+7][(p-2) + (p-55) - 2] + [-3n+7][(p-2) + (p-58) - 2] + [-3n+7][(p-2) + (p-61) - 2] + [-2n+5][(p-2) + (72n-56) - 2] + [-2n+5][(p-2) + (68n-51) - 2] + [-2n+5][(p-2) + (63n-44) - 2] + [-2n+5][(p-2) + (60n-40) - 2] + [-2n+5][(p-2) + (55n-33) - 2] + [-2n$$

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$+ 5][(p-2) + (50n-26) - 2] + [-2n+5][(p-2) + (47n-22) - 2]$
$+ [-2n+5][(p-2) + (42n-15) - 2] + [-2n+5][(p-2) + (38n-10)$
$- 2] + [(p-2) + (34n-4) - 2] + [(p-10) + (76n-32) - 2] + [(p-10)$
$+ (74n-32) - 2] + [(p-10) + (p-37) - 2] + [(p-10) + (p-39) - 2]$
$+ [(p-10) + (p-42) - 2] + [(p-10) + (p-45) - 2] + [-3n+7][(p$
$- 10) + (74n-4) - 2] + [-3n+7][(p-10) + (p-50) - 2] + [-3n$
$+ 7][(p-10) + (p-53) - 2] + [-3n+7][(p-10) + (p-55) - 2] + [-3n$
$+ 7][(p-10) + (p-58) - 2][-3n+7][(p-10) + (p-61) - 2] + [-2n$
$+ 5][(p-10) + (72n-56) - 2] + [-2n+5][(p-10) + (68n-51) - 2]$
$+ [-2n+5][(p-10) + (63n-44) - 2] + [-2n+5][(p-10) + (60n$
$- 40) - 2] + [-2n+5][(p-10) + (55n-33) - 2] + [-2n+5][(p-10)$
$+ (50n-26) - 2] + [-2n+5][(p-10) + (47n-22) - 2] + [-2n+5][(p$
$- 10) + (42n-15) - 2] + [-2n+5][(p-10) + (38n-10) - 2] + [(p$
$- 10) + (34n-4) - 2] + [(p-13) + (p-39) - 2] + [(p-13) + (p-42)$
$- 2] + [(p-13) + (p-45) - 2] + [-3n+7][(p-13) + (74n-45) - 2]$
$+ [-3n+7][(p-13) + (p-50) - 2] + [-3n+7][(p-13) + (p-53) - 2]$
$+ [-3n+7][(p-13) + (p-55) - 2] + [-3n+7][(p-13) + (p-58) - 2]$
$+ [-3n+7][(p-13) + (p-61) - 2] + [-2n+5][(p-13) + (72n-56)$
$- 2] + [-2n+5][(p-13) + (68n-51) - 2] + [-2n+5][(p-13) + (63n$
$- 44) - 2] + [-2n+5][(p-13) + (60n-40) - 2] + [-2n+5][(p-13)$
$+ (55n-33) - 2] + [-2n+5][(p-13) + (50n-26) - 2] + [-2n+5][(p$
$- 13) + (47n-22) - 2] + [-2n+5][(p-13) + (42n-15) - 2] + [-2n$
$+ 5][(p-13) + (38n-10) - 2] + [(p-13) + (34n-4) - 2] + [-3n$
$+ 7][(p-15) + (74n-45) - 2] + [-3n+7][(p-15) + (p-50) - 2]$
$+ [-3n+7][(p-15) + (p-53) - 2] + [-3n+7][(p-15) + (p-55) - 2]$
$+ [-3n+7][(p-15) + (p-58) - 2] + [-3n+7][(p-15) + (p-61) - 2]$
$+ [-2n+5][(p-15) + (72n-56) - 2] + [-2n+5][(p-15) + (63n$
$- 44) - 2] + [-2n+5][(p-15) + (60n-40) - 2] + [-2n+5][(p-15)$
$+ (55n-33) - 2] + [-2n+5][(p-15) + (50n-26) - 2] + [-2n+5][(p$
$- 15) + (47n-22) - 2] + [-2n+5][(p-15) + (42n-15) - 2] + [-2n$
$+ 5][(p-15) + (38n-10) - 2] + [-2n+5][(p-15) + (34n-4) - 2]$

