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PG AND RESEARCH DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE (Autonomous)
Tirupattur - 635 601, Tirupattur District, Tamil Nadu, India.

Organizes an

International Conference on Harmonizing Graph Theory and Differential Equations in the Age of Industry 4.0

On 12th and 13th February 2024

PROCEEDINGS

Editors
Dr. M.K. Angel Jebitha
Dr. J. Befija Minnie
Dr. A. Jancy Vini

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ARTICLES

Dear colleagues and participants,

Warm Greetings to you all.

It is with great pleasure that I express my warmest greetings to all the readers of the International Conference proceedings on Harmonizing Graph Theory and Differential Equations in the Age of Industry 4.0. Over the course of this conference, we will have the opportunity to delve into a wide range of topics, from pure mathematics to its myriad applications in fields as diverse as physics, engineering, finance and beyond. Through presentations, discussions and shared insights, I am confident that we will gain new perspectives, forge meaningful connections and lay the groundwork for future breakthroughs.

Within these pages, you will find a rich tapestry of mathematical research, spanning a diverse array of topics and methodologies. From abstract theory to real-world applications, each article and paper offers valuable insights that contribute to the ever-evolving landscape of mathematical knowledge.

I hope that you find inspiration and enlightenment within these pages and that they serve as a catalyst for further exploration and discovery. Together, let us continue to push the boundaries of mathematical knowledge and make meaningful contributions to the world around us.

My sincere appreciation to the organizers and reviewers whose hard work and support have made this publication possible. I congratulate all the faculty members and students of the department of mathematics for your efforts and commitment in organizing this conference and wish you all a happy learning.

Dr.Sr.M.Mary Gilda
Secretary
Holy Cross College (Autonomous)
Nagercoil- 629004

Esteemed Readers and Delegates,

Mathematics, often referred to as the queen of sciences, serves as the bedrock upon which countless disciplines are built. Its elegance and precision enable us to unravel the mysteries of the universe, solve complex problems and innovate in ways that shape our world.

Within the pages of these proceedings, you will find a wealth of research, insights and discoveries that reflect the diversity and vitality of contemporary mathematics. From theoretical advances to practical applications, each contribution represents a testament to the ingenuity and dedication of our global community of mathematicians. Our conference serves as a platform for the exchange of ideas, the presentation of cutting-edge research and the cultivation of collaborative networks among scholars and practitioners from around the globe. I am immensely proud of the contributions made by the PG and Research Department of Mathematics for organizing the International Conference on Harmonizing Graph Theory and Differential Equations in the Age of Industry 4.0. I thank the esteemed invited speakers, researchers and attendees for sharing your research, insights and experiences which have enriched the discourse surrounding mathematics and its multifaceted applications.

My best wishes to you all. God bless you!

Dr.Sr.S.Sahayaselvi
Principal
Holy Cross College (Autonomous)
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Dear Esteemed Colleagues and Participants,

I am delighted to announce the release of the proceedings for the recent International Mathematics Conference hosted by our esteemed department. Over the course of two days, we witnessed insightful discussions, groundbreaking research presentations, and invaluable exchanges of knowledge among experts and enthusiasts from around the globe.

This compilation encapsulates the essence of our collective efforts, highlighting the diverse array of topics covered during the conference. From pure mathematics to its interdisciplinary applications, each contribution reflects the depth and breadth of contemporary mathematical inquiry.

I extend my heartfelt gratitude to all the keynote speakers, presenters, organizers, and attendees whose dedication and enthusiasm made this event a resounding success. Your passion for mathematics and commitment to academic excellence continue to inspire and drive our community forward.

I encourage everyone to explore the proceedings, engage with the wealth of ideas contained within its pages, and continue fostering collaborations that will shape the future of mathematical research and education. On behalf of the Mathematics Department, I extend my warmest congratulations to all contributors and express my sincere appreciation for your invaluable contributions to the advancement of our field.

With best regards,
Dr.T.Sheeba Helen
Head of the Department of Mathematics

ABSTRACTS OF THE INVITED SPEAKERS

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DELAY DIFFERENTIAL EQUATIONS WITH IMPULSE

Abstract

This talk explores the intricate dynamics of delay differential equations (DDEs) with impulse, a topic gaining significant attention in mathematics. Delving into fundamental principles and applications, the talk will elucidate analytical techniques, stability analysis, and real-world relevance, aiming to inspire further research and collaboration in this dynamic field.

Ultimately, this presentation aims to stimulate further research and foster collaboration among mathematicians and scientists interested in the dynamics of complex systems governed by delay differential equations with impulse. By unraveling the intricacies of these mathematical structures, we can advance our comprehension of natural and engineered systems, paving the way for innovative solutions to pressing scientific challenges.

Dr. G. BRITTO ANTONY XAVIER
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DERIVATIVE GRAPHS OF EXPONENTIAL FUNCTIONS WITH APPLICATIONS

Abstract

This invited talk aims to introduce a new type function termed as generalized exponential function with shift value, by which three major fields differential equation, graph theory and fuzzy theory of mathematical science have being connected. With this background, some applications in chemical graphs are arrived at and are illustrated with examples. Moreover, regular fuzzy derivative graphs are discussed and analyzed.

Keywords: Derivative graphs, Generalized exponential function, Fuzzy derivative graphs.
AMS Subject Classification: 39A70, 05C72, 05C20, 11E81.

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TOPOLOGICAL INDICES AND ENERGY IN GRAPHS

Abstract

Chemical Graph Theory is the topology branch of Mathematical Chemistry which applies graph theory to study molecular structures. A molecular graph or chemical graph is a representation of the structural formula of a chemical compound in terms of graph theory. Topological indices are numerical parameters designed for the transformation of a molecular graph into a number that characterizes the topology of that graph. A graph is completely determined by specifying either its adjacency structure or its incidence structure and these specifications provide far more efficient ways of representing a large or complicated graph than a pictorial representation. Several matrices can be associated with a graph such as the adjacency matrix or the Laplacian Matrix. A concept related to the spectrum of a graph is that of energy. In this talk, we shall discuss the recent advances in topological indices and energy of graphs.

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SOME NEW OSCILLATION CRITERIA OF TRANSVERSELY VIBRATING BEAMS

Abstract

Let G be a simple graph of order n and G^c be its complement. If $\alpha(G)$ is a graph parameter, then the lower and upper bounds on the sum $\alpha(G)+\alpha(G^c)$ in terms of n are of prime importance in graph theory. The first of its kind with reference to chromatic number of a graph was studied by Nordhaus and Gaddum on complementary graphs and published in American Mathematical Monthly in 1956. In this talk, we discuss the recent developments in the theory of domination and extend it to derived graphs.

AMS Subject Classification: 05C

Key words: Chromatic number, domination number.

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The Chromatic Restrained Domination Number of Middle Graphs

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Abstract

Let $G = (V, E)$ be a graph and $M(G)$ be the middle graph of G. A subset D of V is said to be a chromatic restrained dominating set (or crd-set) if D is a restrained dominating set and $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal chromatic restrained dominating sets is called chromatic restrained domination number and is denoted by $\gamma_r^c(G)$. In this paper, we obtain the chromatic restrained domination number for the middle graph of some standard graphs.

Keywords : Domination, Restrained Domination, Chromatic Number, Middle Graphs.
AMS Subject Classification : 05C15, 05C69

1 Introduction

All the graphs $G = (V, E) = (n, m)$ considered here are simple, finite and undirected, with neither loops nor multipe edges. For $D \subseteq V$, the subgraph induced by D is denoted by $\langle D \rangle$. The *open neighborhood* $N(v)$ of the vertex v consists of the set of vertices adjacent to v, that is $N(v) = \{w \in V : vw \in E\}$, and the *closed neighborhod* of v is $N[v] = N(v) \cup \{v\}$. The *degree* of a vertex v in a graph G, denoted by $deg(v)$ is the number of edges incident with v or, eqivalently, $deg(v) = |N(v)|$. A vertex of degree one is called an *end vertex* or a *pendant vertex.* A k-vertex-coloring of a graph, or simply a k-coloring, is an assignment of k-colors to its vertices. The coloring is proper if no two adjacent vertices are assigned the same color. A coloring in which k-colors are used is called a k-coloring. A graph is k-colorable if it has a proper k-coloring. The minimum k for which a graph G is k-colorable is called its chromatic number, and denoted by $\chi(G)$. If $\chi(G) = k$, the graph G is said to be $k - chromatic$. Graph Theory terminologies which are not defined here can be seen in [4] and [11].

The *line graph* $L(G)$ is the graph whose vertices correspond to the edges of G, and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent. The *middle graph* of a graph $G = (V, E)$ is the graph $M(G) = (V \cup E, E^{'})$ where $uv \in E^{'}$ if and only if either u is a vertex of G and v is an edge of G containing u, or u and v are edges in G having a vertex in common. To maintain clarity across the paper, we establish a standardized notation for the vertex set and edge set of the middle graph $M(G)$. Assuming $V(G) = \{v_1, v_2, ..., v_n\}$, we define $V(M(G))$ as $V(G) \cup \mathcal{M}$ where $\mathcal{M} = \{m_{ij}/v_iv_j \in E(G)\}$, and $E(M(G))$ as $\{v_im_{ij}, v_jm_{ij}/v_iv_j \in E(G)\} \cup E(L(G))$. In [5], Farshad Kazemnejad, Behnaz Pahlavsay, Elisa Palezzato and Michele Torielli discussed about the domination number of middle graphs.

A set $D \subseteq V$ of vertices in a graph G is called a *dominating set* if every vertex $u \in V$ is either an element of D or is adjacent to an element of D. The minimum cardinality taken over all minimal dominating sets is called the domination number of G and is denoted by $\gamma(G)$. A set $D \subseteq V$ is a *restrained dominating set* if every vertex in $V - D$ is adjacent to a vertex in D and another vertex in $V - D$ [6]. The minimum cardinality taken over all minimal restrained dominating sets is called the restrained domination number of G and is denoted by $\gamma_r(G)$. A set D is a γ_r - set if D is a restrained dominating set of cardinality $\gamma_r(G)$.

A set $D \subseteq V$ is a *chromatic preserving set* or a cp-set if $\chi(< D >) = \chi(G)$ and the minimum cardinality of a cp-set in G is called the chromatic preserving number or cp-number of G and is denoted by $cpn(G)$. This new concept was defined by T.N. Janakiraman and M. Poobalaranjani [7]. They also defined the concept of dom-chromatic set of a graph. A subset D of V is said to be a *dom-chromatic set* (or dc-set)if D is a dominating set and $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal dom-chromatic sets in G is called the dom-chromatic number and is denoted by $\gamma_{ch}(G)$. In [8], they established dom-chromatic numbers for some classes of graphs and also some main results in this area. Based on this, S. Balamurugan et al [1], [2], [3] introduced and studied the concept of chromatic strong domination, chromatic total domination, chromatic connected domination, chromatic weak domination and so on. Also, J. Joseline Manora et al [9], [10] discussed about connected majority dom-chromatic number of a graph and majority dom-chromatic set of a graph. Like these, several authors worked on different types of dom-chromatic sets. In this paper, we initiate a study of the new domination parameter on chromatic restrained domination number of middle graphs.

2 Main Results

In this section, we obtain the chromatic restrained domination number for the middle graph of some standard graphs.

Definition 2.1 *Let $G = (V, E)$ be a graph and $M(G)$ be the middle graph of G. A subset D of V is said to be a chromatic restrained dominating set (or crd-set) if D is a restrained dominating set and $\chi(< D >) = \chi(G)$. The minimum cardinality taken over all minimal* *chromatic restrained dominating sets is called chromatic restrained domination number and is denoted by* $\gamma_r^c(G)$. *Throughout this paper, we denote the chromatic restrained domination number of middle graphs by* $\gamma_r^c(M(G))$.

Theorem 2.2 *If* $n \geq 3$, *then* $\gamma_r^c(M(K_n)) = n$.

Proof: Let K_n *be the complete graph on* n *vertices. Then* $V(K_n) = \{v_1, v_2, ..., v_n\}$. *Consider* $V(M(K_n)) = V(K_n) \cup \mathcal{M}$ *where* $\mathcal{M} = \{m_{ij}/1 \leq i \leq n, 1 \leq j \leq n, m_{ij} = m_{ji}, i \neq j\}$. *Also* $\chi(K_n) = \chi(M(K_n)) = n$. *Let* $D_i = N_{M(K_n)}[v_i], 1 \leq i \leq n$. *Then* $D_1, D_2, ..., D_n$ *are the subsets of* $M(K_n)$ *with chromatic number* n. *For any* $D_i - \{m_{ij}\}$, $m_{ij} \in N_{M(K_n)}(v_i)$, $\chi(\langle D_i - \{m_{ij}\}\rangle) < n$. *Therefore,* $D_i = N_{M(K_n)}[v_i]$ *are the only sets of* $M(K_n)$ *with chromatic number* n *and with minimum cardinality. Also* $D_i^{'}s$ *are restrained dominating sets of* $M(K_n)$. *Therefore,* D_i *is a chromatic restrained dominating set of* $M(K_n)$ *and* $\gamma_r^c(M(K_n)) = |D_i| = n$.

Theorem 2.3 *For any path* P_n, $\gamma_r^c(M(P_n)) = \begin{cases} \lceil \frac{n}{2} \rceil + 2 & if \quad n \quad is \quad odd\ , n \geq 5 \\ \frac{n}{2} + 3 & if \quad n \quad is \quad even\ , n \geq 4. \end{cases}$

Proof: Let P_n *be a path on* n *vertices. Then* $V(P_n) = \{v_1, v_2, v_3, ..., v_n\}$ *and* $E(P_n) = \{v_1v_2, v_2v_3, ..., v_{n-1}v_n\}$. *Consider* $V(M(P_n)) = V(P_n) \cup \mathcal{M}$ *where* $\mathcal{M} = \{m_{i(i+1)}/1 \leq i \leq n-1\} = \{m_{12}, m_{23}, ..., m_{(n-1)n}\}$.

Case (i): n *is odd and* $n \geq 5$

Let $D = \{v_1, m_{12}, m_{34}, m_{56}, ..., m_{(n-2)(n-1)}, v_n\}$. *Then* D *is a minimum restrained dominating set of* $M(P_n)$ *and* $|D| = \frac{n-1}{2} + 2 = \lceil \frac{n}{2} \rceil + 1$. *Since,* $v_1m_{12} \in E$, $\chi(\langle D\rangle) = 2 \neq \chi(M(P_n))$. *Therefore,* D *is not a chromatic restrained dominating set for* $M(P_n)$. *Consider* $D_1 = (D - \{m_{12}\}) \cup \{m_{23}, v_3\}$. *Then* $\langle D_1\rangle$ *contains* C_3 *as an induced subgraph and so* $\chi(\langle D_1\rangle) = 3 = \chi(M(P_n))$. *Also* D_1 *is a restrained dominating set for* $M(P_n)$ *and* $|D_1| = |(D - \{m_{12}\}) \cup \{m_{23}, v_3\}| = \lceil \frac{n}{2} \rceil + 2$. *Therefore,* $\gamma_r^c(M(P_n)) \leq |D_1| = \lceil \frac{n}{2} \rceil + 2$. *Suppose there exists a chromatic restrained dominating set* S *such that* $|S| < \lceil \frac{n}{2} \rceil + 2$. *Then* $|D| < |S| < |D_1| = |D| + 1$, *which is impossible. Therefore,* $\gamma_r^c(M(P_n)) = \lceil \frac{n}{2} \rceil + 2$, *where* n *is odd and* $n \geq 5$.

Case (ii): n *is even and* $n \geq 4$

Let $D = \{v_1, m_{23}, m_{45}, m_{67}, ..., m_{(n-2)(n-1)}, v_n\}$. *Then* D *is a unique restrained dominating set of* $M(P_n)$ *and* $|D| = \frac{n-2}{2} + 2 = \frac{n}{2} + 1$. *Since* D *is independent,* $\chi(\langle D\rangle) = 1 \neq \chi(M(P_n))$. *Therefore,* D *is not a chromatic restrained dominating set of* $M(P_n)$. *Consider* $D \cup \{v_2, m_{12}\}$. *Then* $\langle D \cup \{v_2, m_{12}\}\rangle$ *contains* C_3 *as an induced subgraph. This implies that,* $\chi(\langle D \cup \{v_2, m_{12}\}\rangle) = 3 = \chi(M(P_n))$ *and so* $D \cup \{v_2, m_{12}\}$ *is a chromatic restrained dominating set of* $M(P_n)$. *Therefore,* $\gamma_r^c(M(P_n)) = |D \cup \{v_2, m_{12}\}| = |D| + 2 = \frac{n}{2} + 3$, *where* n *is even and* $n \geq 4$.

Theorem 2.4 *For any cycle* C_n, $\gamma_r^c(M(C_n)) = \begin{cases} \lceil \frac{n}{2} \rceil + 1 & if \quad n \quad is \quad odd \\ \frac{n}{2} + 2 & if \quad n \quad is \quad even \end{cases}$

Proof: Let C_n be a cycle on n vertices. Then $V(C_n) = \{v_1, v_2, ..., v_n\}$ and $E(C_n) = \{v_1v_2, v_2v_3, v_3v_4, ..., v_{(n-1)}v_n, v_nv_1\}$. Consider $V(M(C_n)) = V(C_n) \cup \mathcal{M}$ where $\mathcal{M} = \{m_{1n}, m_{i(i+1)}/1 \leq i \leq n-1\} = \{m_{12}, m_{23}, ..., m_{(n-1)n}, m_{1n}\}$. Also $\chi(M(C_n)) = 3$.

Case (i): Let $D = \{m_{i(i+1)}, v_n/i = 2k-1, 1 \leq k \leq \lfloor \frac{n}{2} \rfloor\}$ be a restrained dominating set of $M(C_n)$ and $|D| = \lfloor \frac{n}{2} \rfloor + 1 = \lceil \frac{n}{2} \rceil$. But $\chi(\langle D \rangle) = 1$. Therefore, D is not a chromatic restrained dominating set of $M(C_n)$. Consider $D_1 = (D - v_n) \cup \{v_1, m_{1n}\}$. Then $\langle D_1 \rangle$ contains a 3-cycle and so $\chi(\langle D_1 \rangle) = \chi(M(C_n)) = 3$. Also D_1 is a restrained dominating set of $M(C_n)$. Therefore, D_1 is a chromatic restrained dominating set of $M(C_n)$. Therefore, $\gamma_r^c(M(C_n)) \leq |D_1| = \lceil \frac{n}{2} \rceil + 1$. Suppose there exists a chromatic restrained dominating set S such that $|S| < \lceil \frac{n}{2} \rceil + 1$. Then $|D| < |S| < |D_1| = |D| + 1$, which is impossible. Therefore, $\gamma_r^c(M(C_n)) = \lceil \frac{n}{2} \rceil + 1$.

Case (ii) : n is even

Let $D_1 = \{m_{i(i+1)}/i = 2k-1, 1 \leq k \leq \frac{n}{2}\}$ and $D_2 = \{m_{1n}, m_{i(i+1)}/i = 2k, 1 \leq k \leq \frac{n}{2} - 1\}$ are the only restrained dominating sets of $M(C_n)$ and $|D_1| = |D_2| = \frac{n}{2}$. Since D_1 and D_2 are independent, $\chi(\langle D_1 \rangle) = \chi(\langle D_2 \rangle) = 1 \neq \chi(M(C_n))$. Therefore, D_1 and D_2 are not chromatic restrained dominating sets. Consider $D_3 = (D_1 - \{m_{34}\}) \cup \{m_{23}, v_2, v_4\}$ and $D_4 = (D_2 - \{m_{1n}\}) \cup \{m_{12}, v_2, v_n\}$. Then $m_{12}v_2m_{23}m_{12}$ is a 3-cycle in both $\langle D_3 \rangle$ and $\langle D_4 \rangle$. Therefore, $\chi(\langle D_3 \rangle) = \chi(\langle D_4 \rangle) = \chi(M(C_n)) = 3$. Also both D_3 and D_4 are restrained dominating sets. Therefore, D_3 and D_4 are chromatic restrained dominating sets of $M(C_n)$ and $|D_3| = |D_4| = \frac{n}{2} + 2$. Therefore, $\gamma_r^c(M(C_n)) = \frac{n}{2} + 2$ if n is even.

Theorem 2.5 *For $n \geq 4$, $\gamma_r^c(M(W_n)) = n$.*

Proof: Let W_n be the wheel graph on n vertices with $n-1$ outer vertices and one central vertex. Let $V(W_n) = \{v_1, v_2, ..., v_n\}$ where v_n is the central vertex and $deg(v_n) = n-1$. Consider $V(M(W_n)) = V(W_n) \cup \mathcal{M}$ where $\mathcal{M} = \{m_{i(i+1)}, m_{1(n-1)}, m_{jn}/1 \leq i \leq n-2, 1 \leq j \leq n-1\}$. Since $\Delta(W_n) = n-1, \chi(M(W_n)) = n$. Also $N_{M(W_n)}(v_n) = \{m_{jn}/1 \leq j \leq n-1\}$. Therefore, $D = N_{M(W_n)}[v_n]$ is the only set of $M(W_n)$ with chromatic number n. For any $D - \{m_{jn}\}, m_{jn} \in N_{M(W_n)}(v_n), \chi(\langle D - \{m_{jn}\} \rangle) < n$. Also D is a restrained dominating set. Therefore, $D = N_{M(W_n)}[v_n]$ is the only chromatic restrained dominating set of $M(W_n)$ with minimum cardinality. Therefore, $\gamma_r^c(M(W_n)) = |D| = n$.