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$+ [-3n + 7][(p - 18) + (p - 55) - 2] + [-3n + 7][(p - 18) + (p - 58) - 2]$
$+ [-2n + 7][(p - 18) + (p - 61) - 2] + [-2n + 5][(p - 18) + (72n - 56)$
$- 2] + [-2n + 5][(p - 18) + (68n - 51) - 2] + [-2n + 5][(p - 18) + (63n$
$- 44) - 2] + [-2n + 5][(p - 18) + (60n - 40) - 2] + [-2n + 5][(p - 18)$
$+ (55n - 33) - 2] + [-2n + 5][(p - 18) + (51n - 28) - 2] + [-2n + 5][(p$
$- 18) + (48n - 24) - 2] + [-2n + 5][(p - 18) + (44n - 19) - 2] + [-2n$
$+ 5][(p - 18) + (40n - 14) - 2] + [-2n + 5][(p - 18) + (37n - 10) - 2]$
$+ [-2n + 5][(p - 18) + (33n - 5) - 2] + [n + 2][(p - 18) + 29n - 2] + [3n$
$- 2][(p - 18) + (26n + 4) - 2] + [-2n + 5][(p - 21) + (73n - 58) - 2]$
$+ [-2n + 5][(p - 21) + (69n - 53) - 2] + [-2n + 5][(p - 21) + (65n$
$- 48) - 2] + [-2n + 5][(p - 21) + (62n - 44) - 2] + [-2n + 5][(p - 21)$
$+ (58n - 39) - 2] + [-2n + 5][(p - 21) + (54n - 34) - 2] + [-2n + 5][(p$
$- 21) + (51n - 30) - 2] + [-2n + 5][(p - 21) + (47n - 25) - 2] + [-2n$
$+ 5][(p - 21) + (43n - 20) - 2] + [-2n + 5][(p - 21) + (40n - 16) - 2]$
$+ [-2n + 5][(p - 21) + (36n - 11) - 2] + [n + 2][(p - 21) + (32n - 6)$
$- 2] + [n + 2][(p - 21) + (29n - 2) - 2] + [n + 2][(p - 21) + (25n + 3)$
$- 2] + [n + 2][(p - 21) + (21n + 8) - 2] + [3n - 2][(p - 21) + (18n + 12)$
$- 2] + [-2n + 5][(p - 23) + (64n - 48) - 2] + [-2n + 5][(p - 23) + (63n$
$- 46) - 2] + [-2n + 5][(p - 23) + (56n - 38) - 2] + [-2n + 5][(p - 23)$
$+ (53n - 34) - 2] + [-2n + 5][(p - 23) + (49n - 29) - 2] + [-2n + 5][(p$
$- 23) + (45n - 24) - 2] + [-2n + 5][(p - 23) + (42n - 20) - 2] + [-2n$
$+ 5][(p - 23) + (38n - 15) - 2] + [n + 2][(p - 23) + (34n - 10) - 2] + [n$
$+ 2][(p - 23) + (31n - 6) - 2] + [n + 2][(p - 23) + (27n - 1) - 2] + [n$
$+ 2][(p - 23) + (23n + 4) - 2] + [n + 2][(p - 23) + (20n + 8) - 2] + [n$
$+ 2][(p - 23) + (16n + 13) - 2] + [3n - 2][(p - 23) + (12n + 18) - 2]$
$+ [-2n + 5][(p - 26) + (55n - 38) - 2] + [-2n + 5][(p - 26) + (51n$
$- 33) - 2] + [-2n + 5][(p - 26) + (47n - 28) - 2] + [-2n + 5][(p - 26)$
$+ (44n - 24) - 2] + [-2n + 5][(p - 26) + (40n - 19) - 2] + [n + 2][(p$
$- 26) + (36n - 14) - 2] + [n + 2][(p - 26) + (33n - 10) - 2] + [n + 2][(p$
$- 26) + (29n - 5) - 2] + [n + 2][(p - 26) + 25n - 2] + [n + 2][(p - 26)$
$+ (22n + 4) - 2] + [n + 2][(p - 26) + (18n + 9) - 2] + [n + 2][(p - 26)$