Theorem 2.6 *For $n \geq 2$, $\gamma_r^c(M(K_{1,n-1})) = 2n-1$.*

Proof: Let $V(K_{1,n-1}) = \{v_0, v_1, v_2, ..., v_{n-1}\}$ and $E(K_{1,n-1}) = \{v_0v_1, v_0v_2, ..., v_0v_{n-1}\}$. Consider $V(M(K_{1,n-1})) = V(K_{1,n-1}) \cup \mathcal{M}$ where $\mathcal{M} = \{m_i/1 \leq i \leq n-1\}$. Also $\chi(M(K_{1,n-1})) = n$, since $\Delta(K_{1,n-1}) = n-1$. Let $D = V(K_{1,n-1})$ be the restrained dominating set of $M(K_{1,n-1})$. Since D is independent in $M(K_{1,n-1})$, $\chi(\langle D \rangle) = 1 \neq \chi(M(K_{1,n-1}))$. Therefore, D is not a chromatic restrained dominating set of $M(K_{1,n-1})$. Since $\chi(M(K_{1,n-1})) = n, \mathcal{M} \cup \{v_0\}$ is the only set whose induced subgraph has chromatic number n. But $\mathcal{M} \cup \{v_0\}$ is not a restrained dominating

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set of $M(K_{1,n-1})$. Therefore, $V(M(K_{1,n-1}))$ is the only chromatic restrained dominating set of $M(K_{1,n-1})$. Therefore, $\gamma_r^c(M(K_{1,n-1})) = |V(M(K_{1,n-1}))| = n + n - 1 = 2n - 1$.

Theorem 2.7 *For any complete bipartite graph $K_{r,s}$, $\gamma_r^c(M(K_{r,s})) = r + s$ where $s \geq r \geq 2$.*

Proof: *Let $K_{r,s}$ be the complete bipartite graph with $r + s$ vertices. Let $V(K_{r,s}) = \{v_1, v_2, ..., v_r, u_1, u_2, ..., u_s\}$ and $E(K_{r,s}) = \{v_iu_j/1 \leq i \leq r, 1 \leq j \leq s\}$. Consider $V(M(K_{r,s})) = V(K_{r,s}) \cup \mathcal{M}$ where $\mathcal{M} = \{m_{ij}/1 \leq i \leq r, 1 \leq j \leq s\}$. Also $\chi(M(K_{r,s})) = s + 1$ since $\Delta(K_{r,s}) = s$. Let D be a restrained dominating set of $M(K_{r,s})$.*

Case (i): γ_r-set contains only the vertices of $K_{r,s}$

Let $D = \{v_1, v_2, ..., v_r, u_1, u_2, ..., u_s\}$. Then D is a restrained dominating set of $M(K_{r,s})$ with cardinality $|D| = r + s$. Therefore, $\gamma_r(K_{r,s}) \leq r + s$. Suppose there exists a restrained dominating set D' such that $\left|D'\right| < r + s$. Since γ_r-set contains only vertices of $K_{r,s}$, $D' \subset V(K_{r,s})$. Then there exists at least one vertex of $V(K_{r,s})$ in $V - D'$ which is not adjacent to any vertex of D'. Therefore, D' is not a restrained dominating set of $M(K_{r,s})$. Hence, there does not exists a restrained dominating set D' such that $\left|D'\right| < r + s$. Therefore, $\gamma_r(M(K_{r,s})) = r + s$.

Case (ii): γ_r-set contains only the edges of $K_{r,s}$

Since all the $v_i's$ and $u_j's$ are independent and $s \geq r$, every γ_r-set of $M(K_{r,s})$ should contain s edges, from which, r edges connecting different v_i' with different $u_j's$ and the remaining $s - r$ edges connecting the remaining $u_j's$ with any $v_i's$. Hence, it contributes at least s vertices to every γ_r-set. Therefore, $\gamma_r(M(K_{r,s})) \geq s$. Let $D = \{m_{11}, m_{22}, ..., m_{rr}\} \cup \{m_{1(r+1)}, m_{1(r+2)}, ..., m_{1s}\}$ be a restrained dominating set of $M(K_{r,s})$ with cardinality s. Then, $\gamma_r(M(K_{r,s})) \leq s$. Therefore, $\gamma_r(M(K_{r,s})) = s$.

Case (iii): γ_r-set contains both vertices and edges of $K_{r,s}$

Subcase (i): $s > r$

Then every γ_r-set can contain at most $s - r$ vertices from $u_j's, 1 \leq j \leq s$ of $K_{r,s}$ and so there exists at least $2r$ vertices (r vertices from $v_i's$ and at least r vertices from $u_j's$) of $K_{r,s}$ which are not adjacent to any of these $s - r$ vertices. If r vertices from $u_j's$ and r vertices from $v_i's$ are not adjacent to any of these $s - r$ vertices, then there must be r edges of $K_{r,s}$ in every γ_r-set incident with different $v_i's$ and different $u_j's$ where the $u_j's$ should be other than the $s - r$ vertices which are in the γ_r-set. Hence, $\gamma_r(M(K_{r,s})) \geq s$. Consider $D = \{u_1, u_2, ..., u_{s-r}, m_{1(s-r+1)}, m_{2(s-r+2)}, ..., m_{rs}\}$ where $|D| = s$. Then D is a restrained dominating set and $\gamma_r(M(K_{r,s})) \leq s$. Therefore, $\gamma_r(M(K_{r,s})) = s$.

Subcase (ii): $s = r$

Consider $D = \{v_1, u_1, m_{22}, m_{33}, ..., m_{rr}\}$. Then D is a restrained dominating set of $M(K_{r,s})$ with cardinality $s + 1$ and $\gamma_r(M(K_{r,s})) \leq s + 1$. Furthermore, every γ_r-set can contain at most two vertices of $K_{r,s}$. If it contains exactly one vertex of $K_{r,s}$, then there exists $2s - 1$ vertices of $K_{r,s}$ (s vertices from $v_i's$ and $s - 1$ vertices from $u_j's$ or vice versa) which are not adjacent to

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the vertex in the γ_r-set. Then there must be s edges in the γ_r-set incident with different $v_i^{'}s$ and different $u_j^{'}s$. On the other hand, if there exists two vertices (one vertex from $v_i^{'}s$ and another vertex from $u_j^{'}s$) of $K_{r,s}$ in the γ_r-set, then there exits $2(s-1)$ vertices($s-1$ vertices in $v_i^{'}s$ and $s-1$ vertices in $u_j^{'}s$) which are not adjacent to any of the two vertices in the γ_r-set. Hence, there must be $s-1$ edges of $K_{r,s}$ in the γ_r-set incident with different $v_i^{'}s$ and different $u_j^{'}s$ where the $v_i^{'}s$ and $u_j^{'}s$ should be other than the vertices in the γ_r-set. Thus every γ_r-set of $M(K_{r,s})$ must contain at least $s+1$ vertices from $M(K_{r,s})$. Hence, $\gamma_r(M(K_{r,s})) \geq s+1$. Therefore, $\gamma_r(M(K_{r,s})) = s+1$.

From the above three cases, $\gamma_r(M(K_{r,s})) = s$, since s is minimum. Then $D = \{m_{11}, m_{22}, ..., m_{rr}\} \cup \{m_{1(r+1)}, m_{1(r+2)}, ..., m_{1s}\}$ is a restrained dominating set with cardinality s. But D is not a chromatic restrained dominating set of $M(K_{r,s})$, since $\chi(\langle D\rangle) = s-r+1 \neq \chi(M(K_{r,s}))$. Let $N_{M(K_{r,s})}(v_i) = \{m_{i1}, m_{i2}, ..., m_{is}/1 \leq i \leq r\}$. Since $v_i^{'}s$ are vertices of maximum degree, exactly one of $N_{M(K_{r,s})}[v_i]$ is in every γ_r^c-set. Let $D_1 = D \cup N_{M(K_{r,s})}[v_1]$. Since $\chi\langle D_1\rangle) = s+1$ and D_1 is a restrained dominating set of $M(K_{r,s})$, D_1 is a chromatic restrained dominating set of $M(K_{r,s})$ with cardinality $r+s$. Therefore, $\gamma_r^c(M(K_{r,s})) = r+s$.

Theorem 2.8 *For any bistar graph $B_{r,s}$, $\gamma_r^c(M(B_{r,s})) = 2r+s+2$.*

Proof: Let $B_{r,s}$ be the bistar graph obtained by joining the center vertices of two stars $K_{1,r}$ and $K_{1,s}$ where $r \geq s \geq 2$. Let $V(B_{r,s}) = \{v_0, u_0, v_1, v_2, ..., v_r, u_{r+1}, u_{r+2}, ..., u_{r+s}\}$ $= \{v_0, u_0, v_i, u_j/1 \leq i \leq r, r+1 \leq j \leq r+s\}$ where u_0 and v_0 are the center vertices of $K_{1,r}$ and $K_{1,s}$ with $|V(B_{r,s})| = r+s+2$ and $E(B_{r,s}) = \{v_0v_i, u_0u_j, v_0u_0/1 \leq i \leq r, r+1 \leq j \leq r+s\}$. Consider $V(M(B_{r,s})) = V(B_{r,s}) \cup \mathcal{M}$ where $\mathcal{M} = \{m_i/1 \leq i \leq r\} \cup \{m_j/r+1 \leq j \leq r+s\} \cup \{m_0\}$. Let v_0 be a vertex of maximum degree and $\Delta(B_{r,s}) = r+1$. Therefore, $\chi(M(B_{r,s})) = r+2$. Let N be the set of pendant vertices where $N = \{v_i, u_j/1 \leq i \leq r, r+1 \leq j \leq r+s\}$. Consider $D = N \cup \{m_0\}$. Then D is a restrained dominating set of $M(B_{r,s})$ with cardinality $r+s+1$. Since D is independent, $\chi(\langle D\rangle) = 1$. Therefore, D is not a chromatic restrained dominating set of $M(B_{r,s})$. Let $S = N_{M(B_{r,s})}[v_0] = \{m_0, v_0, m_1, m_2, ..., m_r\}$. Consider $D_1 = N \cup S = \{m_0, v_0, v_i, u_j, m_i/1 \leq i \leq r, r+1 \leq j \leq r+s\}$. Then D_1 is a restrained dominating set of $M(B_{r,s})$ with $\chi(\langle D_1\rangle) = r+2$. Therefore, D_1 is a chromatic restrained dominating set of $M(B_{r,s})$ with cardinality $|D| = 2r+s+2$. Therefore, $\gamma_r^c(M(B_{r,s})) \leq 2r+s+2$. Since any chromatic restrained dominating set must contain all the end vertices and the maximum degree vertex together with the edges incident with it, we have $\gamma_r^c(M(B_{r,s})) \geq 2r+s+2$. Therefore, $\gamma_r^c(M(B_{r,s})) = 2r+s+2$.

Theorem 2.9 *Let F_n be the friendship graph where $n \geq 2$. Then $\gamma_r^c(M(F_n)) = 2n+1$.*

Proof: Let $V(F_n) = \{v_0, v_1, v_2, ..., v_{2n}\}$ and

$E(F_n) = \{v_0v_1, v_0v_2, ..., v_0v_{2n}\} \cup \{v_1v_2, v_3v_4, ..., v_{2n-1}v_{2n}\}$. Consider $V(M(F_n)) = V(F_n) \cup \mathcal{M}$ where $\mathcal{M} = \{m_i/1 \leq i \leq 2n\} \cup \{m_{i(i+1)}/1 \leq i \leq 2n-1$ and i is odd $\}$. Let v_0 be a vertex of maximum degree of F_n where $deg(v_0) = 2n$. Since $\Delta(F_n) = 2n$, $\chi(M(F_n)) = 2n+1$. Let $N_{M(F_n)}(v_0) = \{m_i/1 \leq i \leq 2n\}$. Since any chromatic restrained dominating set must contain all the vertices of $N_{M(F_n))}[v_0]$, $\gamma_r^c(M(F_n)) \geq 2n+1$. Let $D = \{v_0, m_i/1 \leq i \leq 2n\}$. Then D is a restrained dominating set of $M(F_n)$. Since $\chi(\langle D\rangle) = \chi(M(F_n))$, D is a chromatic restrained dominating set of $M(F_n)$. Therefore,$\gamma_r^c(F_n) \leq |D| = 2n+1$. Hence, $\gamma_r^c(M(F_n)) = 2n+1$.

3 Conclusion

In this paper, we obtained the chromatic restrained domination number for the middle graph of some standard graphs and observed that $\lceil \frac{n}{2} \rceil \leq \gamma_r^c(M(G)) \leq 2n-1$. Characterizing the extremal graphs associated with both the upper and lower bounds for the chromatic restrained domination number of middle graphs is a prospective avenue for future research.

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Decomposition of Square Harmonic Mean Graphs

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Abstract

Let $G = (V, E)$ be a connected simple graph with p vertices and q edges. If $G_1, G_2, , G_n$ are connected edge disjoint sub graphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup ... \cup E(G_n)$, then $(G_1, G_2, , G_n)$ is said to be a Decomposition of G.If there is an injective function $h : V(G) \to \{1, 2, , q + 1\}$ such that an induced edge function $h^* : E(G) \to \{1, 2, , q\}$ defined by $h^*(e = uv) = \lceil \frac{2h(u)^2 h(v)^2}{h(u)^2 + h(v)^2} \rceil$ or $\lfloor \frac{2h(u)^2 h(v)^2}{h(u)^2 + h(v)^2} \rfloor$ is bijective, then a graph $G = (V, E)$ with p vertices and q edges is called a square harmonic mean labeling. A graph which admits a square harmonic mean labeling is called a square harmonic mean graph. Based on this result, we introduce a new concept namely Decomposition of Square Harmonic Mean Graphs. In this paper, we study Continuous Monotonic Star Decomposition and Even Path Decomposition of Square Harmonic Mean Graphs.

Keywords : Square harmonic mean graphs, Decomposition, Continuous Monotonic Decomposition (CMD), Continuous Monotonic Star Decomposition (CMSD), Even Path Decomposition (EPD).

AMS Subject Classification : 05C38, 05C78

1 Introduction

All graphs considered here are simple, finite, connected and undirected graph. A graph labeling is an assignment of integers to the vertices or edges or both based on certain conditions. The concept of square harmonic mean labeling was introduced by L. S. Bebisha Lenin,M. Jaslin Melbha [1]. We adhere to Hararys [3] conventions for all other terms and notations. The concept of Continuous Monotonic Decomposition was introduced by N. Gnana Dhas and J. Paulraj Joseph [5]. Arithmetic odd decomposition was introduced by E. Ebin Raja Merly and N. Gnana Dhas [2]. The aforementioned studies served as our inspiration as we introduce a new concept Decomposition of Square Harmonic Mean Graphs.The basic definitions and theorems which are needed in the subsequent sections are given below.

Definition 1.1 *A bipartite graph is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. If every* *vertex of V_1 is adjacent with every vertex of V_2, then G is a complete bipartite graph. If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denoted by $K_{m,n}$.*

Definition 1.2 *A Triangular Snake T_n is obtained from a path $u_1, u_2, , u_n$ by joining u_α and $u_{\alpha+1}$ to a new vertex v_i for $1 \leq \alpha \leq n-1$. That is every edge of a path is replaced by a triangle C_3.*

Definition 1.3 *A Diamond Snake graph D_n is obtained by joining vertices u_α and $u_{\alpha+1}$ to two new vertices v_α and w_α for $1 \leq \alpha \leq n$. That is every edge of a path is replaced by a cycle C_4.*

Theorem 1.4 *Path P_n admits a square harmonic mean graph. [1]*

Theorem 1.5 *Triangular Snake T_n admits a square harmonic mean graph. [1]*

Theorem 1.6 *Diamond snake admits a square harmonic mean graph.*

2 CMSD of Square Harmonic Mean Graphs

In this section, we investigate Continuous Monotonic Star Decomposition of complete bipartite graph $K_{2,n}$, complete bipartite graph $K_{3,n}$.

Definition 2.1 *A Decomposition $(G_1, G_2, , G_n)$ of G is said to be a Continuous Monotonic Decomposition (CMD), if each G_i is connected and $|E(G_i)| = i$ for every $i = 1, 2, , n$. Clearly $q = \frac{n(n+1)}{2}$.*

Definition 2.2 *A Continuous Monotonic Decomposition in which each G_i is a star is said to be a Continuous Monotonic Star Decomposition (CMSD).*

Theorem 2.3 *A complete bipartite graph $K_{2,n}$ admits Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph, if $n = 2\alpha - 1$; $\alpha \geq 4$.*
Proof: Let $G = K_{2,n}$ be a complete bipartite graph. Let $V = (V_1, V_2)$ be the bipartition of $K_{2,n}$ where $V_1 = u, v$ and $V_2 = \{u_1, u_2, , u_n\}$.
case (i) *If $n = 2\alpha - 1; \alpha = 2, 3$.*
Decompose the graph G with Continuous Monotonic Star Decomposition satisfies the condition $q = \frac{n(n+1)}{2}$. After decomposition of $K_{2,n}$ and get edge disjoint star sub graphs $S_\alpha; 1 \leq \alpha \leq n$ of G. Each sub graph will have $n+1$ vertices and n edges. Also, label the vertices of each subgraph and get distinct edge labels. Therefore, it satisfies the labeling pattern of square harmonic mean graph. Hence $G = \bigcup_{\alpha=1}^{n} S_\alpha$. Each sub graph is a square harmonic mean graph and G is the union of sub graphs. Also satisfies the condition of Continuous Monotonic Star Decomposition (CMSD). Hence $K_{2,n}$ admits a Continuous Monotonic Star Decomposition of

Square Harmonic Mean Graph.

case (ii) *If* $n = 2\alpha - 1; \alpha \geq 4$.
Decompose the graph G with Continuous Monotonic Star Decomposition satisfies the condition $q = \frac{n(n+1)}{2}$. *After decomposition of* $K_{2,n}$ *and not get edge disjoint star sub graphs* S_α. *Also, it does not satisfy the condition of Continuous Monotonic Star Decomposition (CMSD). Hence* $K_{2,n}$ *does not admits a Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph, if* $n = 2\alpha - 1; \alpha \geq 4$.

Obviously, $K_{2,n}$ *admits a Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph, if* $n = 2\alpha - 1; \alpha = 2, 3$.

Theorem 2.4 *A complete bipartite graph* $K_{3,n}$ *admits Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph, if* $n = \alpha$*;* $\alpha = 1, 2$.
Proof: Let $G = K_{3,n}$ *be a complete bipartite graph. Let* $V = (V_1, V_2)$ *be the bipartition of* $K_{3,n}$ *where* $V_1 = u, v$ *and* $V_2 = \{u_1, u_2, , u_n\}$.
case (i) *If* $n = \alpha; \alpha = 1, 2$.
Decompose the graph G with Continuous Monotonic Star Decomposition satisfies the condition $q = \frac{n(n+1)}{2}$. *After decomposition of* $K_{3,n}$ *and get edge disjoint star sub graphs* $S_\alpha; 1 \leq \alpha \leq n$ *of G. Each sub graph will have* $n + 1$ *vertices and* n *edges. Also, label the vertices of each subgraph and get distinct edge labels. Therefore, it satisfies the labeling pattern of square harmonic mean graph. Hence* $G = \bigcup_{\alpha=1}^{n} S_\alpha$. *Each sub graph is a square harmonic mean graph and G is the union of sub graphs. Also satisfies the condition of Continuous Monotonic Star Decomposition (CMSD). Hence* $K_{3,n}$ *admits a Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph.*

case (ii) *If* $n = \alpha; \alpha \geq 3$.
Decompose the graph G with Continuous Monotonic Star Decomposition satisfies the condition $q = \frac{n(n+1)}{2}$. *After decomposition of* $K_{3,n}$ *and not get edge disjoint star sub graphs* S_α. *Also, it does not satisfy the condition of Continuous Monotonic Star Decomposition (CMSD). Hence* $K_{3,n}$ *does not admits a Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph, if* $n = \alpha; \alpha \geq 3$.

Obviously, $K_{3,n}$ *admits a Continuous Monotonic Star Decomposition of Square Harmonic Mean Graph, if* $n = \alpha; \alpha = 1, 2$.

Example 2.5 *The image below displays a Continuous Monotonic Star Decomposition of Square Harmonic Mean Labeling of* $K_{3,2}$.

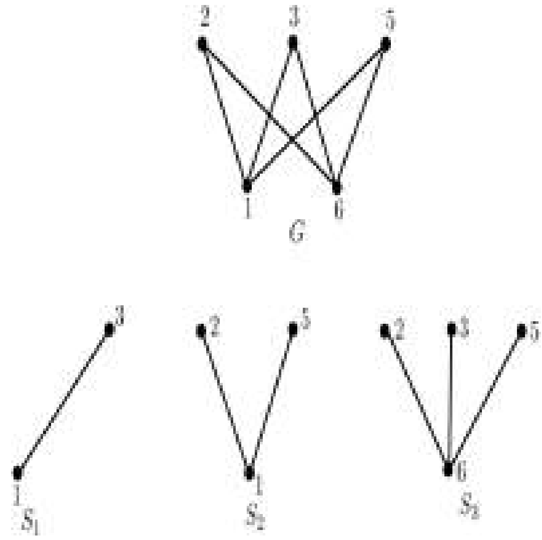

Figure 1: CMSD of (S_1, S_2, S_3) of $K_{3,2}$

In this example, Number of sub graphs, $n = 3$. Therefore $\frac{n(n+1)}{2} = \frac{3(3+1)}{2} = 6$ which is equal to the number of edges of $K_{3,6}$. Clearly it satisfies the condition of Continuous Monotonic Star Decomposition (CMSD) of Square Harmonic Mean Graph.

3 Even Path Decomposition of Square Harmonic Mean Graphs

In this section, we investigate Even Path Decomposition of Triangular snake nC_3, Diamond snake D_n.

Definition 3.1 *If $a = 2$ and $d = 2$ in Arithmetic Decomposition, then $q = n(n + 1)$. That is, the number of edges of G is the sum of first n even numbers $2, 4, 6, ..., 2n$. Thus, we call this decomposition as Even Decomposition (ED). Since, the number of edges of each sub graphs of G is even, we denote the Even Decomposition as $G_2, G_4, , G_2n$.*

Definition 3.2 *An Even Decomposition (ED) as $G_2, G_4, , G_2n$ of G is said to be an Even Path Decomposition (EPD) if each G_2i is a path of size $2i$ and it is denoted by $P_2, P_4, , P_2n$.*

Theorem 3.3 *Triangular snake nC_3 admits Even Path Decomposition (EPD) of Square Harmonic Mean graph, if $n = \dfrac{\alpha(\alpha+1)}{3}$ where $\dfrac{\alpha(\alpha+1)}{3}$ is a natural number.*
Proof: Let $G = nC_3$. Let $u_1, u_2, , u_n$ be the path of length n and let $v_1, v_2, , v_n$ be new vertices with joining the path $u_\alpha, u_(\alpha + 1)$ respectively. That is every edge of a path is replaced by C_3.

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Then $|V(T_n)| = 2n + 1$ and $|E(T_n)| = 3n$. By theorem 1.2, G admits a square harmonic mean graph.

case (i) *If $n = \frac{\alpha(\alpha+1)}{3}$, where $\frac{\alpha(\alpha+1)}{3}$ is a natural number*

Decompose the graph G with Even Path Decomposition and get distinct edge labels for each sub graphs of G be $P_\alpha; 2 \leq \alpha \leq 2n$ of G. Each sub graph will have $n+1$ vertices and n edges. Also, it satisfies the labeling pattern of square harmonic mean graph. By theorem 1.1., each subgraph P_α is a square harmonic mean graph. Hence, $G = \bigcup_{\alpha=1}^{n} P_2\alpha$. Therefore G is the union of sub graphs, also it satisfies the condition of Even Path Decomposition (EPD), that is $q = n(n+1)$. Hence, nC_3 admits Even Path Decomposition (EPD) of Square Harmonic Mean Graph.

case(ii) *If $n = \dfrac{\alpha(\alpha+1)}{3}$ where $n \notin \mathbb{N}$*

Decompose the graph G with Even Path Decomposition, but there is no existence of distinct edge labels for each sub graphs of G be P_α. Hence, it does not satisfy the condition of Even Path Decomposition. Thus, nC_3 does not admits Even Path Decomposition (EPD) of Square Harmonic Mean Graph, if $n \neq \frac{\alpha(\alpha+1)}{3}$, where $\frac{\alpha(\alpha+1)}{3}$ is a natural number

Therefore, nC_3 admits Even Path Decomposition (EPD) of Square Harmonic Mean Graph, $n = \frac{\alpha(\alpha+1)}{3}$, where $\frac{\alpha(\alpha+1)}{3}$ is a natural number

Example 3.4 *The image below displays an Even Path Decomposition of Square Harmonic Mean Labeling of $4C_3$.*

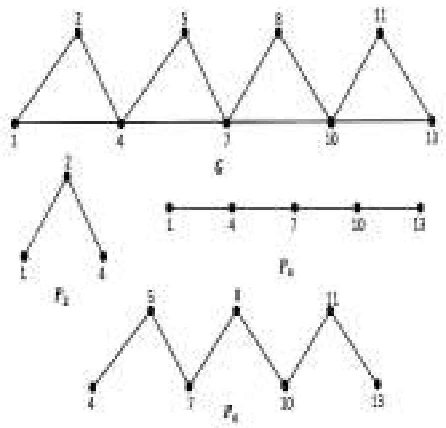

Figure 2: CMSD of (P_2, P_4, P_6) of $4C_3$

In this example, Number of sub graphs, $n = 3$. Therefore $n(n+1) = 3(3+1) = 12$ which is equal to the number of edges of $4C_3$. Clearly it satisfies the condition of Even Path Decomposition

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(EPD) of Square Harmonic Mean Graph.

Theorem 3.5 *Diamond snake D_n admits Even Path Decomposition (EPD) of Square Harmonic Mean graph, if $n = \dfrac{\alpha(\alpha+1)}{4}$ where $\dfrac{\alpha(\alpha+1)}{4}$ is a natural number.*

Proof: Let $G = D_n$. Consider a path $v_1, v_2, , v_n$ of size n by joining vertices v_α and $v_{\alpha+1}$ to two new vertices u_α and w_α. That is every edge of a path can be replaced by a cycle C_4. Then $|V(G)| = 3n + 1$ and $|E(G)| = 4n$, where n is the number of blocks in G. By theorem 1.3., G admits a square harmonic mean graph.

case (i) *If $n = \frac{\alpha(\alpha+1)}{4}$, where $\frac{\alpha(\alpha+1)}{4}$ is a natural number*

Decompose the graph G with Even Path Decomposition and get distinct edge labels for each sub graphs of G be $P_\alpha; 2 \leq \alpha \leq 2n$ of G. Each sub graph will have $n+1$ vertices and n edges. Also, it satisfies the labeling pattern of square harmonic mean graph. By theorem 1.1., each subgraph P_α is a square harmonic mean graph. Hence, $G = \bigcup_{\alpha=1}^{n} P_2\alpha$. Therefore G is the union of sub graphs, also it satisfies the condition of Even Path Decomposition (EPD), that is $q = n(n+1)$. Hence, D_n admits Even Path Decomposition (EPD) of Square Harmonic Mean Graph.

case(ii) *If $n = \dfrac{\alpha(\alpha+1)}{4}$ where $n \notin \mathbb{N}$*

Decompose the graph G with Even Path Decomposition, but there is no existence of distinct edge labels for each sub graphs of G be P_α. Hence, it does not satisfy the condition of Even Path Decomposition. Thus, D_n does not admits Even Path Decomposition (EPD) of Square Harmonic Mean Graph, if $n \neq \frac{\alpha(\alpha+1)}{4}$, where $\frac{\alpha(\alpha+1)}{4}$ is a natural number

Therefore, D_n admits Even Path Decomposition (EPD) of Square Harmonic Mean Graph, $n = \frac{\alpha(\alpha+1)}{4}$, where $\frac{\alpha(\alpha+1)}{4}$ is a natural number

4 Conclusion

The study of labeled graph and their decomposition is important due to its diversified applications. All graphs are not Square Harmonic Mean Graphs. It is very interesting to investigate the decomposition of graphs that admits Square Harmonic Mean Labeling. All Square Harmonic Mean Graphs cannot be decomposed by Continuous Monotonic Star Decomposition and Even Path Decomposition. The derived results are demonstrated by means of sufficient illustrations which provide better understanding.

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Results on Relatively Prime Domination Number of Some Standard and Vertex Switching Graphs

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Abstract

A set $S \subseteq V$ has at least two members and every pair of vertices u and v is such that $(d(u), d(v)) = 1$, then it is said to be a relatively prime dominating set. The relatively prime domination number, represented by $\gamma_{rpd}(G)$, is the lowest cardinality of a relatively prime dominating set. The switching of a finite undirected graph by a subset is defined as the graph $G^\sigma(V, E')$, which is obtained from G by removing all edges between σ and its complement $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. In this paper, we compute the relatively prime domination number of some standard graphs like Shell Graph, Wheel Graph, Barbell Graph, Jewel Graph, Extended Jewel Graph and Comb Graph. Apart from these, the relatively prime domination number of some vertex switching graphs such as Wheel Graph and Comb Graph are also evaluated .

Keywords : Dominating Set, Domination Number, Relatively Prime Dominating Set, Relatively Prime Dominating Number, Vertex switching.

AMS Subject Classification : 05C69

1 Introduction

By a graph $G = (V, E)$ we mean a finite undirected graph without loops and multiple edges. For graph theoretical terms, we refer to Harary [2] and for terms related to domination we refer to Haynes [3]. A subset S of V is said to be a dominating set in G if every vertex in $V - S$ is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G. Berge [1] and Ore [8] formulated the concept of domination in graphs. It was further extended to define many othermdomination related parameters in graphs. Let G be a non-trivial graph. A set S of V is said to be a relatively prime dominating set if it is a dominating set and for every pair of vertices u and v in S such that (d(u), d(v)) = 1. The minimum cardinality of a relatively prime dominating set is called the relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. Switching in graphs was introduced by Lint and Seidel [7]. For a finite undirected graph $G(V, E)$ and $v \in V$, the vertex switching [6] of G by v is the graph G^v which is obtained from G by removing all edges incident to v and adding edges which are not adjacent to v. In this paper we determine the relatively prime domination number of some standard graphs which includes shell graph $C(n, n-3)$, wheel graph W_n, $(2, n)$ - barbell graph $B(K_n, K_n)$, jewel graph J_n, extended jewel graph $E(J^*n, m)$, Comb graph $PnK1$.

The relatively prime domination number of some vertex switching graphs such as Wheel graph and Comb graph are also evaluated.

2 Definitions and examples

Definition 2.1 *For $n \geq 4$, the wheel W_n is defined to be the graph $K_1 + C_{n-1}$*

Definition 2.2 *Shell Graph $C(n, n-3)$ is defined as a cycle C_n with n-3 chords sharing a common end point called apex. Shell graphs are denoted by $C(n, n-3)$, $n \geq 4$.*

Definition 2.3 *Jewel Graph J_n is a graph with vertex set $V(J_n) = \{u, x, v, y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n)$=$\{ux, vx, uy, vy, xy, uu_i, vu_i : 1 \leq i \leq n\}$.*

The prime edge in a jewel graph J_n is defined to be the edge joining the vertices x and y.

Definition 2.4 *Jewel Graph J_n^* without the prime edge is defined as the graph in which the prime edge, that is the edge joining the vertices x and y is removed.*

Definition 2.5 *Extended Jewel Graph $E(J_{n,m}^*)$ without the prime edge is the graph with the vertex set $V(E(J_{n,m}^*)) = \{u, x, v, y, u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(E(J_{n,m}^*)) = \{ux, vx, uy, vy, xy, uu_i, vu_i, uv_i, vv_i : 1 \leq i \leq n\}$.*

Definition 2.6 *Comb Graph is a graph obtained by joining a single pendant edge to each vertex of a path P_n. It is denoted by $P_n K_1$.*

Definition 2.7 *The (2,n) - Barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge and it is denoted by $B(K_n, K_n)$.*

3 Main Results

Theorem 3.1 *Let G be a wheel graph W_n for $n \geq 4$.*

Then $\gamma_{rpd}(W_n) = \begin{cases} 2 & if\ n \equiv 1\ (mod\ 3) \\ 0 & if\ n \equiv 1\ (mod\ 3) \end{cases}$

Proof: Let u be the centre, and $v_1, v_2, \ldots\ldots, v_{n-1}$ be the vertices of the outer cycle C_{n-1} of W_n. The resultant graph G is W_n with $V(G) = \{u, v_i/1 \leq i \leq n-1\}$ and $E(G) = \{uv_i, v_ivi+1, v_1vn/1 \leq i \leq n-1\}$. Then W_n has n vertices and $2n-2$ edges and $d(u) = n-1$ and $d(v_i) = 3, 1 \leq i \leq n-1$.

Case 1. $n \not\equiv 1\ (mod\ 3)$

Suppose that n is either odd or even. In this case $\{u, v_i\}$ for $1 \leq i \leq n-1$ is a minimal relatively prime dominating set of W_n. Also $d(u) = n-1$ and $d(v_i) = 3, 1 \leq i \leq n-1$. Then $(d(u), d(v_i)) = (n-1, 3) = 1$. Hence $\gamma_{rpd}(W_n) = 2$.

Case 2. $n \equiv 1 \ (mod\ 3)$

In this case $(d(u), d(v_i)) = (n-1, 3) \neq 1$. *This implies that any dominating set of* W_n *must contain atleast two vertices having degree which is divisible by 3. Hence* $\gamma_{rpd}(W_n) = 0$.

Theorem 3.2 *Let G be a shell graph* $C(n, n-3)$, $n \geq 4$.

Then $\gamma_{rpd}(G) = \begin{cases} 2 & if\ n \not\equiv 1 (mod\ 6) \\ 0 & if\ n \equiv 1\ (mod\ 6) \end{cases}$.

Proof: Let $v_1, v_2, \ldots\ldots, v_n$ be the vertices of the cycle C_n. Fix the vertex v_1 to be the apex of the graph and construct $(n-3)$ chords sharing a common vertex with the apex. The resultant graph G is $C(n, n-3)$ with $V(G) = \{v_1, v_2, \ldots\ldots, v_n\}$ and $E(G) = \{v_i v_{i+1}, 1 \leq i \leq n-1, v_n v_1, v_1 v_i, 3 \leq i \leq n-1\}$. Hence $C(n, n-3)$ has n vertices and $2n-3$ edges.

Case 1. $n \not\equiv 1 (mod\ 6)$

Suppose that n is either odd or even and let $n \equiv 1 (mod\ 6)$. In this case either $\{v_{1,} v_2\}$ or $\{v_{1,} v_3\}$ is a minimal relatively prime dominating set of $C(n, n-3)$. Also $d(v_1) = n-1$ and $d(v_2) = d(v_n) = 2$ and $d(v_i) = 3, 3 \leq i \leq n-1$. Then either $(d(v_1), d(v)_2)) = (n-1, 2) = 1$ or $(d(v_1), d(v_2)) = (n-1, 3) = 1$. Therefore $\gamma_{rpd}(G) = 2$.

Case 2.. $n \equiv 1\ (mod\ 6)$

Here $(d(v_1), d(v_2)) = (n-1, 2) \neq 1$ and $(d(v_1), d(v_2)) = (n-1, 3) \neq 1$. This implies that any dominating set of G must contain atleast two vertices having degree which is divisible by either 3 or 2. Hence $\gamma_{rpd}(G) = 0$ if $n \equiv 1\ (mod\ 6)$.

Example 3.3 *For* $C(n, n-3)$, v_1, v_2 *is a minimal relatively prime dominating set. Hence* $\gamma_{rpd}(C(6, 3) = 0$

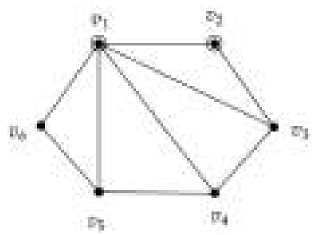

Figure 1: C(6, 3)

Theorem 3.4 *Let G be a jewel graph* J_n. *Then* $\gamma_{rpd}(J_n) = \begin{cases} 2 & if\ n+2 \not\equiv 0\ (mod\ 6) \\ 0 & if\ n+2 \equiv 0\ (mod\ 6). \end{cases}$

Proof: Let J_n be the jewel graph where n denotes the number of jewels in the graph. Then $V(J_n) = \{u, x, v, y, u_i : 1 \leq i \leq n\}$ and $E(J_n) = \{ux, vx, uy, vy, xy, uu_i, vu_i : 1 \leq i \leq n\}$. The prime edge in a jewel graph J_n is defined to be the edge joining the vertices x and y. Then $d(x) = 3 = d(y)$, $d(u) = d(v) = n + 2$ and $d(u_i) = 2$. Hence J_n has $n + 4$ vertices and $2n + 5$ edges.

Case 1. $n + 2 \not\equiv 0(mod\ 6)$

Suppose that n is either odd or even and let $n+2 \not\equiv 1(mod\ 6)$. Clearly either $\{u, u_i : 1 \leq i \leq n\}$ or $\{u, x\}$ is a minimal relatively prime dominating set of J_n. Also $d(u) = n + 2$ and $d(x) = 3$ and $d(u_i) = 2, 2 \leq i \leq n$. Then either $(d(u), d(u_i)) = (n+2, 2) = 1$ or $(d(u), d(x)) = (n+2, 3) = 1$. Hence $\gamma_{rpd}(J_n) = 2$.

Case 2. $n + 2 \equiv 0\ (mod\ 6)$

In this case $(d(u), d(u_i)) = (n + 2, 2) \neq 1$ or $(d(u), d(x)) = (n + 2, 3) \neq 1$. This implies that any dominating set of J_n must contain atleast two vertices having degree which is divisible by either 3 or 2. Hence $\gamma_{rpd}(J_n) = 0$.

Example 3.5 *For J_5, $\{u,\ u_i\}$ for $1 \leq i \leq 5$ is a minimal relatively prime dominating set. Hence $\gamma_{rpd}(C(6, 3)\ = 0$*

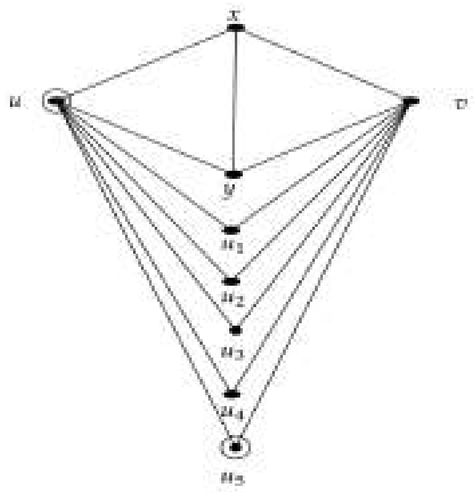

Figure 2: J_5

Theorem 3.6 *Let G be a jewel graph J_n^* without the prime edge.*
Then $\gamma_{rpd}(J_n^*) = \begin{cases} 2 & if\ n\ is\ odd \\ 0 & if\ n\ is\ even. \end{cases}$

Proof: Let J_n^* be the jewel graph without the prime edge where n denotes the number of jewels in the graph. The prime edge which joins the vertices x and y is removed. Consider J_n^* with the vertex set $V(J_n^*) = \{u, x, v, y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n^*) =$

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$\{ux, vx, uy, vy, xy, uu_i, vu_i : 1 \leq i \leq n\}$. Hence J_n^* has $n+4$ vertices and $2n+4$ edges. Also $d(u) = n+2 = d(v)$ and $d(x) = d(y) = d(u_i) = 2, 1 \leq i \leq n$.

Case 1. Assume that n is odd. Clearly $\{u, u_i\}$, $\{u, x\}$, $\{u, y\}$, $\{v, u_i\}$, $\{v, x\}$, $\{v, y\}$ are minimal relatively prime dominating set of J_n^*. Also $(d(u), d(u_i)) = (n+2, 2) = 1, 1 \leq i \leq n$ where n is odd. Hence $\gamma_{rpd}(J_n^*) = 2$.

Case 2. Suppose that n is even. In this case $(d(u), d(u_i)) = (n+2, 2) \neq 1, 1 \leq i \leq n$. Clearly any dominating set must contain atleast two vertices whose degree must be divisible by 2. Hence $\gamma_{rpd}(J_n^*) = 0$.

Theorem 3.7 *Let G be a extended jewel graph $J_{n,m}^*$ without the prime edge. Then $\gamma_{rpd}(E(J_{n,m}^*)) = 2$ if and only if n and m are of opposite parity.*

Proof: Let $(E(J_{n,m}^*))$ be the extended jewel graph without the prime edge. The total number of vertices are given by $V|E(J_{n,m}^*)| = n+m+4$. The total number of edges are given by E $|E(J_{n,m}^*)| = 2(m+n)+4$. Consider $E(J_{n,m}^*)$ with the vertex set $V(J_{n,m}^*)) = \{u, x, v, y, u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(E(J_{n,m}^*)) = \{ux, vx, uy, vy, xy, uu_i, vu_i, uv_i, vv_i : 1 \leq i \leq n\}$. Also $d(u) = d(v) = n+m+2$, $d(x) = d(y) = 2$ and $d(u_i) = d(v_j) = 2, 1 \leq i \leq n, 1 \leq j \leq n$. Suppose that n and m are of opposite parity. That is one is odd and the other is even. Clearly $\{u, u_i : 1 \leq i \leq n\}$, $\{u, x\}$, $\{u, y\}$, $\{v, u_i\}$, $\{v, x\}$, $\{v, y\}$ are the minimal relatively prime dominating set of $E(J_{n,m}^*)$. Also $(d(u), d(x)) = (n+m+2, 2) = 1$. Hence, $\gamma_{rpd}(E(J_{n,m}^*)) = 2$. If n and m are of same parity, then $(d(u), d(x)) = (n+m+2, 2) = 1$. Any dominating set of $E(J_{n,m}^*)$ must contain atleast two vertices whose degree is divisible by 2. Therefore, $\gamma_{rpd}(E(J_{n,m}^*)) = 0$.

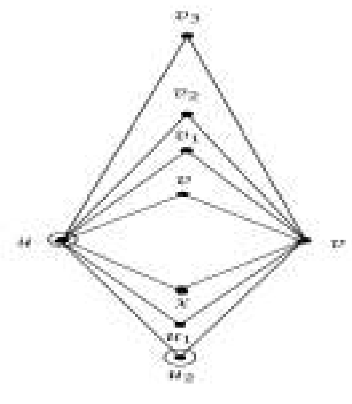

Figure 3: $E(J^*3, 2)$

Theorem 3.8 *Let G be a comb graph, $G = P_n K_1, n \geq 2$. Then $\gamma_{rpd}(P_n \odot K_1) = n$.*

Proof: Let v_1, v_2,......, v_n be the n vertices of a path P_n. Join a single pendant edge to each vertex of the path P_n. The resultant graph G is $P_n \odot K_1$ with the vertex set $V(G) =$

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$\{u_i, v_i/1 \leq i \leq n\}$, where u_i is a leaf for i $\in$ [1, n], $d(v_1) = d(vn) = 2$, $d(v_i) = 3$ for i $\in$ [2, n-1] and $E(G) = \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{u_iv_i/1 \leq i \leq n\}$. Clearly $\{u_1, u_2, \ldots.., u_n\}$ is a minimal relatively prime dominating set of $P_n \odot K_1$. Also $d(u_i) = 1, 1 \leq i \leq n$. That is the leaves of $P_n \odot K_1$ serves as the minimal relatively prime dominating set of $P_n \odot K_1$. Then $(d(u_i), d(ui+1)) = 1, 1 \leq i \leq n-1$. Hence, $\gamma_{rpd}(P_n \odot K_1) = n$.