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$+ (14n + 14) - 2] + [n + 2][(p - 26) + (11n + 18) - 2] + [3n - 2][(p$
$- 26) + (7n + 23) - 2] + [-2n + 5][(p - 29) + (49n - 31) - 2] + [-2n$
$+ 5][(p - 29) + (42n - 23) - 2] + [n + 2][(p - 29) + (38n - 18) - 2] + [n$
$+ 2][(p - 29) + (35n - 14) - 2] + [n + 2][(p - 29) + (31n - 9) - 2] + [n$
$+ 2][(p - 29) + (27n - 4) - 2] + [n + 2][(p - 29) + 24n - 2] + [n + 2][(p$
$- 29) + (20n + 5) - 2] + [n + 2][(p - 29) + (16n + 10) - 2] + [n + 2][(p$
$- 29) + (13n + 14) - 2] + [n + 2][(p - 29) + (9n + 19) - 2] + [n + 2][(p$
$- 29) + (5n + 24) - 2] + [3n - 2][(p - 29) + (2n + 28) - 2] + [n + 2][(p$
$- 37) + (19n + 2) - 2] + [n + 2][(p - 37) + (15n + 7) - 2] + [n + 2][(p$
$- 37) + (11n + 12) - 2] + [n + 2][(p - 37) + (8n + 16) - 2] + [n + 2][(p$
$- 37) + (4n + 21) - 2] + [-2n + 5][(p - 37) + (2n + 24) - 2] + 3[(p - 37)$
$+ (3n + 24) - 2] + 3[(p - 37) + (6n + 22) - 2] + 3[(p - 37) + (4n + 25)$
$- 2] + [2n - 1][(p - 37) + (5n + 25) - 2] + [n + 2][(p - 39) + (10n + 12)$
$- 2] + [n + 2][(p - 39) + (6n + 17) - 2] + [-2n + 5][(p - 39) + (7n + 17)$
$- 2] + 3[(p - 39) + (5n + 20) - 2] + 3[(p - 39) + (5n + 21) - 2] + 3[(p$
$- 39) + (6n + 21) - 2] + 3[(p - 39) + (6n + 22) - 2] + 3[(p - 39) + (6n$
$+ 23) - 2] + [2n - 1][(p - 39) + (6n + 24) - 2] + 3[(p - 42) + (6n + 17)$
$- 2] + 3[(p - 42) + (6n + 18) - 2] + 3[(p - 42) + (6n + 19) - 2] + 3[(p$
$- 42) + (6n + 20) - 2] + 3[(p - 42) + (6n + 21) - 2] + 3[(p - 42) + (6n$
$+ 22) - 2] + 3[(p - 42) + (6n + 23) - 2] + [2n - 1][(p - 42) + (6n + 24)$
$- 2] + 3[(p - 45) + (6n + 18) - 2] + 3[(p - 45) + (6n + 19) - 2] + 3[(p$
$- 45) + (6n + 20) - 2] + 3[(p - 45) + (6n + 21) - 2] + 3[(p - 45) + (6n$
$+ 22) - 2] + 3[(p - 45) + (6n + 23) - 2] + [2n - 1][(p - 45) + (6n + 24)$
$- 2] + 3[(p - 50) + (6n + 20) - 2] + 3[(p - 50) + (6n + 21) - 2] + 3[(p$
$- 50) + (6n + 22) - 2] + 3[(p - 50) + (6n + 23) - 2] + [2n - 1][(p - 50)$
$+ (6n + 24) - 2] + 3[(p - 53) + (6n + 21) - 2] + 3[(p - 53) + (6n + 22)$
$- 2] + 3[(p - 53) + (6n + 23) - 2] + [2n - 1][(p - 53) + (6n + 24) - 2]$
$+ 3[(p - 55) + (6n + 24) - 2] + 3[(p - 55) + (6n + 23) - 2] + [2n - 1][(p$
$- 55) + (6n + 24) - 2] + 3[(p - 58) + (6n + 23) - 2] + [2n - 1][(p - 58)$
$+ (6n + 24) - 2] + [2n - 1][(p - 61) + (6n + 24) - 2] + [(76n - 6) + (p$
$- 15) - 2] + [(76n - 6) + (p - 18) - 2] + [(76n - 6) + (p - 21) - 2]$