Example 3.9 *For P_4K_1, $\{u_1, u_2, u3, u4\}$ is a minimal relatively prime dominating set. Also $d(u_i) = 1, 1 \leq i \leq 4$ and $(d(u_i), d(ui+1)) = 1, 1 \leq i \leq 3$. Hence $\gamma_{rpd}(P_4K_1) = 4$*

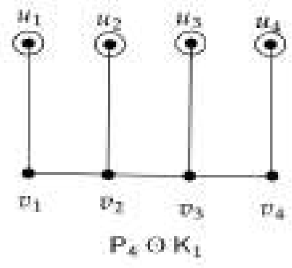

Figure 4:

Theorem 3.10 *Let G be a $(2, n)$ Barbell graph denoted by $B(K_n, K_n)$. Then for any n, $\gamma_{rpd}(G) = 2$.*

Proof: Let $v_1, v_2, \ldots, vn$ and $u_1, u_2, \ldots, un$ be the vertices of two copies of a complete graph K_n. Join u_1 and v_1 by a bridge. The resultant graph G is $B(K_n, K_n)$ with the vertex set $V(B(K_n, K_n)) = \{u_i, v_i, 1 \leq i \leq n\}$ and $E(B(K_n, K_n)) = \{u_1v_1, v_ivi+1, vnv_1, u_iui+1, unu_1, 1 \leq i \leq n\}$ and hence $B(K_n, K_n)$ has 2n vertices. Also $d(u_1) = d(v_1) = n$ and $d(u_i) = d(v_i) = n-1, 2 \leq i \leq n$. For any n, clearly $\{u_1, v_i\}$ and $\{v_1, u_i\}, 2 \leq i \leq n$ is a minimal relatively prime dominating set of $B(K_n, K_n)$. Also $d(u_1) = d(v_1) = n$ and $d(u_i) = d(v_i) = n-1, 2 \leq i \leq n$. Then $(n, n-1) = 1$. Hence $\gamma_{rpd}(G) = 2$.

Example 3.11 *For $n = 5$, clearly $\{v_1, u_i\}$ and $\{u_1, v_i\}, 2 \leq i \leq 5$ is a minimal relatively prime dominating set of B(K_5, K_5) of size 2. Therefore, $\gamma_{rpd}(G) = 2$.*

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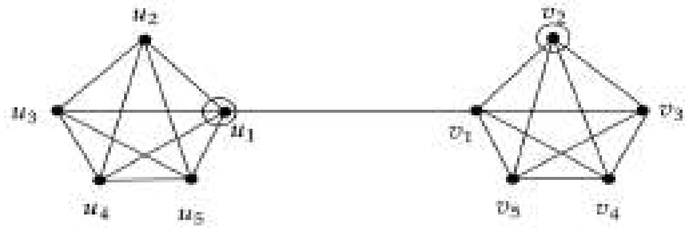

Figure 5: B(K_5, K_5)

4 RPD Number of Vertex Switching in Some Standard Graphs

Theorem 4.1 *Let G be a wheel graph W_n for $n \geq 4$ and let v be any vertex of W_n.*
i) If $d_G(v) = 3$, then $\gamma_{rpd}(W_n^v) = \begin{cases} 2 & if\ n\ is\ odd \\ 0 & otherwise \end{cases}$
ii) If dG(v) = n-1, then $\gamma_{rpd}({W_n}^v) = 0$.

Proof: Let u be the centre, and $v_1, v_2, \ldots\ldots, v_{n-1}$ be the vertices of the outer cycle C_{n-1} of W_n. The resultant graph G is W_n with $V(G) = \{u, v_i/1 \leq i \leq n-1\}$ and $E(G) = \{uv_i, v_iv_{i+1}, v_1v_n/1 \leq i \leq n-1\}$. Then W_n has n vertices and $2n-2$ edges.

Case 1. $dG(v) = 3$

Sub case 1. 1. n is odd.

Suppose that n is odd. Here v is $v_i, 1 \leq i \leq n$. Clearly, $G^{v_1} \cong G^{v_2} \cong \ldots \cong G^{v_{n-1}}$. Let v be v_2. In G, v_2 is adjacent to v_1, v_3 and u. Hence v_2 is adjacent to all vertices except v_1, v_3 and u in G^{v_2}. Therefore, $\{u, v_2\}$ is a minimal relatively prime dominating(RPD) set of G^{v_2}. And $(d^{'}(v_2), d^{'}(u)) = (n-4, n-2) = 1$. This implies that $\gamma_{rpd}(G^{\text{v}}) = 2$.

Sub Case 1. 2. n is even.

Suppose that n is even. Now, $(d^{'}(\text{v}_2), d^{'}\text{u})$=(n-2, n-4) $\neq 1$. Then $\{u, v_2\}$ is a dominating set of G^{v_2}. Clearly, any minimal relatively prime dominating set of G^{v_2} must contain atleast two vertices having degree which is divisible by 2. Hence, γ_{rpd} $({\text{W}_n}^{\text{v}}) = 0$.

Case 2. $dG(v) = n-1$

Here v is u. Clearly, $G^{\text{u}} \cong C_{n-1}$ with a isolated vertex u. Also $d^{'}(\text{v}_i) = 2 forall$i$\in$ [1,n-1], Then $\gamma_{rpd}(W_n^u) = 0$.

Example 4.2 *In Figure 4.1, $\{u, v_1\}$ is a minimal relatively prime dominating set of ${W_5}^{v_1}$. Also $(d(u), d(v_1)) = 1$. Hence $\gamma_{rpd}({W_5}^{v_1}) = 2$.*

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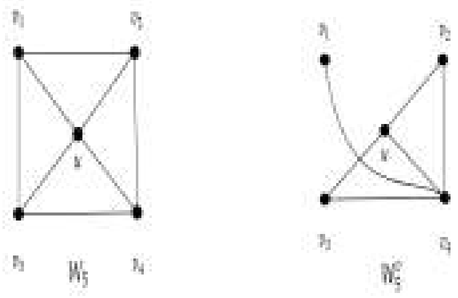

Figure 6:

Theorem 4.3 *Let G be a comb graph $P_n \odot K_{1,n} \geq 2$. Then $\gamma_{rpd}(G_v) = 2$ if and only if v is either a leaf u_1 or u_n.*

Proof: Let $v_1,\ v_2, \ldots\ldots, v_n$ be the n vertices of a path P_n. Join a single pendant edge to each vertex of the path P_n. The resultant graph G is $P_n \odot K_1$ with the vertex set $V(G) = \{u_i, v_i/1 \leq i \leq n\}$, where u_i is a leaf for i $\in$ [1, n], $d(v_1) = d(v_n) = 2$, $d(v_i) = 3$ for i $\in$ [2, n-1] and $E(G) = \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{u_i v_i/1 \leq i \leq n\}$. Here v is either u_1 or u_n. Clearly, $G^{u_1} \cong G^{u_n}$. Let v be u_1. In G, u_1 is adjacent to v_1. Then in G^{u_1}, u_1 is adjacent to all the vertices except v_1. Therefore, $\{v_1, u_1\}$ is a minimal relatively prime dominating set of G^{u_1}. Also $d(u_1) = 2n-2$, $d(v_1) = 1$ and $(2n-2, 1) = 1$. Hence $\gamma_{rpd}(G^v) = 2$.

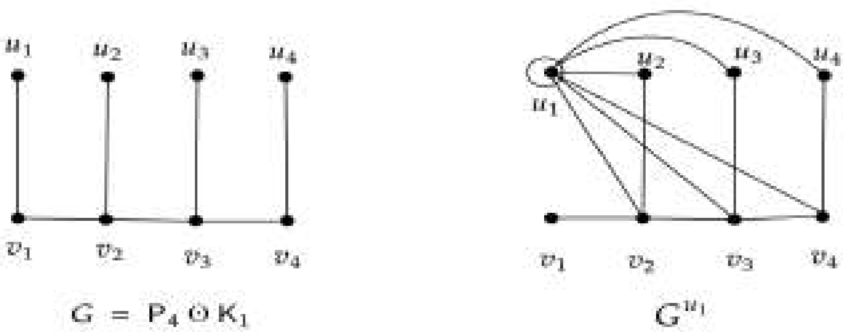

Figure 7:

5 Conclusion

Domination theory is one of the most flourishing branches of graph theory today. In this paper we have discussed the relatively prime domination number of some standard graphs which includes Shell graph, Wheel graph, Barbell graph, Jewel graph, Extended jewel graph and Comb

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graph . The relatively prime domination number of some vertex switching graphs such as Wheel graph and Comb graph are also evaluated.

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Intersection graph and Gamma graph of a Power graph

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Abstract

In this paper,we study about the structure of a power graph $\Gamma_p(\mathbb{Z}_n)$ .We investigate,the interplay between the graph theoretic properties of $\gamma.\Gamma_p(\mathbb{Z}_n)$ and $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n)$.Further ,we prove that $\gamma.\Gamma_p(\mathbb{Z}_n)$ and $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n)$ are planar and also $\gamma.\Gamma_p(\mathbb{Z}_n)$ is self-centered and has a perfect matching.Also,we obtain the value of clique number and the number of ω-set of $\Gamma_p(\mathbb{Z}_n)$.

Keywords : Power graph,Intersection graph,Gamma graph,Planar,Self-centered,Perfect matching,Clique.

AMS Subject Classification : 05C25,05C69,05C10

1 Introduction

The study of algebraic structures, using the properties of graphs, became an exciting research topic in the past twenty years, leading to many fascinating results and questions. In the literature, there are many papers assigning graphs to rings, groups and semigroups.Also investigation of algebraic properties of groups or rings using the associated graph becomes an exciting topic. In 2002, The directed power graph of a semi group $\mathbb{S}$ was defined by Kelarev and Quinn [7] as the digraph $\vec{\mathcal{G}}$ with vertex set $\mathbb{S}$, in which there is an arc from x to y if and only if $x \neq y$ and $y = x^m$ for some positive integer m. Motivated by this, Chakrabarty et al.[2] defined the undirected power graph $\Gamma_P(G)$ of a group G. Actually the power graph $\Gamma_P(G)$ of G is the graph with vertex set $V(\Gamma_P(G))$ and two distinct vertices x, y $\in$ G are adjacent in $\Gamma_P(G)$ if and only if either $x^i = y$ or $y^j = x$, where i and j are integers and $2 \leq i, j \leq n$.

By a graph $\Gamma_P(G)$ = (V,E), we mean an undirected graph Γ with vertex set V , edge set E and has no loops or multiple edges.A set D $\subseteq$ V of vertices in a graph G= (V,E) is called a dominating set if for every vertex u $\in V - D$, there exists a vertex v $\in$ D such that v is adjacent to u. A dominating set D is minimal if no proper subset of D is a dominating set. The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a minimal dominating set of G. A dominating set D in a graph G with cardinality γ is called γ-set of G.Let D be the collection of γ-sets in G.The gamma graph of G, denoted by γ.G is the graph with vertex set D and any two vertices D_1 and D_2 are adjacent if $|D_1 \cap D_2| = \gamma(G) - 1$. The intersection graph of G, denoted by $\mathbb{I}_G$ is the graph with vertex set D and any two vertices D_1 and D_2 are adjacent if $D_1 \cap D_2 \neq \emptyset$.

A planar graph is a graph that can be embedded in the plane so that no two edges intersect geometrically except at a vertex which both are incident.Any set M of independent lines of a graph G is called a matching of G.A matching M is called a perfect matching if every point of G is M-saturated.A graph is said to be self-centered graph if the eccentricity of every vertex of the graph is the same.The clique number is the number of vertices in a largest complete subgraph of G and is denoted by ω(G).Let G be a group with identity e. The number of elements of a group is called its order and it is denoted by o(G). The order of an element g in a group is the smallest positive integer n such that g^n = e. If no such integer exists, we say g has infinite order. The order of an element g is denoted o(g).

2 Structures of a power graph

In this section we drive the structure of a power graphs,which will be used for further study.

Theorem 2.1 *Let n=pq where p is a even prime and q ≥3 .Then the number of ω-sets in $\Gamma_p(\mathbb{Z}_n)$ is 1 and the clique number is n-1.*

Proof: Let $\mathbb{Z}_n$ be a cyclic group of order pq where p is a even prime and q ≥3 is prime. Then $\Gamma_p(\mathbb{Z}_n) \cong (K_{p-1} \cup K_{q-1}) + K_{\phi(n)+1}$ where ϕ is the Euler function.

Clearly the elements of order p and the elements of order q are not adjacent. The number of elements of order p is $p-1$ and the number of elements of order q is $q-1$.But we have p is a even prime and hence $\omega(\Gamma_p(\mathbb{Z}_n)) = n-1$ and the number of ω-set in $\Gamma_p(\mathbb{Z}_n)$ is 1.

Theorem 2.2 *Let n= p^k where k ≥1,p is prime .Then the number of ω-sets in $\Gamma_p(\mathbb{Z}_n)$ is 1 and the clique number is n.*

Proof: Let x,y $\neq$ e $\in \mathbb{Z}_n$.Then o(x)= p^i and o(y)= p^j for some $1 \leq i, j \leq n$. From this we have either $o(x)|o(y)$ or $o(y)|o(x)$ and so either $< x > \in < y >$ or $< y > \in < x >$. Since the identity element is adjacent to all other elements of $\mathbb{Z}_n$, $\Gamma_p(\mathbb{Z}_n) \cong K_1 + K_{p^k-1}$. Hence the number of ω-set in $\Gamma_p(\mathbb{Z}_n)$ is 1 and the clique number is n.

Theorem 2.3 *Let n= $p^k q$, where $k > 1$, p<q, p and q are two distinct primes. Then the number of ω-sets in $\Gamma_p(\mathbb{Z}_n)$ is 1 and the clique number is n-(p^k-1).*

Proof: Since $\mathbb{Z}_n$ is a cyclic group,$\mathbb{Z}_n$ has a unique subgroup H of order p^k .Then

$$\Gamma_p(H) \cong K_{p^k}$$

The number of elements which are relatively prime to $\mathbb{Z}_n$ is $\phi(n)$ where ϕ is a Euler function and it is denoted by M .Let o(x)= $p^k q$ and o(y)= $p^k q$.From this we have both $o(x)|o(y)$ and $o(y)|o(x)$

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and so both $<x>\in<y>$ *and* $<y>\in<x>$.
Thus we get, $\Gamma_p(M)\cong K_{\phi(n)}$
Consider the remaining elements of $\mathbb{Z}_n$*-H-M and it is denoted by Q .Let* $x_1,y_1\in Q$.*Then* $o(x_1)= p^i q$ *and* $o(y_1)= p^j q$,*for some* $o\leq i,j\leq n$. *From this we have either* $o(x_1)|o(y_1)$ *or* $o(y_1)|o(x_1)$ *and so either* $<x_1>\in<y_1>$ *or* $<y_1>\in<x_1>$.
Thus we get, $\Gamma_p(Q)\cong K_{(p^{k-1})(q-1)}$. *Since the identity element is adjacent to every element in* $\mathbb{Z}_n,(K_{(p^k-1)}\cup K_{(p^{k-1})(q-1)})+K_{\phi(n)+1}$ *is a spanning subgraph of* $\Gamma_p(\mathbb{Z}_n)$.
Hence $\omega(\Gamma_p(\mathbb{Z}_n))$ = n - (p^k-1) *and the number of* ω*-set in* $\Gamma_p(\mathbb{Z}_n)$ *is 1.*

Theorem 2.4 *Let n=p* q^k, *where* $k>1$,$p<q$, *p and q are two distinct primes. Then the number of* ω*-sets in* $\Gamma_p(\mathbb{Z}_n)$ *is 1 and the clique number is n-(* q^{k-1}*)(p-1).*
Proof: Since $\mathbb{Z}_n$ *is a cyclic group ,*$\mathbb{Z}_n$ *has a unique subgroup H of order* q^k .*Then* $\Gamma_p(H)\cong K_{q^k}$.
The number of elements which are relatively prime to $\mathbb{Z}_n$ *is* $\phi(n)$ *where* ϕ *is a Euler function and it is denoted by M .Let o(x)=p* q^k *and o(y)=p* q^k.*From this we have both* $o(x)|o(y)$ *and* $o(y)|o(x)$ *and so both* $<x>\in<y>$ *and* $<y>\in<x>$. *Thus we get,* $\Gamma_p(M)\cong K_{\phi(n)}$
Consider the remaining elements of $\mathbb{Z}_n$*-H-M and it is denoted by Q .Let* $x_1,y_1\in Q$.*Then* $o(x_1)$*=p* q^i *and* $o(y_1)$*=p* q^j,*for some* $o\leq i,j\leq n$. *From this we have either* $o(x_1)|o(y_1)$ *or* $o(y_1)|o(x_1)$ *and so either* $<x_1>\in<y_1>$ *or* $<y_1>\in<x_1>$.
Thus we get, $\Gamma_p(Q)\cong K_{(q^{k-1})(p-1)}$. *Since the identity element is adjacent to every element in* $\mathbb{Z}_n,(K_{(q^{k-1})(p-1)}\cup K_{(q^k-1)}+K_{\phi(n)+1}$ *is a spanning subgraph of* $\Gamma_p(\mathbb{Z}_n)$.
Hence $\omega(\Gamma_p(\mathbb{Z}_n))$ = n - (q^{k-1}*)(p-1) and the number of* ω*-set in* $\Gamma_p(\mathbb{Z}_n)$ *is 1.*

3 Intersection graph and Gamma graph of a power graph

Definition 3.1 *The Intersection graph of gamma sets in the power graph of a commutative ring* $\mathbb{Z}_n$ *with vertex set as the collection of all gamma sets of the power graph of* $\mathbb{Z}_n$ *and two distinct vertices A and B are adjacent if and only if* $|A\cap B|\neq\emptyset$.*This graph is denoted by* $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n)$.

Definition 3.2 *The Gamma graph of a power graph of a finite commutative ring* $\mathbb{Z}_n$ *is a graph with vertex set as the collection of all gamma sets of the power graph of* $\mathbb{Z}_n$ *and two distinct vertices A and B are adjacent if and only if* $|A\cap B|=\gamma(\Gamma_p(\mathbb{Z}_n))-1$.*This graph is denoted by* $\gamma.(\Gamma_p(\mathbb{Z}_n))$.

Theorem 3.3 *Let n be an positive integer.Then*
$$\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n)=\begin{cases}\overline{K_n} & \text{if } n=p^k, k\geq 1, p \quad \text{is} \quad \text{prime}\\ \overline{K_{\phi(n)+1}} & \text{otherwise}\end{cases}$$
Proof: If $n=p^k$, *then* $\Gamma_p(\mathbb{Z}_n)\cong K_1+K_P^{k-1}$ *and* $\gamma(\Gamma_p(\mathbb{Z}_n))=1$.
Hence $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n)=\overline{K_n}$

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If $n \neq p^k$ *is an integer, then* $\Gamma_p(\mathbb{Z}_n)$ *has a subgraph of* $K_{\phi(n)+1}$ *, where* ϕ *is a Euler function.Also* $\gamma(\Gamma_p(\mathbb{Z}_n)) = 1$.

Hence $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n) = \overline{K_{\phi(n)+1}}$.

Theorem 3.4 *Let n be an positive integer.Then*

$$\gamma.(\Gamma_p(\mathbb{Z}_n)) = \begin{cases} K_n & \text{if } n = p^k, k \geq 1, p \text{ is prime} \\ K_{\phi(n)+1} & \text{otherwise} \end{cases}$$

Proof: If n= p^k, *then* $\Gamma_p(\mathbb{Z}_n) \cong K_1 + K_{P^k-1}$ *and* $\gamma(\Gamma_p(\mathbb{Z}_n)) = 1$.

Hence $\gamma.(\Gamma_p(\mathbb{Z}_n)) = K_n$

If $n \neq p^k$ *is an integer, then* $\Gamma_p(\mathbb{Z}_n)$ *has a subgraph of* $K_{\phi(n)+1}$, *where* ϕ *is a Euler function.Also* $\gamma(\Gamma_p(\mathbb{Z}_n)) = 1$.

Hence $\gamma.(\Gamma_p(\mathbb{Z}_n)) = K_{\phi(n)+1}$.

Theorem 3.5 *Let n be an positive integer. Then* $\gamma.(\Gamma_p(\mathbb{Z}_n))$ *is self-centered.*

Proof: If $n = p^k$, *then* $\gamma.(\Gamma_p(\mathbb{Z}_n)) = K_n$, *a complete graph and so* $e(v) = 1$ *for all* $v \in V$.

ie., $\gamma.(\Gamma_p(\mathbb{Z}_n))$ *is self-centered.*

If $n \neq p^k$ *is an integer, then* $\gamma.(\Gamma_p(\mathbb{Z}_n)) = K_{\phi(n)+1}$, *a complete graph and so* $e(v) = 1$ *for all* $v \in V$.

ie., $\gamma.(\Gamma_p(\mathbb{Z}_n))$ *is self-centered.*

Theorem 3.6 *Let n be an positive integer. Then* $\gamma.(\Gamma_p(\mathbb{Z}_n))$ *has a perfect matching.*

Proof: If $n = p^k$, *then* $\gamma.(\Gamma_p(\mathbb{Z}_n)) = K_n$, *a complete graph and so has a perfect matching.*

If $n \neq p^k$ *is an integer, then* $\gamma.(\Gamma_p(\mathbb{Z}_n)) = K_{\phi(n)+1}$, *a complete graph and so has a perfect matching.*

Theorem 3.7 *Let n be an positive integer. Then* $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n)$ *is planar.*

Proof: If $n = p^k$, *then* $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n) = \overline{K_n}$, *which is planar.*

If $n \neq p^k$, *then* $\mathbb{I}_{\Gamma_p}(\mathbb{Z}_n) = \overline{K_{\phi(n)+1}}$, *which is also a planar.*

Theorem 3.8 *Let n be an positive integer.Then* $\gamma.(\Gamma_p(\mathbb{Z}_n)$ *is planar if* $n = p^k$, $n < 5$ *otherwise* $n \neq p^k$, $\phi(n) + 1 < 5$.

Proof: If $n = p^k$, *then* $\gamma.(\Gamma_p(Z)_n = K_n$.*Hence* $\gamma.(\Gamma_p(Z)_n$ *is planar for* $n < 5$.

If $n \neq p^k$, *then* $\gamma.(\Gamma_p(Z)_n = K_{\phi(n)+1}$.*Hence* $\gamma.(\Gamma_p(Z)_n$ *is planar for* $\phi(n) + 1 < 5$.

4 Conclusion

In this paper, we have discussed about the strucutre of power graphs and intersection graph and gamma graph of a power graph.It is planned to explore different graph properties in future work regarding to this concept.

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Edge 4-Product Cordial Labeling of Some Graphs

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Abstract

For a graph $G = (V(G), E(G))$ having no isolated vertex, an edge labeling function $f : E(G) \to \{0, 1, ..., k-1\}$ is said to be an edge k-product cordial labeling if it induces a vertex labeling $f^\star : V(G) \to \{0, 1, ..., k-1\}$ defined by $f^\star(v) = (\prod_{uv \in E(G)} f(uv))(mod\ k)$ satisfies $|v_{f^\star}(i) - v_{f^\star}(j)| \leq 1$, and $|e_f(i) - e_f(j)| \leq 1$ for $i, j \in \{0, 1, ..., k-1\}$ where $e_f(i)$ and $v_{f^\star}(i)$ denote the number of edges and vertices respectively having label i for $i = 0, 1, ..., k-1$. A graph that admits edge k-product cordial labeling is called an edge k-product cordial graph. In this paper, we establish edge 4-product cordial labeling of path, cycle, comb, and fan graphs.

Keywords : Cordial Labeling, Edge Product Cordial Labeling, Edge 4-Product Cordial Labeling, Path, Cycle, Comb, Fan.
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1 Introduction

All graphs under consideration are finite, connected, undirected, and adhere to the fundamental notations and terminology of graph theory, as outlined in Harary [4]. During the past six decades, the concept of graph labeling has attracted considerable attention in graph theory due to its diverse range of applications. Graph labeling involves assigning real numbers, typically positive integers, to the elements of a graph through a function. In 1967, Rosa [5] made a significant contribution to the field of graph theory with a pioneering paper on the graph labeling problem. Since then, numerous types of graph labeling techniques have been investigated by various researchers. Gallian [2], in his comprehensive survey, elegantly categorized these labelings into graceful labelings, variations of harmonious labelings, magic-type labelings, anti-magic-type labelings, and miscellaneous labelings. Cordial labeling, a refined version of graceful and harmonious labeling, was introduced by Cahit [1]. Drawing from the concept of cordial labeling, Sundaram et al.[6] introduced product cordial labeling in 2004. In 2012, Vaidya et al.[7] introduced edge product cordial labeling as an edge-based counterpart to product cordial labeling. Building upon previous concepts, we advance further by introducing a new extension called edge k-product cordial labeling. For a graph $G = (V(G), E(G))$ having no isolated vertex, an edge labeling function $f : E(G) \to \{0, 1, ..., k-1\}$ is said to be an edge k-product cordial labeling if it induces a vertex labeling $f^\star : V(G) \to \{0, 1, ..., k-1\}$ defined by $f^\star(v) = (\prod_{uv \in E(G)} f(uv))(mod\ k)$ satisfies $|v_{f^\star}(i) - v_{f^\star}(j)| \leq 1$, and $|e_f(i) - e_f(j)| \leq 1$ for $i, j \in \{0, 1, ..., k-1\}$ where $e_f(i)$ and $v_{f^\star}(i)$ denote the number of edges and vertices respectively having label i for $i = 0, 1, ..., k-1$. This paper concentrates on investigating the behavior of edge 4-product cordial labeling of path, cycle, fan graphs, and comb graphs. A fan graph F_n [3], is obtained by joining all vertices of P_n to a new vertex which is known as the center. The graph formed by attaching a single pendant edge to each vertex of a path is referred to as a Comb $P_n \odot K_1$.

2 Main Results

Theorem 2.1 *The Path P_n is edge 4-product cordial if and only if $5 \leq n \leq 11$.*
Proof: Let $V(P_n) = \{u_i : 1 \leq i \leq n\}$ and $E(P_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\}$.
Assume $5 \leq n \leq 11$.
An edge 4-product cordial labeling of $P_5, P_6, P_7, P_8, P_9, P_{10}$, and P_{11} are shown in Table 1.

n	v_1v_2	v_2v_3	v_3v_4	v_4v_5	v_5v_6	v_6v_7	v_7v_8	v_8v_9	v_9v_{10}	$v_{10}v_{11}$
5	0	2	3	1						
6	0	2	3	3	1					
7	0	1	3	2	3	1				
8	1	3	2	0	2	3	1			
9	1	3	2	0	0	2	3	1		
10	0	0	2	3	1	2	3	3	1	
11	0	0	2	3	1	2	3	3	1	1

Table 1

From the above labeling pattern we have $|v_{f^\star}(i) - v_{f^\star}(j)| \leq 1$, and $|e_f(i) - e_f(j)| \leq 1$ for $i, j \in \{0, 1, 2, 3\}$.
Conversely, Assume $n \leq 4$ or $n \geq 12$.
If possible, let there be an edge 4-product cordial labeling f of P_n for $n \leq 4$ or $n \geq 12$.
Case (i): $n \leq 4$.
Subcase (i): $n = 2$.
Clearly, $v_{f^*}(i)$ and $e_f(i)$ are either 0 or 1 $(i = 0, 1, 2, 3)$. If $e_f(i) = 1$, then $v_{f^\star}(i) = 2$ which is a contradiction. So, $e_f(i) = 0$ for all $i (i = 0, 1, 2, 3)$ which is absurd. Hence, P_2 is not an edge 4-product cordial graph.

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Subcase (ii): $n = 3$.
Thus, $e_f(i)$ *and* $v_{f^\star}(i)$ *are either* 0 *or* 1 $(i = 0, 1, 2, 3)$. *Clearly,* $e_f(0) = 0$ *otherwise* $v_{f^\star}(0) = 2$. *Also,* $e_f(2) = 0$ *otherwise* $v_{f^\star}(2) = 2$. *Now,* $e_f(1) = e_f(3) = 1$ *imply* $v_{f^\star}(3) = 2$. *Therefore,* $|v_{f^*}(0) - v_{f^*}(3)| > 1$ *which is a contradiction. Hence, the path* P_3 *is not edge 4-product cordial.*
Subcase (iii): $n = 4$.
Thus, $e_f(i) = 0$ *or* 1 $(i = 0, 1, 2, 3)$ *and* $v_{f^\star}(i) = 1$ *for all* $i = 0, 1, 2, 3$. *Clearly,* $e_f(0) = 0$ *otherwise* $v_{f^\star}(0) = 2$. *Also,* $e_f(1) = e_f(2) = e_f(3) = 1$ *imply* $v_{f^\star}(2) = 2$. *Therefore,* $|v_{f^*}(i) - v_{f^*}(2)| > 1$ *for some* $i \in \{0, 1, 3\}$ *which is a contradiction. Hence,* P_4 *is not an edge 4-product cordial graph.*
Case (ii): $n \geq 12$.
Subcase (i): $n \equiv 0(mod\ 4)$.
Let $|V(P_n)| = 4t$ *and* $|E(K_n)| = 4t - 1(t \geq 3)$. *Thus,* $e_f(i) = t$ *or* $t - 1(i = 0, 1, 2, 3)$ *and* $v_{f^*}(i) = t$ *for all* $i = 0, 1, 2, 3$. *Clearly,* $e_f(0) = t - 1$ *and 0 must be assigned consecutively otherwise* $v_{f^*}(0) \geq t + 1$. *Hence,* $e_f(i) = t$ *for all* i $(i = 1, 2, 3)$. *Also, two adjacent edges cannot labeled by 2 otherwise* $v_{f^*}(0) > t$. *Since* $e_f(2) = t$, $v_{f^*}(2) > t$. *Therefore,* $|v_{f^*}(i) - v_{f^*}(2)| > 1$ *for some* $i \in \{0, 1, 3\}$ *which is a contradiction. Hence, the path* P_n *is not edge 4-product cordial if* $n \equiv 0(mod\ 4)$.
Subcase (ii): $n \equiv 1(mod\ 4)$.
Let $|V(P_n)| = 4t + 1$ *and* $|E(K_n)| = 4t(t \geq 3)$. *Thus,* $e_f(i) = t$ *for all* i $(i = 0, 1, 2, 3)$ *and* $v_{f^*}(i)$ *is either* t *or* $t + 1$ *for* $i = 0, 1, 2, 3$. *By counting* $v_{f^*}(2)$ *as in above subcase, we get a contradiction. Hence,* P_n *is not an edge 4-product cordial graph if* $n \equiv 1(mod\ 4)$.
Subcase (iii):$n \equiv r(mod\ 4); 2 \leq r \leq 3$.
Let $|V(P_n)| = 4t + r$ *and* $|E(K_n)| = 4t + r - 1(t \geq 3)$. *Thus,* $e_f(i)$ *and* $v_{f^*}(i)$ *are either* t *or* $t + 1(i = 0, 1, 2, 3)$. *Clearly,* $e_f(0) = t$, *0 must be assigned consecutively and two adjacent edges cannot labeled by 2 otherwise* $v_{f^*}(0) \geq t + 1$. *If* $e_f(2) = t$ *or* $t + 1$, $v_{f^*}(2) > t + 1$. *Therefore,* $|v_{f^*}(i) - v_{f^*}(2)| > 1$ *for some* $i \in \{0, 1, 3\}$ *which is a contradiction. Hence, the path* P_n *is not edge 4-product cordial if* $n \equiv 2, 3(mod\ 4)$.

Theorem 2.2 *The cycle* C_n *is edge 4-product cordial if and only if* $5 \leq n \leq 10$ *except* $n = 8$.
Proof: Let $V(C_n) = \{u_i : 1 \leq i \leq n\}$ *and* $E(C_n) = \{u_iu_{i+1} : 1 \leq i \leq n - 1\} \cup \{u_nu_1\}$.
Assume $5 \leq n \leq 10$ *except* $n = 8$.
An edge 4-product cordial labeling of C_5, C_6, C_7, C_9, *and* C_{10} *are shown in Table 2.*

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n	v_1v_2	v_2v_3	v_3v_4	v_4v_5	v_5v_6	v_6v_7	v_7v_8	v_8v_9	v_9v_{10}	$v_{10}v_1$
5	*0*	*1*	*1*	*3*	*2*					
6	*0*	*1*	*3*	*3*	*1*	*2*				
7	*0*	*2*	*3*	*3*	*1*	*1*	*2*			
9	*0*	*0*	*2*	*3*	*3*	*1*	*1*	*3*	*2*	
10	*0*	*0*	*2*	*1*	*3*	*3*	*1*	*1*	*3*	*2*

Table 2

From the above labeling pattern we have $|v_{f^\star}(i) - v_{f^\star}(j)| \leq 1$, *and* $|e_f(i) - e_f(j)| \leq 1$ *for* $i, j \in \{0, 1, 2, 3\}$.

Conversely, Assume $3 \leq n$ *or* $n \geq 11$.

If possible, let there be an edge 4-product cordial labeling f *of* C_n *for* $3 \leq n$ *or* $n \geq 11$.

Case (i): $n \equiv 0(mod\ 4)$.

Let $|V(C_n)| = |E(C_n)| = 4t$. *Thus,* $e_f(i) = v_{f^*}(i) = t$ *for all* i $(i = 0, 1, 2, 3)$. *But* $e_f(0) = t$ *implies* $v_{f^*}(0) > t$. *Therefore,* $|v_{f^*}(0) - v_{f^*}(j)| > 1$ *for* $j = 1, 2, 3$ *which is a contradiction. Hence* C_n *is not an edge 4-product cordial graph if* $n \equiv 0(mod\ 4)$.

Case (ii): $n \equiv r(mod\ 4); 1 \leq r \leq 3$.

$|V(C_n)| = |E(C_n)| = 4t + r$. *Thus,* $e_f(i)$ *and* $v_{f^*}(i)$ *are either* t *or* $t + 1$ *for* $i = 0, 1, 2, 3$. *Clearly, 0 must be assigned consecutively and two adjacent edges cannot labeled by 2 otherwise* $v_{f^*}(0) > t+1$. *So,* $e_f(2) = t$ *or* $t+1$ *implies* $v_{f^*}(2) > t+1$. *Therefore,* $|v_{f^*}(i) - v_{f^*}(2)| > 1$ *for some* $i \in \{0, 1, 3\}$ *which is a contradiction. Hence,* C_n *is not an edge 4-product cordial graph if* $n \equiv 1, 2, 3(mod\ 4)$.

Theorem 2.3 *The Fan graph* F_n *is not edge 4-product cordial.*

Proof: Let the vertex set of F_n *be* $V(F_n) = \{u_i, u : 1 \leq i \leq n\}$ *and the edge set be* $E(F_n) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{uu_i : 1 \leq i \leq n\}$. *If possible, let there be an edge 4-product cordial labeling* f *of* F_n. *Then we have the following six cases.*

Case (i): $n = 1, 2$.

Clearly, $F_1 \cong P_2$ *and* $F_2 \cong C_3$ *which are not edge 4-product cordial graphs.*

Hence, the fans F_1 *and* F_2 *are not edge 4-product cordial.*

Case (ii): $n = 4$.

Thus, $e_f(i)$ *and* $v_{f^*}(i)$ *are either 1 or 2* $(i = 0, 1, 2, 3)$. *Clearly,* $e_f(0) = 1$ *otherwise* $v_{f^*}(0) > 2$. *Hence,* $e_f(i) = 2$ *and* $v_{f^*}(i) = 1$ *for all* i $(i = 1, 2, 3)$. *But,* $e_f(2) = 2$ *implies* $v_{f^*}(2) > 1$. *Therefore,* $|v_{f^*}(0) - v_{f^*}(2)| > 1$ *which is a contradiction. Hence,* F_4 *is not an edge 4-product cordial graph.*

Case (iii): $n = 5, 6$.

Thus, $e_f(i) = 2$ *or* 3 *for* $i(i = 0, 1, 2, 3)$ *and* $v_{f^*}(i) = 1$ *or* 2 *for* $i(i = 0, 1, 2, 3)$. *Clearly,*

$e_f(2) = 2$ or 3 implies $v_{f^*}(2) > 2$. Therefore, $|v_{f^*}(i) - v_{f^*}(2)| > 1$ for $i = 0, 1, 3$ which is a contradiction. Hence, F_5 and F_6 are not edge 4-product cordial graphs.

Case (iv): *$n \equiv 0(mod\ 4)$ for $n \geq 8$.*

Let $|V(F_n)| = 4t + 1$ and $|E(F_n)| = 8t - 1$ where $t \geq 2$. Thus, $e_f(i) = 2t$ or $2t - 1$ for $i = 0, 1, 2, 3$ and $v_{f^*}(i) = t$ or $t + 1$ for $i = 0, 1, 2, 3$. Clearly, $e_f(0) = 2t$ or $2t - 1$ implies $v_{f^*}(0) > t + 1$. Therefore, $|v_{f^*}(0) - v_{f^*}(j)| > 1$ for $j = 1, 2, 3$ which is a contradiction. Hence, F_n is not an edge 4-product cordial graph for $n \equiv 0(mod\ 4)$ and $n \geq 8$.

Case (v): *$n \equiv r(mod\ 4)$ for $n \geq 9$ and $1 \leq r \leq 2$.*

Let $|V(F_n)| = 4t + r + 1$ and $|E(F_n)| = 8t + 2r - 1$ where $t \geq 2$. Thus, $e_f(i) = 2t$ or $2t + 1$ for $i = 0, 1, 2, 3$ and $v_{f^*}(i) = t$ or $t + 1$ for $i = 0, 1, 2, 3$. Clearly, $e_f(0) = 2t$ or $2t + 1$ implies $v_{f^*}(0) > t + 1$. Therefore, $|v_{f^*}(0) - v_{f^*}(j)| > 1$ for $j = 1, 2, 3$ which is a contradiction. Hence, the fan F_n is not edge 4-product cordial for $n \equiv 1, 2(mod\ 4)$ and $n \geq 9$.

Case (vi): *$n \equiv 3(mod\ 4); n \geq 3$.*

Let $|V(F_n)| = 4t + 4$ and $|E(F_n)| = 8t + 5$ where $t \geq 0$. Thus, $e_f(i) = 2t + 1$ or $2t + 2$ for $i = 0, 1, 2, 3$ and $v_{f^*}(i) = t + 1$ for all i $(i = 0, 1, 2, 3)$. Clearly, $e_f(0) = 2t + 1$ or $2t + 2$ implies $v_{f^*}(0) > t + 1$. Therefore, $|v_{f^*}(0) - v_{f^*}(j)| > 1$ for $j = 1, 2, 3$ which is a contradiction. Hence, F_n is not edge 4-product cordial for $n \equiv 3(mod\ 4)$ and $n \geq 3$.

Theorem 2.4 *The Comb $P_n \odot K_1$ is edge 4-product cordial if and only if $n \geq 3$.*

Proof: Let $V(P_n \odot K_1) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Then we have the following two cases.

Define $f : E(P_n \odot K_1) \to \{0, 1, 2, 3\}$ as follows:

Case (i): *$n \equiv 0, 1(mod\ 4)$.*

Define $f(u_iu_{i+1}) = 0$ if $2\lfloor \frac{n}{4} \rfloor + 1 \leq i \leq n - 1$ and $f(u_iv_i) = 2$ if $2\lfloor \frac{n}{4} \rfloor + 1 \leq i \leq n$.

Subcase (i): *$\lfloor \frac{n}{4} \rfloor$ is odd.*

For $1 \leq i \leq 2\lfloor \frac{n}{4} \rfloor$, define

$$f(u_iu_{i+1}) = f(u_iv_i) = \begin{cases} 1 & \text{if } i \equiv 2,\ 3(mod\ 4) \\ 3 & \text{if } i \equiv 0,\ 1(mod\ 4). \end{cases}$$

Subcase (ii): *$\lfloor \frac{n}{4} \rfloor$ is even.*

For $1 \leq i \leq 2\frac{n}{4}$, define

$$f(u_iu_{i+1}) = f(u_iv_i) = \begin{cases} 1 & \text{if } i \equiv 0,\ 1(mod\ 4) \\ 3 & \text{if } i \equiv 2,\ 3(mod\ 4). \end{cases}$$

From the above labeling,

$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) = \frac{n}{2}$ and $v_{f^*}(0) = v_{f^*}(1) = v_{f^*}(2) = v_{f^*}(3) = \frac{n}{2}$ if

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$n \equiv 0 (mod\ 4)$.
$e_f(0) = e_f(1) = e_f(2) - 1 = e_f(3) = 2\lfloor \frac{n}{4} \rfloor$ and $v_{f^*}(0) - 1 = v_{f^*}(1) = v_{f^*}(2) - 1 = v_{f^*}(3) = 2\lfloor \frac{n}{4} \rfloor$ if $n \equiv 1 (mod\ 4)$.
Case (ii): *$n \equiv 2, 3 (mod\ 4)$.*
Define $f(u_iu_{i+1}) = 0$ if $2\lfloor \frac{n}{4} \rfloor + 2 \leq i \leq n - 1$ and $f(u_iv_i) = 2$ if $2\lfloor \frac{n}{4} \rfloor + 2 \leq i \leq n$.
Subcase (i): *$\lfloor \frac{n}{4} \rfloor$ is odd.*
For $1 \leq i \leq 2\lfloor \frac{n}{4} \rfloor + 1$, define

$$f(u_iu_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1,\ 2 (mod\ 4) \\ 3 & \text{if } i \equiv 0,\ 3 (mod\ 4) \quad and \end{cases}$$

$$f(u_iv_i) = \begin{cases} 1 & \text{if } i \equiv 0,\ 1 (mod\ 4) \\ 3 & \text{if } i \equiv 2,\ 3 (mod\ 4). \end{cases}$$

Subcase (ii): *$\lfloor \frac{n}{4} \rfloor$ is even.*
For $1 \leq i \leq 2\lfloor \frac{n}{4} \rfloor + 1$, define

$$f(u_iu_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 0,\ 3 (mod\ 4) \\ 3 & \text{if } i \equiv 1,\ 2 (mod\ 4) \quad and \end{cases}$$

$$f(u_iv_i) = \begin{cases} 1 & \text{if } i \equiv 1,\ 2 (mod\ 4) \\ 3 & \text{if } i \equiv 0,\ 3 (mod\ 4). \end{cases}$$

From the above labeling,
$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) = 2\lfloor \frac{n}{4} \rfloor + 1$ and $v_{f^*}(0) = v_{f^*}(1) = v_{f^*}(2) = v_{f^*}(3) = 2\lfloor \frac{n}{4} \rfloor + 1$ if $n \equiv 2 (mod\ 4)$.
$e_f(0) = e_f(1) = e_f(2) - 1 = e_f(3) = 2\lfloor \frac{n}{4} \rfloor + 1$ and $v_{f^*}(0) - 1 = v_{f^*}(1) = v_{f^*}(2) - 1 = v_{f^*}(3) = 2\lfloor \frac{n}{4} \rfloor + 1$ if $n \equiv 3 (mod\ 4)$.
In all the cases, $|v_{f^\star}(i) - v_{f^\star}(j)| \leq 1$, and $|e_f(i) - e_f(j)| \leq 1$ for $i, j \in \{0, 1, 2, 3\}$. Hence, the comb $P_n \odot K_1$ is edge 4-product cordial.

An example of edge 4-product cordial labeling of $P_8 \odot K_1$ is shown in Figure 1.

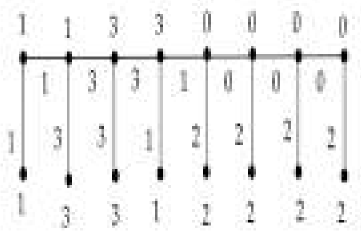

Figure 1: Edge 4 – product cordial labeling of $P_8 \odot K_1$

3 Conclusion

In this paper, we study the edge 4-product cordial labeling behavior of P_n, C_n, Fn, and $P_n \odot K_1$. In the future, we propose to find the edge k-product cordial behavior of these graphs.