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$+ [(76n - 6) + (p - 23) - 2] + [(76n - 6) + (p - 26) - 2] + [(76n - 6)$
$+ (p - 29) - 2] + [(76n - 6) + (76n - 32) - 2] + [(76n - 6) + (74n - 32)$
$- 2] + [(76n - 6) + (p - 37) - 2] + [(76n - 6) + (p - 39) - 2] + [(76n$
$- 6) + (p - 42) - 2] + [(76n - 6) + (p - 45) - 2] + [-3n + 7][(76n - 6)$
$+ (74n - 45) - 2] + [-3n + 7][(76n - 6) + (p - 50) - 2] + [-3n$
$+ 7][(76n - 6) + (p - 53) - 2] + [-3n + 7][(76n - 6) + (p - 55) - 2]$
$+ [-3n + 7][(76n - 6) + (p - 58) - 2] + [-3n + 7][(76n - 6) + (p - 61)$
$- 2] + [-2n + 5][(76n - 6) + (72n - 56) - 2] + [-2n + 5][(76n - 6)$
$+ (68n - 51) - 2] + [-2n + 5][(76n - 6) + (63n - 44) - 2] + [-2n$
$+ 5][(76n - 6) + (60n - 40) - 2] + [-2n + 5][(76n - 6) + (55n - 33) - 2]$
$+ [-2n + 5][(76n - 6) + (50n - 26) - 2] + [-2n + 5][(76n - 6) + (47n$
$- 22) - 2] + [-2n + 5][(76n - 6) + (42n15) - 2] + [-2n + 5][(76n - 6)$
$+ (38n - 10) - 2] + [(76n - 6) + (34n - 4) - 2] + [(76n - 8) + (p - 23)$
$- 2] + [(76n - 8) + (p - 26) - 2] + [(76n - 8) + (p - 29) - 2] + [(76n$
$- 8) + (76n - 32) - 2] + [(76n - 8) + (74n - 32) - 2] + [(76n - 8) + (p$
$- 37) - 2] + [(76n - 8) + (p - 39) - 2] + [(76n - 8) + (p - 42) - 2]$
$+ [(76n - 8) + (p - 45) - 2] + [-3n + 7][(76n - 8) + (74n - 45) - 2]$
$+ [-3n + 7][(76n - 8) + (p - 50) - 2] + [-3n + 7][(76n - 8) + (p - 53)$
$- 2] + [-3n + 7][(76n - 8) + (p - 55) - 2] + [-3n + 7][(76n - 8) + (p$
$- 58) - 2] + [-3n + 7][(76n - 8) + (p - 61) - 2] + [-2n + 5][(76n - 8)$
$+ (72n - 56) - 2] + [-2n + 5][(76n - 8) + (68n - 51) - 2] + [-2n$
$+ 5][(76n - 8) + (63n - 44) - 2] + [-2n + 5][(76n - 8) + (60n - 40) - 2]$
$+ [-2n + 5][(76n - 8) + (55n - 33) - 2] + [-2n + 5][(76n - 8) + (50n$
$- 26) - 2] + [-2n + 5][(76n - 8) + (47n - 22) - 2] + [-2n + 5][(76n$
$- 8) + (42n - 15) - 2] + [-2n + 5][(76n - 8) + (38n - 10) - 2] + [(76n$
$- 8) + (34n - 4) - 2] + [n + 2][(79n - 35) + (37n - 18) - 2] + [n$
$+ 2][(79n - 35) + (33n - 13) - 2] + [n + 2][(79n - 35) + (29n - 8) - 2]$
$+ [n + 2][(79n - 35) + (26n - 4) - 2] + [n + 2][(79n - 35) + (22n + 1)$
$- 2] + [n + 2][(79n - 35) + (18n + 6) - 2] + [n + 2][(79n - 35) + (15n$
$+ 10) - 2] + [n + 2][(79n - 35) + (11n + 15) - 2] + [n + 2][(79n - 35)$
$+ (7n + 20) - 2] + [n + 2][(79n - 35) + (4n + 24) - 2] + [n + 2][(79n$