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Triameter of the Zero Divisor Graph for the ring of integer mod n

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Abstract

Let $\mathbb{Z}_n$ be the ring of integer modulo n and $\Gamma(\mathbb{Z}_n)$ be the corresponding zero divisor graph. The zero divisor graph of a commutative ring is the graph whose vertices are the nonzero zero divisors of the commutative ring, and two vertices are connected by an edge if and only if their product is zero. Also, a distance parameter triameter of a connected graph G, which is defined as for $|V(G)| \geq 3$, $tr(G)$= max{ $d(u,v) + d(v,w) + d(u,w); u, v, w \in$ V(G) }. This article focuses on the triameter of zero divisor graph for the ring of integer modulo n. More precisely, we completely characterize the triameter of $\Gamma(\mathbb{Z}_n)$.

Keywords : Commutative ring, Ring of integers, Diameter, Zero divisor graph, Triameter
AMS Subject Classification : 05C12, 13A70

1 Introduction

One of the most rapidly growing areas of mathematics is graph theory. Understanding how to quantify the nodal connections in a network to build communities is one of the challenges with the clustering process. Investigating the distance between the vertices will help us to find the solution. The diameter and radius of the graph are two of the most frequently seen properties. Angsuman Das[6] first proposed the idea of a distance parameter triameter of a connected graph in 2018, which is stated as follows: For a connected graph $G = (V, E)$, the distance parameter triameter of G, $tr(G)$ is equal to the maximum of $\{d_G(u, v) + d_G(v, w) + d_G(u, w)$; u,v,w $\in$ V(G)}. The triameter was originally used as a parameter in [11], but it wasn't formally termed until [7]. Metric polytopes are another field that utilizes the idea of triameter [9]. The study of algebraic structures using the properties of graph theory has been an exciting research topic in recent days. Let R be a commutative ring with identity and $Z(R)$ the set of zero divisors of R. The zero divisor graph of the ring R, denoted by $\Gamma(R)$, is the simple graph with vertex set $Z(R)$, and two distinct vertices x and y are adjacent if and only if $xy = 0$, I. Beck[5] first proposed this idea in 1988, however since every element would be adjacent to the element 0, the structure of these graphs is not particularly interesting. The current definition of $\Gamma(R)$ was provided by Anderson and Livingston in 1999 in [3]. They associate a simple graph with vertices as components of $Z(R)^* = Z(R) - \{0\}$, the set of nonzero zero divisors of R, and the adjacency between two distinct vertices is determined in the same manner as Beck's zero-divisor graph. Anderson and Livingston[3] and Anderson and Mulay[2] investigated the diameter of the zero divisor graph of a commutative ring. It was proven that the zero divisor graph of a commutative ring is always connected with diameter at most three. The study of zero divisor graphs was continued by S. Akbari and B. Mohamadian[1], T. G. Lucas[10] who found solely on nonzero zero divisors. In [8] Kim et.al focus on the diameter and girth of the zero-divisor graph for the ring of integer modulo n, and they completely characterized the diameter of zero divisor graph of $\mathbb{Z}_n$. Let $\mathbb{Z}_n$ be the ring of integer modulo n and $\Gamma(\mathbb{Z}_n)$ be the corresponding zero divisor graph. If n is a prime number, then $\mathbb{Z}_n$ has no zero divisors, so $\Gamma(\mathbb{Z}_n)$ is the empty graph. Hence, In this paper, we only consider the case that n is a composite. We use the following definitions and results in the subsequent section.

Definition 1.1 *[12] A graph G is called connected if there is a path between any two distinct vertices in G.*

Definition 1.2 *[13] A graph G is said to be complete if every pair of distinct vertices are adjacent. A complete graph of n vertices is denoted by K_n.*

Definition 1.3 *[13] A graph $G = (V, E)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty disjoint subsets A and B such that every edge in G has one end in A and the other end in B, denoted by $V(G) = (A, B)$. The sets A and B are called a bipartition of G. If every vertex of A is adjacent with all the vertices of B, then G is said to be complete bipartite graph, and it is denoted by $K_{|A|,|B|}$. A graph is called a star graph if it is a complete bipartite graph with one partition a singleton set.*

Definition 1.4 *[13] For vertices u and v in a graph G, $d(u, v)$ denotes the length of the shortest path from u to v. If there is no such path, then $d(u, v)$ is defined to be infinity; and $d(u, u)$ is defined to be zero.*

Definition 1.5 *[13] The diameter of a graph G is the supremum of $\{d(u, v) \mid u$ and v are vertices of G $\}$ denoted by diam(G).*

Definition 1.6 *[13] The girth of a graph G is the length of a smallest cycle contained in the graph, and $gr(G) = \infty$ if G contains no cycles.*

Definition 1.7 *[13] A set S of vertices of G is a dominating set of G if every vertex in $V(G) - S$ is adjacent to some vertex in S. The cardinality of minimum dominating set is called the domination number of G and is denoted by $\gamma(G)$.*

Definition 1.8 *[14] A ring R in which $uv = vu$ for all $u, v \in R$, is called a commutative ring.*

Definition 1.9 *[14] A commutative ring R which contains no nonzero zero divisors is called an integral domain.*

Definition 1.10 *[6] Let $G = (V, E)$ be a connected graph on $n \geq 3$ vertices. The triameter of G is defined as $max\{d(u, v) + d(v, w) + d(u, w) : u, v, w \in V\}$ and is denoted by $tr(G)$.*

Definition 1.11 *[4] A triple of vertices $u, v, w \in V(G)$ is triametral if $d(u, v, w) = tr(G)$.*

Definition 1.12 *[3] Let R be a commutative ring with identity and $Z(R)^*$ the set of nonzero zero divisors of R. The zero divisor graph of R, denoted by $\Gamma(R)$, is the simple graph with vertex set $Z(R)^*$, and for distinct $a, b \in Z(R)^*$, a and b are adjacent if and only if $ab = 0$. Clearly, $\Gamma(R)$ is the empty graph if and only if R is an integral domain.*

Theorem 1.13 *Let R be a commutative ring. Then $\Gamma(R)$ is connected and $diam(\Gamma(R)) \leq 3$.*
Proof: see Theorem 2.3 in [3].

Theorem 1.14 *For any connected graph G, $2.diam(G) \leq tr(G) \leq 3.diam(G)$.*
Proof: see Theorem 3.1 in [6].

2 Main Results

In this section, we completely characterize the triameter of the zero divisor graph of the ring of integer modulo n

Theorem 2.1 *For any connected graph $\Gamma(\mathbb{Z}_n)$, then $tr(\Gamma(\mathbb{Z}_n)) \leq 9$.*
Proof: This result holds from Theorem 1.13 and Theorem 1.14, i.e., $tr(\Gamma(\mathbb{Z}_n)) \leq 3.3$. Therefore, $tr(\Gamma(\mathbb{Z}_n)) \leq 9$.

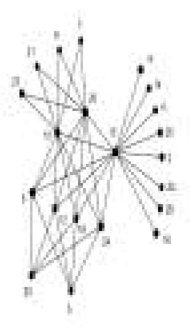

Figure 1: Zero divisor graph of Z_{30}

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Theorem 2.2 *Let p_i be a prime number and $\alpha_i \in \mathbb{N}$ for $i = 1, ..., m$. Then the following statements hold true:*

(i) *$tr(\Gamma(\mathbb{Z}_n)) = 0$ if and only if $n = 4$ or $n = 9$.*
(ii) *$tr(\Gamma(\mathbb{Z}_n)) = 3$ if and only if n is a prime square with a prime of more than 3 non zero zero divisors.*
(iii) *$tr(\Gamma(\mathbb{Z}_n)) = 4$ if and only if $n = 6$ or $n = 8$.*
(iv) *$tr(\Gamma(\mathbb{Z}_n)) = 6$ if and only if $n = p_1p_2$ or $n = p_1^{\alpha_1}$ with $\alpha_1 \geq 3$. In either case $Z(\mathbb{Z}_n)^*$ has more than three zero divisors.*
(v) *$tr(\Gamma(\mathbb{Z}_n)) = 8$ if and only if $n = p_1^{\alpha_1}p_2^{\alpha_2}$ with at least $\alpha_1 \geq 2$ or $\alpha_2 \geq 2$.*
(vi) *$tr(\Gamma(\mathbb{Z}_n)) = 9$ if and only if $n = p_1^{\alpha_1}p_2^{\alpha_2}...p_m^{\alpha_m}$ with $3 \leq m \in \mathbb{N}$.*
Proof:
(i) *If $n = 4$, $Z(\mathbb{Z}_n)^* = \{ 2 \}$. Hence, $\Gamma(\mathbb{Z}_n)$ contains only one vertex. Therefore, $tr(\Gamma(\mathbb{Z}_n)) = 0$. If $n = 9$, $Z(\mathbb{Z}_n)^* = \{ 3,6 \}$. Hence, $\Gamma(\mathbb{Z}_n)$ is a line segment with length one. Therefore, $tr(\Gamma(\mathbb{Z}_n)) = 0$.*
(ii) *If n is a prime square with a prime of more than 3 nonzero zero divisors. Then, all the elements in $Z(\mathbb{Z}_n)^*$ are the multiples of p. So the product of any two elements of $Z(\mathbb{Z}_n)^*$ is zero, i.e., $uv = 0$ for all $u, v \in Z(\mathbb{Z}_n)^*$. Implies that $\Gamma(\mathbb{Z}_n)$ is a complete graph. Therefore, $tr(\Gamma(\mathbb{Z}_n)) = 3$.*
(iii) *Suppose $n = 6$ or 8, then $\Gamma(\mathbb{Z}_n)$ is a line segment with length two. Therefore, $tr(\Gamma(\mathbb{Z}_n)) = 4$.*
(iv) *Suppose $n = p_1p_2$ with $Z(\mathbb{Z}_n)^* > 3$ then $Z(\mathbb{Z}_{p_1p_2})^* = \{p_1, 2p_1, ..., (p_2 - 1)p_1, p_2, 2p_2, ..., (p_1 - 1)p_2\}$ implies $(ip_1) \times (jp_2) = 0$ $\forall$ $i = 1, 2, 3, ..., p_2 - 1$ and $j = 1, 2, 3, ..., p_1 - 1$. For any $a, b \in \{p_1, 2p_1, ...(p_2 - 1)p_1\}$ and $c, d \in \{p_2, 2p_2, ...(p_1 - 1)p_2\}$ where $ab \neq 0$ and $cd \neq 0$. Every vertex of $\{p_1, 2p_1, ...(p_2 - 1)p_1\}$ is a multiples of p_1, and every vertex of $\{p_2, 2p_2, ...(p_1 - 1)\}$ is a multiples of p_2. Thus, every element of $\{p_1, 2p_1, ...(p_2 - 1)p_1\}$ shares an edge with every element of $\{p_2, 2p_2, ...(p_1 - 1)p_2\}$. Then $d(u, v, w)$ is a triametral triple if u, v, w belongs to either $\{p_1, 2p_1, ...(p_2 - 1)p_1\}$ or $\{p_2, 2p_2, ...(p_1 - 1)p_2\}$. Therefore, $tr(\Gamma(\mathbb{Z}_{p_1p_2})) = 6$.*
Suppose, $n = p_1^{\alpha_1}$ with $\alpha_1 \geq 3$ and $Z(\mathbb{Z}_n)^* > 3$. Then $Z(\mathbb{Z}_{p_1^{\alpha_1}})^* = \{p_1, 2p_1, ...(p_1^{\alpha_1 - 1} - 1)p_1\}$. Here, $ap_1^{\alpha_1-1}$ =0 $\forall$ $a \in Z(\mathbb{Z}_{p_1^{\alpha_1}}))^*$. Then, $d(u, v) \leq 2$ for all $u, v \in \Gamma(\mathbb{Z}_{p_1^{\alpha_1}})$ implies that $tr(\Gamma(\mathbb{Z}_{p_1^{\alpha_1}})) \leq 6$. Note that, $p_1(p_1^{\alpha_1-1} - 1) \times p_1 \neq 0$ and $p_1 \times (2p_1) \neq 0$ also $p_1(p_1^{\alpha_1-1} - 1) \times p_1 \neq 0$. Thus, $d(p_1, 2p_1) \geq 2$, $d(2p_1, (p_1^{\alpha_1-1} - 1)p_1) \geq 2$, $d(p_1, (p_1^{\alpha_1-1} - 1)p_1) \geq 2$. Since $d(u, v) \leq 2$ for all $u,v \in \Gamma(\mathbb{Z}_{p_1^{\alpha_1}})$, then $d(p_1, 2p_1) = 2$, $d(2p_1, (p_1^{\alpha_1-1} - 1)p_1) = 2$, $d(p_1, (p_1^{\alpha_1-1} - 1)p_1) = 2$. By the definition of triameter, $tr(\Gamma(\mathbb{Z}_{p_1^{\alpha_1}})) = 6$.
(v) *Suppose that $n = p_1^{\alpha_1}p_2^{\alpha_2}$ with at least $\alpha_1 \geq 2$ or $\alpha_2 \geq 2$. Then $p_1, p_2 \in Z(\mathbb{Z}_{p_1^{\alpha_1}p_2^{\alpha_2}})^*$ with $p_1 \times p_2 \neq 0$. So that $d(p_1, p_2) \geq 2$. Also, p_1— $p_1^{\alpha_1-1}p_2^{\alpha_2}$— $p_1^{\alpha_1}p_2^{\alpha_2-1}$ — p_2 is a path. Therefore,*

$d(p_1, p_2) = 3$. There exists a vertex q_1 such that $(n, q_1) = p_1$. Then q_1 — $p_1^{\alpha_1 -1}p_2^{\alpha_2}$ — p_1 and q_1— $p_1^{\alpha_1-1}p_2^{\alpha_2}$— $p_1^{\alpha_1}p_2^{\alpha_2-1}$ — p_2 are the paths, i.e, $d(q_1, p_1) = 2$ and $d(q_1, p_2) = 3$. If both p_1 and p_2 divides q_1, then either $p_1q_1 = 0$ or $p_2q_1 = 0$. Therefore, $tr(\Gamma(\mathbb{Z}_{p_1^{\alpha_1}p_2^{\alpha_2}})) = 8$.

(vi) *If $n = p_1^{\alpha_1}p_2^{\alpha_2}...p_m^{\alpha_m}$ with $3 \leq m \in \mathbb{N}$. Then $p_i, p_j \in Z(\mathbb{Z}_n)^*$ and $p_i \times p_j \neq 0$ where $i \neq j$. Therefore, $d(p_i, p_j) \geq 2$. Also, p_i — $p_1^{\alpha_1}p_2^{\alpha_2}...p_i^{\alpha_i-1}...p_m^{\alpha_m}$— $p_1^{\alpha_1}p_2^{\alpha_2}...p_j^{\alpha_j-1}...p_m^{\alpha_m}$ — p_j is a path. Implies that $d(p_i, p_j) = 3$. Also, $p_k \in Z(\mathbb{Z}_n)^*$. note that $d(p_i, p_k) \geq 2$ and $d(p_j, p_k) \geq 2$. Also, p_i — $p_1^{\alpha_1}p_2^{\alpha_2}...p_i^{\alpha_i-1}...p_m^{\alpha_m}$— $p_1^{\alpha_1}p_2^{\alpha_2}...p_k^{\alpha_k-1}...p_m^{\alpha_m}$ — p_k and p_j — $p_1^{\alpha_1}p_2^{\alpha_2}...p_j^{\alpha_j-1}...p_m^{\alpha_m}$— $p_1^{\alpha_1}p_2^{\alpha_2}...p_k^{\alpha_k-1}...p_m^{\alpha_m}$ — p_k are the paths. Therefore, $d(p_i, p_k) = 3$ and $d(p_j, p_k) = 3$. Therefore, $tr(\Gamma(\mathbb{Z}_{p_1^{\alpha_1}p_2^{\alpha_2}...p_m^{\alpha_m}})) \geq 9$. Hence $tr(\Gamma(\mathbb{Z}_{p_1^{\alpha_1}p_2^{\alpha_2}...p_m^{\alpha_m}})) = 9$*

Theorem 2.3 *Let $n = p_1^{\alpha_1}p_2^{\alpha_2}...p_m^{\alpha_m}$ where p_i's are distinct prime and $\alpha_i \in \mathbb{N}$ for $i = 1, ..., m$ and if $\Gamma(\mathbb{Z}_n)$ has triameter three then. $gr(\Gamma(\mathbb{Z}_n)) = 3$.*

Proof: Assume that $\Gamma(\mathbb{Z}_n)$ has triameter three. Then, by theorem 2.2, n is a prime square with a prime of more than three nonzero zero divisors. Moreover, $\Gamma(\mathbb{Z}_n)$ is a complete graph. Then clearly, $gr(\Gamma(\mathbb{Z}_n)) = 3$.

Remark 2.4 *Consider the ring $\mathbb{Z}_{27}$. The corresponding zero divisor graph has given below. And $\Gamma(\mathbb{Z}_{27})$ has 3 – 9 – 18 – 3 as a triangle, i.e., $gr(\Gamma(\mathbb{Z}_{27})) = 3$. But $tr(\Gamma(\mathbb{Z}_{27})) = 6$, which says that converse of the above theorem is not true.*

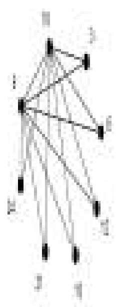

Figure 2: Zero divisor graph of Z_{27}

Theorem 2.5 *Let $n = p_1^{\alpha_1}p_2^{\alpha_2}...p_m^{\alpha_m}$ where p_i's are distinct prime and $\alpha_i \in \mathbb{N}$ for $i = 1, ..., m$ and if $\Gamma(\mathbb{Z}_n)$ is triangle free with $|\Gamma(\mathbb{Z}_n)| > 3$, then $tr(\Gamma(\mathbb{Z}_n)) \in \{ 6,8 \}$.*

Proof:If $\Gamma(\mathbb{Z}_n)$ is triangle free with $|\Gamma(\mathbb{Z}_n)| > 3$, then there are two cases.

case (i): *If $n = p_1p_2$ with $|\Gamma(\mathbb{Z}_n)| > 3$. Then, by theorem 2.2, $tr(\Gamma(\mathbb{Z}_n)) = 6$.*

case (ii): *If $n = 2^2p_1$ with $|\Gamma(\mathbb{Z}_n)| > 3$. Then, 2, $p_1 \in Z(\mathbb{Z}_n)^*$ such that $2p_1 \neq 0$. Therefore*

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$d(2,p_1) = 3$, *since the vertex 2 is connected only to* $2p_1$, *which is not connected to* p_1. *The vertex* $2p_1$ *is connected only to vertices divisible by* 2^2, *any of which are in turn connected to the vertex* p_1. *Moreover, there is a vertex* q_1 *such that* $(n, q_1) = 2$. *Then, 2 –* $2p_1$ *–* q_1, *and* q_1 *–* $2p_1$ *– 2 –* p_1 *are the paths, i.e.,* $d(2, q_1) = 2$, *and* $d(q_1, p_1) = 3$. *Since the vertex* q_1 *is connected only to* $2p_1$. *Therefore* $tr(\Gamma(\mathbb{Z}_n)) = 8$.

Theorem 2.6 *Let* $n = p_1^{\alpha_1} p_2^{\alpha_2} ... p_m^{\alpha_m}$ *where* p_i*'s are distinct prime and* $\alpha_i \in \mathbb{N}$ *for* $i = 1, ..., m$. *Then* $tr(\Gamma(\mathbb{Z}_n)) = 9$ *if and only if* $\gamma(\Gamma(\mathbb{Z}_n)) = 3$.

Proof: Assume $tr(\Gamma(\mathbb{Z}_n)) = 9$. *Let* u, v *and* w *be the triametral triple of vertices in* $\Gamma(\mathbb{Z}_n)$, *i.e.,* $d(u, v, w) = 9$. *Clearly,* $d(u, v) = 3, d(v, w) = 3, d(u, w) = 3$. *Suppose* $\gamma(\Gamma(\mathbb{Z}_n)) < 3$ *then,* $D = \{x, y\}$ *be the minimum dominating set of* $\Gamma(\mathbb{Z}_n)$. *If* $x = u$, *then exactly one of the following held,* v *or* $w = y$, *or vertices* v *and* w *belong to the neighborhood of* y, *implies* $d(v, w) \leq 2$, *a contradiction. Therefore, the vertex* w *must be in* D, *i.e.,* $\gamma(\Gamma(\mathbb{Z}_n)) = 3$.

For the converse part assume $\gamma(\Gamma(\mathbb{Z}_n)) = 3$. *Let* $D = \{x, y, z\}$ *where* $x, y, z \in \Gamma(\mathbb{Z}_n)$. *Suppose* $tr(\Gamma(\mathbb{Z}_n)) < 9$, *let* $d(u, v, w) = tr(\Gamma(\mathbb{Z}_n))$ *where* u, v *and* w *be any arbitrary vertices such that* $d(u, v) = 3, d(u, w) = 3, d(v, w) = 2$. *If* $u = x$ *or* $u \in N(x)$, *then* $v \neq x$ *and* $v \notin N(x)$. *Equivalently,* v *has dominated by some vertex in* $D - \{x\}$. *Since* w *is any arbitrary vertex and* $d(v, w) = 2$, w *would dominated by an element that dominates* v, *say* y. *Then* $\{x, y\}$ *is a minimum dominating set, a contradiction. Therefore,* $tr(\Gamma(\mathbb{Z}_n)) = 9$.

observation 2.7 *Let* $n = p_1^{\alpha_1} p_2^{\alpha_2} ... p_m^{\alpha_m}$ *where* p_i*'s are distinct prime and* $\alpha_i \in \mathbb{N}$ *for* $i = 1, ..., m$. $\gamma(\Gamma(\mathbb{Z}_n)) = 2$, *then* $tr(\Gamma(\mathbb{Z}_n)) < 9$, *i.e.,* $tr(\Gamma(\mathbb{Z}_n)) \leq 8$

3 Conclusion

In this paper, we study the triameter of the zero divisor graph for the ring of integer modulo n. In the future, we have to determine the triameter of the zero divisor graph of a commutative ring.