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$$- 35) + 29 - 2] + [(79n - 35) + (-2n + 32) - 2] + [n + 2][(74n - 32)$$
$$+ (28n - 8) - 2] + [n + 2][(74n - 32) + (24n - 3) - 2] + [n + 2][(74n$$
$$- 32) + (20n + 2) - 2] + [n + 2][(74n - 32) + (17n + 6) - 2] + [n$$
$$+ 2][(74n - 32) + (13n + 11) - 2] + [n + 2][(74n - 32) + (9n + 16) - 2]$$
$$+ [n + 2][(74n - 32) + (6n + 20) - 2] + [n + 2][(74n - 32) + (2n + 25)$$
$$- 2] + [-2n + 5][(74n - 32) + 28 - 2] + 3[(74n - 32) + (n + 28) - 2]$$
$$+ (2n - 1)[(74n - 32) + (n + 29) - 2] + [13n - 13][(p - 1) + (p - 60)$$
$$- 2] + [52n - 52][(p - 1) + (p - 93) - 2] + [52n - 52][(p - 1) + (p - 95)$$
$$- 2] + [52n - 52][(p - 1) + (p - 98) - 2] + [52n - 52][(p - 1) + (p$$
$$- 101) - 2] + [52n - 52][(p - 1) + (p - 103) - 2] + [52n - 52][(p - 1)$$
$$+ (p - 106) - 2] + (52n - 52)[(p - 1) + (p - 109) - 2] + (52n - 52)[(p$$
$$- 1) + (p - 111) - 2] + (52n - 52)[(p - 1) + (p - 114) - 2] + (52n$$
$$- 52)[(p - 1) + (p - 117) - 2] + (52n - 52)[(p - 1) + (p - 119) - 2]$$
$$+ (52n - 52)[(p - 1) + (p - 122) - 2] + (13n - 13)[(p - 1) + (p - 123)$$
$$- 2] + (39n - 39)[(p - 1) + (p - 121) - 2] + (39n - 39)[(p - 1) + (p$$
$$- 120) - 2] + (39n - 39)[(p - 1) + (p - 118) - 2] + (39n - 39)[(p - 1)$$
$$+ (p - 116) - 2] + (39n - 39)[(p - 1) + (p - 115) - 2] + (39n - 39)[(p$$
$$- 1) + (p - 113) - 2] + (39n - 39)[(p - 1) + (p - 112) - 2] + (39n$$
$$- 39)[(p - 1) + (p - 110) - 2] + (39n - 39)[(p - 1) + (p - 108) - 2]$$
$$+ (39n - 39)[(p - 1) + (p - 107) - 2] + (39n - 39)[(p - 1) + (p - 105)$$
$$- 2] + (39n - 39)[(p - 1) + (p - 104) - 2] + (39n - 39)[(p - 1) + (p$$
$$- 102) - 2] + (39n - 39)[(p - 1) + (p - 100) - 2] + (39n - 39)[(p - 1)$$
$$+ (p - 99) - 2] + (39n - 39)[(p - 1) + (p - 97) - 2] + (39n - 39)[(p - 1)$$
$$+ (p - 96) - 2] + (39n - 39)[(p - 1) + (p - 94) - 2] + (13n - 13)[(p - 1)$$
$$+ (p - 92) - 2] + (n - 1)[(p - 2) + (p - 90) - 2] + (4n - 4)[(p - 2) + (p$$
$$- 93) - 2] + (4n - 4)[(p - 2) + (p - 95) - 2] + (4n - 4)[(p - 2) + (p$$
$$- 98) - 2] + (4n - 4)[(p - 2) + (p - 101) - 2] + (4n - 4)[(p - 2) + (p$$
$$- 103) - 2] + (4n - 4)[(p - 2) + (p - 106) - 2] + (4n - 4)[(p - 2) + (p$$
$$- 109) - 2] + (4n - 4)[(p - 2) + (p - 111) - 2] + (4n - 4)[(p - 2) + (p$$
$$- 114) - 2] + (4n - 4)[(p - 2) + (p - 117) - 2] + (4n - 4$$

Simplifying the above expression, we get

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$$\sum_{uv\epsilon E(G_{Nr})}(d_u+d_v-2)=-16445n^2-438851n+7768pn-6139p+457298$$

(g)

$$\sum_{uv\epsilon E(G_{Nr})}(d_u+d_v)=-16445n^2-429961n+7768pn-6139p+446995 \quad \text{(h)}$$

[7]

From (g) and (h) $ABC(G_{Nr})=\sqrt{\frac{-16445n^2-438851n+7768pn-6139p+457298}{-16445n^2-429961n+7768pn-6139p+446995}}$

Therfore, $ABC(G_{Nr})=(-16445n^2-438851n+7768pn-6139p+457298)^{\frac{1}{2}}(-16445n^2-429961n+7768pn-6139p+446995)^{-\frac{1}{2}}$

3 Numercal Representation

The below table displays numerical values of the atom-bond connectivity index on different step Values

Step Value k	$ABC(G_{Nr})$
0.1	0.9461
0.09	0.9489
0.08	0.9599
0.07	0.9653
0.06	−0.1319
0.05	−0.6458
0.04	−1.2977
0.03	−1.7570
0.02	1.0057
0.01	1.4017

Table:1 Numerical values of the atom-bond connectivity index

4 Conclusion

In this article, this study explored the degree-based topological index namely the atom-bond connectivity (ABC) index, for the Nanorod grapg G_{Nr}. Numerical values of the ABC index are computed for step values of k ranging from 0.1 to 0.01 in increments of 0.01.

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