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Restrained Domination Value in Graphs

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Abstract

In a graph $G = (V, E)$, a set $S \subseteq V$ is restrained dominating set of G if every vertex in $V \setminus S$ is adjacent to a vertex in S as well as another vertex in $V \setminus S$. The restrained domination number $\gamma_r(G)$ is the smallest cardinality of a restrained dominating sets of G. For each vertex $v \in V(G)$, we define the restrained domination value of v to be the number of $\gamma_r(G)$-sets to which v belongs. In this paper, we study some basic properties of the restrained domination value function, thus initiating a local study of restrained domination in graphs. Further, we characterize restrained domination value of cycle and path graphs.

Keywords : Restrained domination, restrained domination value, $\gamma_r(G)$-set, cycles, paths.

AMS Subject Classification : 05C69, 05C38

1 Introduction

Let $G = (V(G), E(G)$ be a simple, undirected and nontrivial graph with order $|V(G)|$ and size $|E(G)|$. For $S \subseteq V(G)$, We denote by $< S >$ the subgraph of G induced by S. The degree of a vertex v in G, denoted by $deg_G(v)$, is the number of edges that are incident to v in G, and end-vertex is a vertex of degree one. We denote $\Delta(G)$ the maximum degree of a graph G. A set $S \subseteq V$ is a restrained dominating set of G if every vertex $V \setminus S$ is adjacent to a vertex S as well as another vertex in $V \setminus S$. The restrained domination number $\gamma_r(G)$, is the smallest cardinality of a restrained dominating set of G; a RDS of G of smallest cardinality is called $\gamma_r(G)$-set. Throughout the paper, we denote by P_n, C_n, and K_n the path, the cycle and the complete graph on n vertices, respectively.

In this paper, we will use $\tau_r(G)$ to denote the total number of $\gamma_r(G)$-sets, and by $RDM(G)$ the collection of all $\gamma_r(G)$-sets. For each vertex $v \in V(G)$, we define the domination value of v denoted by $RDV_G(v)$, to be the number of $\gamma_r(G)$-sets to which v belongs. In this paper, we study some basic properties of the domination value function, thus initiating a local study of restrained domination in graphs.

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2 Basic Properties of Restrained Domination Values

Theorem 2.1 *Let G be a connected graph with $n > 2$ vertices. Then every end vertex is contained in each γ_r-set of G.*

Proof: let v be an end vertex of a graph G. Let S be a γ_r-set of G. Suppose v is not in S, then v belongs to $V \setminus S$. Since v is an end vertex, v is adjacent to either a vertex in S or a vertex in $V \setminus S$, which is a contradiction to the definition of restrained domination, and so $v \in S$. Thus every end vertex is contained in each γ_r-set of G.

Observation 2.2 $\sum_{v \in V(G)} RDV_G(v) = \tau_r(G).\gamma_r(G)$.

Observation 2.3 *If there is an isomorphism of graphs carrying a vertex v in G to a vertex v' in G' then $RDV_G(v) = RDV_{G'}(v')$.*

Observation 2.4 *Let G be a disjoint union of two graphs G_1 and G_2. Then $\gamma_r(G) = \gamma_r(G_1) + \gamma_r(G_2)$ and $\tau_r(G) = \tau_r(G_1).\tau_r(G_2)$. For $v \in V(G_1)$, $RDV_G(v) = RDV_{G_1}(v).\tau_r(G_2)$*

3 Restrained Domination Value for some Graph Families

Theorem 3.1 *If G is a complete graph K_n $(n \geq 1)$, then $\tau_r(G) = n$ and $RDV = 1$, for all $v \in V(K_n)$.*

Remark 3.2 *If $n = 2$, then $G = K_{m_1,m_2}$ is a complete bipartite graph.*

Theorem 3.3 *If G is a complete bipartite graph K_{m_1,m_2}, then*

$$\tau_r(G) = \begin{cases} m_1.m_2 & \text{if } m_1, m_2 \geq 2 \\ 1 & \text{if } m_1 = m_2 = 1 \\ 1 & \text{if } \{m_1, m_2\} = \{1, x\} \textit{ where } x > 1. \end{cases}$$

If $m_1, m_2 \geq 2$, then $RDV(v) = \begin{cases} m_2 & \text{if } v \in V_1 \\ m_1 & \text{if } v \in V_2. \end{cases}$

If $m_1 = m_2 = 1$, then $RDV = 1$ for any v in $K_{1,1}$.

If $\{m_1, m_2\} = \{1, x\}$ with $x > 1$, say $m_1 = 1$ and $m_2 = x$, then $RDV(v) = 1$ if $v \in V_1, V_2$.

4 Restrained Domination Value on Cycles

Let the vertices of the path P_n be labelled 1 through nLet the vertices of the cycle C_n be labeled 1 through n consecutively in counterclockwise order, where $n \geq 3$. Observe that the

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restrained domination value is constant on the vertices of C_n, for each n, by vertex transitivity. Recall that $\gamma_r(C_n) = n - 2\lfloor \frac{n}{3} \rfloor$ where $n \geq 3$ [2].

Example.

1. $RDM(C_4) = \{\{1,2\},\{2,3\},\{3,4\},\{1,4\}\}$. Since $\gamma_r(C_4) = 2$. So $\tau_r(C_4) = 4$ and $RDV(i) = 2$ for each $i \in V(C_4)$.

2. $RDM(C_6) = \{\{1,4\},\{2,5\},\{3,6\}\}$. Since $\gamma_r(C_6) = 2$. So $\tau_r(C_6) = 3$ and $RDV(i) = 1$ for each $i \in V(C_6)$.

Theorem 4.1 *For $n \geq 3$,* $\tau_r(C_n) = \begin{cases} 3 & \text{if } n \equiv 0(mod\ 3) \\ n & \text{if } n \equiv 1(mod\ 3) \\ \frac{1}{2}n(1 + \lfloor \frac{n}{3} \rfloor) & \text{if } n \equiv 2(mod\ 3). \end{cases}$

Proof: ***Case (i):*** *$n \equiv 0(mod\ 3)$. Let $n = 3k$, where $k \geq 1$. $\gamma_r(C_n) = k$. A $\gamma_r(C_n)$-set Γ comprises k, $K_1's$ and Γ is fixed by the choice of the first K_1. There exist exactly one $\gamma_r(C_n)$-set containing the vertex 1, and there are two $\gamma_r(C_n)$-sets omitting the vertex 1 such as Γ containing the vertex 2 and Γ containing the vertex n. Thus $\tau(C_n) = 3$.*

Case (ii): *$n \equiv 1(mod\ 3)$. $< \Gamma > \cong (k-1)K_1 \cup K_2$. Let $n = 3k + 1$ where $n \geq 1$. Here $\gamma_r(C_n) = k + 1$ a $\gamma_r(C_n)$-set Γ comprises of one K_2 and $(k-1)K_1$'s. Note that Γ is fixed by the choice of single K_2. Choosing K_2 is same as choosing its initial vertex in the counterclockwise order. If we choose first two vertex in Γ then Γ omits the next two vertices. Likewise we can choose $3k + 1$ vertices $\tau_r(C_n) = 3k + 1 = n$.*

Case (iii): *$n \equiv 2(mod\ 3)$.*

Let $n = 3k + 2$, where $k \geq 1$. Hence $\gamma_r(C_n) = k + 2$ a $\gamma_r(C_n)$-set Γ is either comprises of $(k-1)K_1 \cup P_3$ and or of $(k-2)K_1 \cup 2P_2$.

Subcase (i): *$< \Gamma > \cong (k-1)K_1 \cup P_3$.*

Note that Γ is fixed by the placement of the single P_3. Choosing P_3 is the same as choosing its initial vertex in the counterclockwise order. P_3 can be placed in end vertices or intermediate vertices. Thus we can choose $\tau_r(C_n) = 3k + 2$.

Subcase (ii) *$< \Gamma > \cong (k-2)K_1 \cup 2P_2$.*

Note that Γ is fixed by the placement of the two P_2. Every end vertices belongs to every Γ set. There are $n = 3k + 2$ ways of choosing the first P_2 as discussed. The initial vertex of the second P_2 may be placed in any slot of any of $k - 1$ subintervals. Thus $\tau(C_n) = \frac{(3k+2)(k-1)}{2}$.

Summing over the two disjoint cases we get

$\tau_r(C_n) = 3k + 2 + \frac{(3k+2)(k-1)}{2} = (3k+2)(1 + \frac{k-1}{2}) = \frac{n}{2}(1 + \lfloor \frac{n}{3} \rfloor)$.

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5 Restrained Domination Values on Paths

Let the vertices of the path P_n be labelled 1 through n consecutively. Recall that $\gamma_r(P_n) = n - 2\lfloor \frac{n-1}{3} \rfloor$ where $n \geq 1$ [2].

Example.

1. $\gamma_r(P_4) = 2$. $RDM(P_4) = \{\{v_1, v_4\}\}$. So $\tau_r(P_4) = 1$ and $RDV(i) = \begin{cases} 1 & \text{if } i = 1, 4 \\ 0 & \text{if } i = 2, 3. \end{cases}$

2. $\gamma_r(P_5) = 3$. $RDM(P_5) = \{\{v_1, v_4, v_5\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_4\}, \{v_3, v_4, v_5\}, \{v_1, v_2, v_3\}\}$. So $\tau_r(P_5) = 5$ and $RDV(i) = 3$ for each $i \in V(P_5)$.

Theorem 5.1 *For $n \geq 2$,* $\tau_r(P_n) = \begin{cases} \frac{1}{2}\lceil \frac{n}{3} \rceil (1 + \lceil \frac{n}{3} \rceil) & \text{if } n \equiv 0 (mod\ 3) \\ 1 & \text{if } n \equiv 1 (mod\ 3) \\ \lceil \frac{n}{3} \rceil & \text{if } n \equiv 2 (mod\ 3). \end{cases}$

Proof: ***Case (i):*** $n \equiv 0(mod\ 3)$. *Let $n = 3k$, where $k \geq 1$. Then $\gamma_r(P_n) = k + 2$ and $\gamma_r(P_n)$-set Γ is constituted in exactly one of the following ways (i) $(k-1)K_1 \cup P_3$, (ii) $(k-2)K_1 \cup 2P_2$.*

Subcase (i): $< \Gamma > \cong (k-1)K_1 \cup P_3$.

Note that Γ is fixed by the placement of P_3. Every end vertices P_n belongs to every Γ-set. We can take P_3 as initial, terminal or intermediate vertices. If we choose P_3 as initial vertex in Γ, then the next two vertices omit Γ. Thus we can choose k number of Γ-set in this case. Thus $\tau_r(P_n) = k$.

Subcase (ii) $< \Gamma > \cong (k-2)K_1 \cup 2P_2$.

Note that Γ is fixed by the placement of the $2P_2$. Every end vertices belongs to every Γ set. Γ is fixed by the placement of two P_2's into the k available slots. Thus $\tau_r(P_n) = \binom{k}{2} = \frac{k(k-1)}{2}$.

Summing the two cases $\tau_r(P_n) = k + \frac{k(k-1)}{2} = \frac{k(k+1)}{2} = \frac{1}{2}\lceil \frac{n}{3} \rceil (1 + \lceil \frac{n}{3} \rceil)$.

Case (ii): $n \equiv 1(mod\ 3)$. *Let $n = 3k + 1$, where $k \geq 1$. Here $\gamma_r(P_n) = k + 1$ and a $\gamma_r(P_n)$-set Γ comprises kK_1's. In this case the end vertices of P_n belongs to every Γ-set. Every vertices in Γ restrainly dominate $\lfloor \frac{n}{3} \rfloor$ vertices. The first vertex is fixed by vertex 1. Thus $\tau_r(P_n) = 1$.*

Case (iii): $n \equiv 2(mod\ 3)$ *where $k \geq 1$. Let $n = 3k + 2$. Here $\gamma_r(P_n) = k + 2$ and $\gamma_r(P_n)$-set Γ comprises of kK_1's and one K_2. Note that every Γ contains both the end vertices. Γ is fixed by the placement of one K_2. Thus $\tau_r(P_n) = k = \lceil \frac{n}{3} \rceil$.*

6 Conclusion

This paper introduces a new parameter known as restrained domination value in graphs. Further this concept can be extend to various types of graphs.

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Exploring a Novel Ranking Approach for Elevating Fuzzy Goal Programming Solutions

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Abstract

The main aspect of this paper is to maximize the solution of a fuzzy goal programming problem. Where fuzzy quantities are in the form of triangular fuzzy numbers with its membership value. The triangular fuzzy number is converted into a crisp value using a novel suggested ranking method and an existing ranking method. Fuzzy simplex procedure for goal programming is then applied to obtain the solution. By comparing the results, we can achieve better results with the novel suggested ranking method.

Key words::Triangular fuzzy number, Fuzzy goal programming problem, A novel suggested ranking method, F.Reubens ranking method.
AMS classification: 90C29 Multi-objective and goal programming

1 Introduction

Bellman and Zadeh proposed the concept of decision-making in a Fuzzy environment [2]. In traditional goal programming, all parameters are assumed to be precise and deterministic, but in real-world scenarios, various factors may involve vagueness or ambiguity [4]. In FGPP, decision variables, coefficients, and constraints can be described using fuzzy sets, which provide a more flexible framework for modeling the inherent uncertainty in decision-making. Goal Programming was first addressed by Charnes and Cooper [3], and Tamiz et al. [7]. Zangiabadai and Maleki [8,9] also applied fuzzy goal programming approach to solve multiobjective transportation problems with linear and nonlinear membership functions. Goal Programming is a mathematical technique and a variation of Linear Programming. It is an approach that is capable to handle the decision-making problems having multiple conflicting goals and the objective function. By considering fuzzy parameters, FGPP enables a more realistic representation of complex systems.

The outline of this work is described below. In section 2: Basic Definitions, Proposed ranking technique. In section 3: Procedure for solving a fuzzy goal programming problem. In section 4: Numerical example. Finally, a conclusion in section 5.

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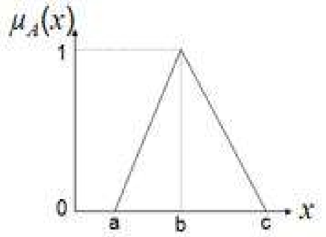
2 Preliminaries

Definition 2.1 *For a triangular fuzzy number A it can be represented by A= (a,b,c) with membership function $\mu_A(x)$ given by*

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, a \leq x \leq b \\ 1, x = b \\ \dfrac{c-x}{c-b}, b \leq x \leq c \\ 0, otherwise \end{cases}$$



The Arithmetic Operations on Fuzzy Numbers

Let $A_1 = (a, b, c)$ and $A_2 = (d, e, f)$ be two non-negative triangular fuzzy numbers then

(i) $A_1 \oplus A_2 = (a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f)$

(ii) $A_1 - A_2 = (a, b, c) - (d, e, f) = (a - f, b - e, c - d)$

(iii) $-A_1 = -(a, b, c) = (-c, -b, -a)$

(iv) $A_1 \otimes A_2 = (a, b, c) \otimes (e, f, g) = (ae, bf, cg)$

Ranking function

Ranking fuzzy numbers are an important aspect of decision-making in a fuzzy environment. Since 1965, many authors have proposed different methods for ranking fuzzy numbers.

Ranking function for triangular fuzzy numbers [5]

The ranking function for $A = (a_1, a_2, a_3)$ denoted $\mathbb{R}(A)$ proposed by F.Reubens is defined by :

$$\mathbb{R}(A) = \frac{1}{2}\int_0^1 \left(a_\alpha^L + a_\alpha^U\right) d\alpha, \quad where \begin{cases} a_\alpha^L = a_1 + (a_2 - a_1)\alpha \\ a_\alpha^U = a_3 + (a_3 - a_2)\alpha \end{cases}$$

Result: If $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be triangular fuzzy numbers, Then

$$\begin{cases} A < B \ \ if \ \ and \ \ only \ \ if \ \ \mathbb{R}(A) < \mathbb{R}(B) \\ A = B \ \ if \ \ and \ \ only \ \ if \ \ \mathbb{R}(A) = \mathbb{R}(B) \\ A > B \ \ if \ \ and \ \ only \ \ if \ \ \mathbb{R}(A) > \mathbb{R}(B) \end{cases}$$

Proposed Ranking Technique

This paper proposes a method that ranks triangular fuzzy numbers which is simple in calculation.

Let $A = (a_1, a_2, a_3)$ be the triangular fuzzy number. If a triangular fuzzy number with its membership value is $\langle A, \omega_A \rangle$, then new ranking function is defined as, $R(A)/\omega_A$.

Mathematical Formulation of Fuzzy Goal Programming Problem:[6]

Minimize $z = \sum_{i=1}^{m} \omega_i^{'}(d_i^- + d_i^+)$ subject to

$$\sum_{j=1}^{n} a_{ij}^{'} x_j^{'} + d_i^- - d_i^+ = b_i^{'}; i = 1, 2, \cdots, m$$

$$d_i^-, d_i^+ \geq 0$$

m-goals are expressed by as m-component column $b_i^{'}$

$a_{ij}^{'}$- represents the coefficient for the j^{th} decision variable in the i^{th} constraint.

$x_j^{'}$- represents a derision variable

$\omega_i^{'}$- represents the weights of each goal.

d_i^-, d_i^+- deviational variables, represents the amount of under achievement and over achievement respectively.

3 Fuzzy simplex procedure

The procedure of Fuzzy Simplex Method

Step-1: In a Fuzzy linear programming problem, convert fuzzy values to crisp values using the ranking technique.

Step-2: Express the LPP in the standard form of goal programming problem. By introducing the slack/surplus variables in each of the constraints, introduce deviational variables in the objective function equate to a maximum value which we assume and convert it as an additional constraint. Convert the objective function to minimization type using the deviational variables.

Step-3: Obtain an initial basic feasible solution and compute net evaluation

$$z_j - c_j = \sum_{i=1}^{m} c_{B_i} a_{ij} - c_j \ \ where \ j = 1, 2, \cdots, m + n$$

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(i) If all net evaluations are non-negative, then the initial basic feasible solution is an optimal solution.

(ii) If at least one net evaluation is negative, proceed to next step.

Step-4: Choose the most negative of net evaluation. The corresponding column is the entering column. If all values in the column are less than zero, then problem has unbounded solution.
Step-5: Compute the ratio (X_B/Entering column) and choose the minimum of these ratio. The corresponding row is the leaving row. The intersection of entering column and leaving row is called key element.
Step-6: Convert the key element to unity by dividing its row by key element and all other elements in remaining rows by using elementary row transformations.
Step-7: Go to step-3 and repeat the procedure until an optimal solution is obtained or there is an indication of an unbounded solution.

4 Numerical Example [1]

Consider, Maximize $z = \langle(1,6,9),0.9\rangle\, x_1 + \langle(2,3,8),0.8\rangle\, x_2$ subject to

$$\langle(2,3,4),0.7\rangle\, x_1 + \langle(1,2,3),0.8\rangle\, x_2 \leq \langle(6,16,30),0.5\rangle$$
$$\langle(-1,1,2),0.4\rangle\, x_1 + \langle(1,3,4),0.6\rangle\, x_2 \leq \langle(1,17,30),0.8\rangle$$

$$x_1, x_2 \geq 0$$

Applying the F.Reubens ranking technique, we get
Maximize $z = 5.5x_1 + 4x_2$ subject to

$$3x_1 + 2x_2 \leq 17$$

$$0.75x_1 + 2.75x_2 \leq 16.25$$

$$x_1, x_2 \geq 0$$

Standard form, We introduce two slack variables $s_1 \geq 0$ and $s_2 \geq 0$
Minimize $z^{'} = d^-$ subject to

$$3x_1 + 2x_2 + s_1 = 17$$

$$0.75x_1 + 2.75x_2 + s_2 = 16.25$$

$5.5x_1 + 4x_2 + d^- - d^+ = 100$ (Here,100 is our assumption) $d^-, d^+ \geq 0$

Basis		c_j	0	0	−1	0	0	0	
C_B	B	X_B	x_1	x_2	d^-	d^+	s_1	s_2	End of the simplex procedure
0	x_1	2.114	1	0	0	0	0.4062	0.295	
0	x_2	5.333	0	1	0	0	−0.11	0.444	
-1	d^-	67.057	0	0	1	−1	−1.796	−0.148	
Optimality is attained		$z'_j=$ −67.057	0	0	−1	1	1.796	0.148	
		$z'_j - c_j$	0	0	0	1	1.796	0.148	

Now, all the $z_j^{'} - c_j \geq 0$. Hence optimality is reached and the optimal solution is $x_1 = 2.114, x_2 = 5.333, d^- = 67.057, z^{'} = -67.057$ and the maximum value of z = 100 - 67.057= 32.943.

Applying the proposed ranking technique, we get

Maximize $z = 6.111x_1 + 5x_2$ subject to

$$4.285x_1 + 2.5x_2 \leq 34$$

$$1.875x_1 + 4.583x_2 \leq 20.312$$

$$x_1, x_2 \geq 0$$

Standard form,

We introduce two slack variables $s_1 \geq 0$ and $s_2 \geq 0$

Minimize $z^{'} = d^-$ subject to

$$4.285x_1 + 2.5x_2 + s_1 = 34$$

$$1.875x_1 + 4.583x_2 + s_2 = 20.312$$

$6.111x_1 + 5x_2 + d^- - d^+ = 100$ (Here, 100 is our assumption) $d^-, d^+ \geq 0$

Basis		c_j	0	0	−1	0	0	0	
C_B	B	X_B	x_1	x_2	d^-	d^+	s_1	s_2	End of the simplex procedure
0	x_1	7.026	1	0	0	0	0.305	−0.167	
0	x_2	1.557	0	1	0	0	−0.124	0.2866	
-1	d^-	49.277	0	0	1	−1	−1.245	−0.411	
Optimality is attained		$z'_j=$ −49.277	0	0	−1	1	1.245	0.411	
		$z'_j - c_j$	0	0	0	1	1.245	0.411	

Now, all the $z_j^{'} c_j \geq 0$. Hence optimality is reached and the optimal solution is $x_1 = 7.026, x_2 = 1.557, d^- = 49.277, z^{'} = -49.277$ and the maximum value of z = 100-49.277= 50.723.

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5 Applications

Fuzzy goal programming finds application in diverse fields where decision-making involves dealing with imprecise goals and uncertainties. In the financial planning, it aids in optimizing investment portfolios by considering the fuzzy nature of returns and risks, thus accommodating the uncertainty inherent in financial markets. Additionally, in production planning, it addresses the challenges of fluctuating demand, resource availability, and quality requirements by providing a framework that allows for imprecision in setting and achieving production goals. Its versatility extends to supply chain management, environmental decision-making, healthcare resource allocation, project scheduling, and education planning, among other domains. By incorporating fuzzy logic, this approach enables decision-makers to navigate complex scenarios where goals and constraints may not be precisely defined, providing a more realistic and robust framework for optimal decision outcomes.

6 Conclusion

In this paper, we considered numerical examples with values as triangular fuzzy numbers. Then they were transformed into crisp values using a novel suggested ranking technique and an existing ranking method. Fuzzy simplex procedure for goal programming is then applied to obtain the solution. By comparing the results, we can achieve better results with the novel suggested ranking method.

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Numerical Simulation of Atmospheric Turbulence using Fourier Transform and Sub-Harmonics methods

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Abstract

Observations with ground-based telescopes suffer from atmospheric turbulence while looking through the sky. The best option to minimize the atmospheric effects is to launch a telescope into space to avoid the atmospheric problems altogether, but it has its limitations in launching technology for big telescopes and cost of operation. The more economical solution is to build an Adaptive Optics (AO) system that senses the distortions and compensates them in a ground-based telescope. In this paper, the simulation of atmospheric turbulence was carried out numerically using the Kolmogorov turbulence model with the Fourier Transform method and the Sub-Harmonics method.

Key words:: Numerical Simulation, Atmospheric turbulence, Fourier Transform, Sub-Harmonics.

AMS classification: 42A38,42B10

1 Introduction

The atmospheric turbulence can be considered as a random process and can be estimated using variances and co-variances of local refractive index fluctuations [1]. Due to changes in the refractive indices of the different layers, the planar wavefront, from the distant star, propagating through the turbulent atmosphere, gets distorted. So, both the amplitude and phase of the incoming beam fluctuate during its passage and change with time. Thus, the random process of atmospheric turbulence affects the image-forming capabilities of a telescope.

1.1 Effect of Atmospheric Turbulence

The effects of turbulence on light that passes through the atmosphere are three types. a. It creates intensity fluctuations or scintillations which are observed as the twinkling of the stars. b. The position of the star wanders when the varying refractive index of the atmosphere alters the angle of arrival of the starlight. c. There is a spreading effect created by the higher order aberrations which causes stars to appear as small discs of light and not sharply defined point sources. Figure 1 shows the simulated point source images of a diffraction-limited case in the presence of strong turbulence. The intensity is normalized to the peak intensity of the Point

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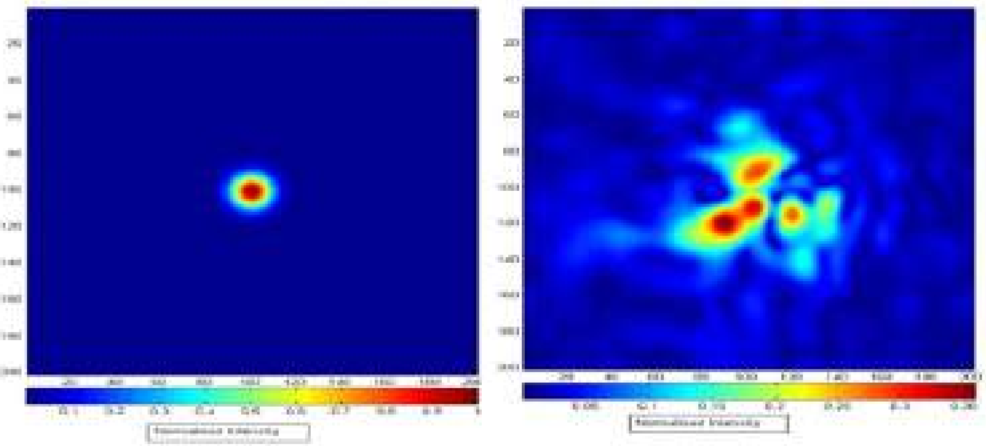
Spread Function (PSF) in the absence of turbulence. This light spreads over a larger area and demonstrates high resolution, high contrast imaging is difficult.



(a) No Turbulence (b) Strong Turbulence

1.2 Wavefront Correcting System

Adaptive Optics is the adaptation of the telescope optical system and it works in such a way that it measures the incoming light from natural stars and gives information on the nature of the atmosphere at a certain point in time. A distorted wavefront comes into the system through the telescope aperture. It is reflected from a Deformable Mirror to a beam splitter that divides the beam to a WFS and a scientific camera. The measurements from WFS are fed into computers to compute the required instructions for the DM. The mirror is deformed using actuators, each of them having its own control voltage. After calculating wavefront errors with WFS, they can be appropriately corrected with spatial correction devices such as Tip Tilt and Deformable Mirror.

1.2.1 Tip-tilt Mirror

The simplest form of Adaptive Optics is Tip-Tilt correction [2,3]. Tip-Tilt mirror is an Opto-Electronic device used to correct the tilts of the wavefront in two dimensions. In Adaptive optics, Tip-Tilt mirror can correct 87% of distortion which is introduced by the atmosphere.

1.2.2 Deformable Mirror

Once the wavefront aberrations are measured with a Wavefront Sensor, they have to be somehow corrected. A mirror with its surface locally bent is called a Deformable Mirror (DM), which is usually used for this purpose. DM is an important component in a wavefront compensation system [4]. A DM is a flexible structure and its surface can be shaped dynamically into a custom form. The incoming light falls onto the mirror which in turn is deformed into the shape producing a straight wavefront leaving the mirror. A DM is also an Opto-Electronic device that corrects the distortions in the wavefront by deforming the mirror. It consists of an array of actuators that control the mirror such that it is perfectly conjugate to the incoming aberrated wavefront.

2 Objective

Once the wavefront aberrations are measured with a Wavefront Sensor, they have to be somehow corrected. A mirror with its surface locally bent is called a Deformable Mirror (DM), which is usually used for this purpose. DM is an important component in a wavefront compensation system [4]. A DM is a flexible structure and its surface can be shaped dynamically into a custom Various Wavefront Sensing techniques have been developed for use in a variety of applications ranging from measuring the wavefront aberrations of human eyes [5] to Adaptive Optics in astronomy [6]. The most commonly used Wavefront Sensors are the Shack-Hartmann (SH) [7, 8], Curvature sensing [9], Lateral Shearing Interferometry (LSI) [10, 11 and 12], Phase Retrieval methods [13] and Pyramid Wavefront Sensor [14]. Among the Wavefront Sensors, the Shack-Hartmann Wavefront Sensor (SHWS) is the most commonly used technique for the measurement of turbulence-induced phase distortions for various applications in atmospheric studies and Adaptive Optics. However, the dynamic range of the SHWS is limited by the optical parameters of its micro lenses, namely, the spacing and the focal length of the microlens array. Development of AO requires a better understanding of the characteristics of turbulent atmospheres and their effects on the wavefront aberrations. So, in this paper, the simulation of atmospheric turbulence was carried out numerically using the Kolmogorov turbulence model with LabVIEW routines for different D/ro ratios with the Fourier Transform method and Sub-Harmonics method.

3 Main Results

3.1 Shack Hartmann Wavefront Sensor

Shack-Hartmann Wavefront Sensor is an optical instrument that senses local gradients through aperture sub-division with a lenslet array. This is the most common Wavefront Sensor in Astronomy and Ophthalmology. In SHWS an image of the telescope exit pupil is projected onto a lenslet array of small identical lenses. Each lens takes a small part of the aperture, called a sub-pupil, and forms an image of the source on back focal plane of array. A CCD detector is placed at the back focal plane of the lenslet array. An array of images is formed at

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the detected plane. To measure the positional accuracy of each image spot, the center of the mass method is used. A reference source is introduced in the optical path to record reference coordinates. Measuring the difference from the reference position, the local slopes are calculated and from these slopes, the wavefront is reconstructed.

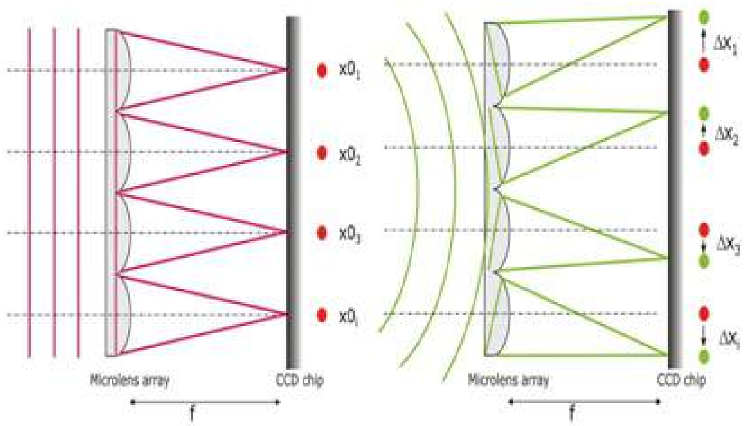

Figure 2: Schematic of Shack Hartmann Wavefront Sensor

Deviation of spot position from a perfectly square grid measures the shape of the incoming wavefront. The shift may be in the x direction or y direction or in both directions. From the knowledge of the focal length of the Shack- Hartmann lens and the distance of the centroid, the slope can be determined. Centroid algorithms are used to determine the spot centroids. The accuracy of SHWS is mainly depending on the centroid calculation accuracy and reconstruction accuracy.

3.2 Numerical simulation of atmospheric turbulence

To understand the adaptive optics technology, it is essential to simulate atmospheric turbulence numerically and experimentally. For testing and calibrating a complex adaptive optics (AO) system it is useful to have an artificial turbulence generator with known, realistic, and repeatable characteristics. Turbulent flow is very complicated and still it is not entirely understood. The most widely accepted theory of turbulence flow, due to consistent agreement with observation and statistical model of the wavefront aberrations induced by the turbulent atmosphere was first put forward by Andrei Kolmogorov [15]. Kolmogorov statistics provide a suitable theoretical model for atmospheric turbulence. This model is based on the idea that energy is fed into the system at large scales and propagates down to smaller structures, where it eventually dissipates into heat. Kolmogorov turbulence model is valid for atmospheric turbulence and it is experimentally proved [16]. However, the Kolmogorov model is only useful between the largest (the outer scale L0), and the smallest structures (the inner scale l_0) of the turbulence.The theory based on Kolmogorov turbulence has been reviewed by Roddier [17].

3.3 Kolmogorov Model of Atmospheric Turbulence

Kolmogorov model suggested that the energy injected into turbulent medium on large spatial scales (outer scale, $Ł_0$ is of the order of a few tens of meters) [18] forms eddies. The outer scale $Ł_0$ limits the contribution of low spatial frequencies to the wavefront aberrations. Since these spatial frequencies dominate the overall wavefront distortions, $Ł_0$ has a significant influence on the achievable performance and image quality of telescope [19]. These large eddies cascade the energy into small scale eddies until they become small enough (small scale l_0) that the energy is dissipated by the viscous properties of the medium. For the inertial range between inner and outer scales, Kolmogorov predicted a power law distribution of the turbulent power with spatial frequency $k^{(-11)/3}$. The outer scale is denoted by $Ł_0$, the inner scale by $Ł_0$. Eddies between these limits form the inertial subrange. Energy is injected by wind shear and convection is transferred until it is dissipated to heat. Atmospheric turbulence is a random process. Tatarski [20] shows the three dimensional power spectrums, $\phi_N(k)$ of the refractive index variations is,

$$\phi_N(k) = 0.033c_N^2(k)^{(-11/3)} \tag{1}$$

where is the scalar wave number vector (K_x, K_y, K_z) The outer scale is an important parameter in turbulence statistics and its range of values are much debated in astronomical databases [21]. The standard spectrum of Kolmogorov turbulence is usually written with infinite outer scale and the effect of infinite outer scales is to reduce the lower spatial frequency contributions.

3.4 Fourier Transform - based Phase Screen

Fourier Transform (FT) method proposed by McGlamery [22, 23] has been used to simulate the phase screens numerically. The FT methods are most common since very large phase screens can be generated quickly. One way of describing the phase statistically is by means of its power spectrum. The phases of the Fourier Transform of the phase map are independent with frequency, uniformly randomly distributed in $-\pi to + \pi$ interval. Based on these the phase map is generated using a complex array of Gaussian random numbers and the array are multiplied by the square root of the power spectrum. The array is subjected to a discrete Fourier Transform and the resulting complex array is separated into its real and imaginary components, each of these arrays represent an independent instantaneous phase map realization. The power spectrum is only valid within the inertial range between the inner and outer scale as it tends to infinity at larger spatial separations. There are other modified models for the atmospheric power spectral density, like

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the Tatarski [20], Von Karman [24], and modified Von Karman [25] which are commonly used. These models are much more sophisticated and include various inner-scale and outer-scale factors that improve the agreement between theory and experimental measurements. To accommodate the finite inner and outer scales, the Kolmogorov power spectrum was modified by Von Karman power spectrum [26] which is given by,

$$\phi_N(k) = 0.033C_N^2(K^2 + K_0^2)^{(-11)/6)}exp(-K^2/K_i^2) \quad (2)$$

Where $k_o = 2\pi/L_0, k_i = 5.92/l_0$ and $k = 2\pi/L$. It can be expressed in another form with Fried parameter r_0,

$$\phi_N(k) = 0.023(D/r_0)^{(5/3)}\frac{exp(-k^2)/(k_i^2)}{(k^2 + k_0^2)^{(11/6)}} \quad (3)$$

For infinite outer scale ($k_0 = 0$) and zero inner scale($k_i = \infty$) above equation reduces to,

$$\phi_N(k) = 0.023(D/r_0)^{(5/3)}\kappa^{(-11)/3} \quad (4)$$

The Power spectral density (PSD) and phase screen $f(r)$are related as,

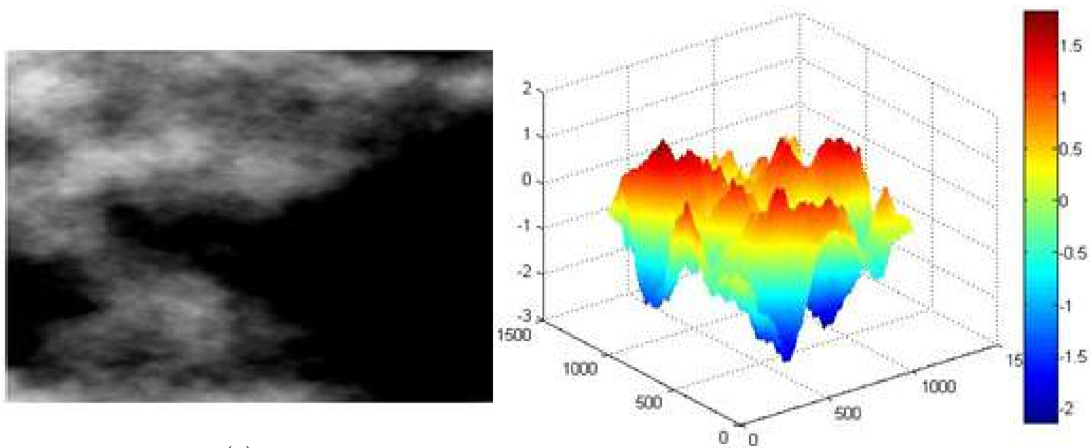
$$\phi_N(k) = \left|\int_{-\infty}^{\infty} f(r)e^{(-ikr)}dr\right|^2 \quad (5)$$

From the above equation phase screen is derived by:

$$f(r) = \int_{-\infty}^{\infty} \sqrt{(\phi_N(k))}e^{ikr}(dk) \quad (6)$$

where f(r) is the 2D- Kolmogorov phase screen. It is obtained from Inverse Fourier Transform (IFT) of square root of Von Karman power spectrum of turbulent atmosphere. The randomness of atmospheric turbulence is implemented with random numerical function. The Figure 4.1. A presents the typical atmospheric phase screen simulated by Fourier Transform method with $D/r_0 = 2, D/r_0 = 1, 2$ where D is the telescope diameter, r_0 is the Frieds parameter, $L_0 = 50$ m and $l_0 = 0.01m$. The Figure 4.2 demonstrate 3D representation of phase screen. In this Figure one can observe that the low spatial frequencies are not sampled well (i.e., no tip / tilt).

This method has a disadvantage of under sampling at low spatial frequencies due to limited low sampling of Fourier Transform technique. This leads to lower-order aberrations such as tilt which are often under-represented. These lower-order aberrations contribute a majority of the atmospheric energy spectrum and must be included to produce realistic models.



(a)

(b)

Figure 3: Sample Phase Screens obtained by Fourier Transform Method

3.5 Sub-Harmonics Method - based Phase Screen

Sub - Harmonics method [27, 28,29] is a simple technique for modelling the effects of lower frequencies by generating additional random frequencies and adds their effects to the sampled frequencies using equation 1.7. It modifies the usual Fourier Transform method of generating phase screens for atmospheric propagation to allow low-frequency turbulence effects. This method consists of generating realizations of turbulence on two different size grids and uses a trigonometric interpolation to introduce low frequency effects on the smaller (propagation) grid. It is proved that the phase screens generated by this method give a better representation of Kolmogorov turbulence since they include effects from the low spatial frequency part of the spectrum. This method can be considerably more efficient than direct implementation of the FT method on a very large grid. It provides a low frequency screen $p(x, y)$ generated by a sum of different number (Np) of phase screens. The low frequency screen as a Fourier series is given by,

$$p(x, y) = \sum_{(g=1)}^{(N_g)} \sum_{(n=-1)}^{1} \sum_{(m=-1)}^{1} c_(n, m) \exp\left[i2\pi(f_{xn}(x) + f_{ym}y)\right] \tag{7}$$

where the sums over n and m are over discrete frequencies and each value of the index g corresponds to a different grid. The phase screen generated in this simulation is derived by addition of two-phase screens obtained with Fourier Transform method and Sub-Harmonics method. The sample phase screens thus simulated are shown in Figure 4. (a) presents the typical atmospheric phase screen simulated by Sub-Harmonics method with $D/ro = 5, Dr_0 = 1, 2, L_0 = 50m$ and $l_0 = 0.01m$. The Figures 4. (b) demonstrates 3D representations of phase screen. In these Figures it is clearly seen that low spatial frequencies are well sampled.

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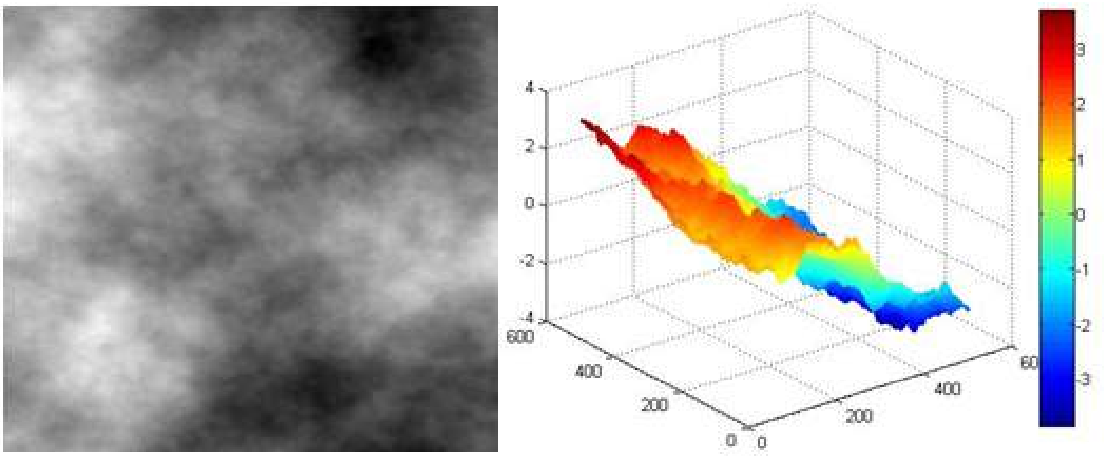

(a) (b)

Figure 4: Sample Phase Screens obtained by Fourier Transform Method

3.6 Numerical Simulation of Shack Hartman in the presence of noise

The atmospheric turbulence affected Shack-Hartmann spot patterns are generated. Initially Airy pattern spots are generated by considering pre-fixed sub lenslet diameter. The generated Airy spot intensity array multiplied with phase screen array exponentially as phase information. Then the resulted image is Fourier transformed. As a result, we get the turbulence distorted SH spot pattern. The spot pattern (7×7) at the focal plane of a Shack Hartmann sensor is simulated. In figure 6 , the Shack Hartmann back focal plane spot intensities is shown when the turbulence at the order of $Dr_o = 15$. This images are generated with the Airy array spot pattren which is corrupted by Kolmogorov atmospheric turbulence.

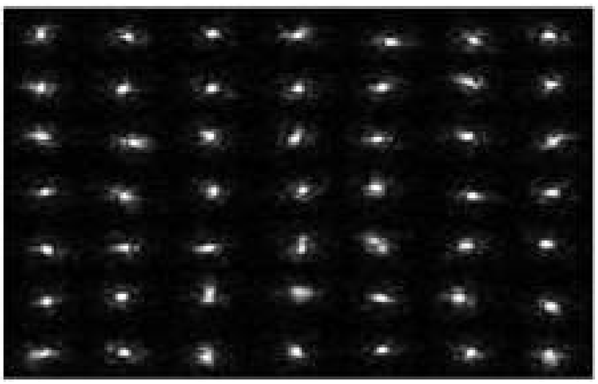

Figure 5: Airy pattern in the presence of turbulence (at $D/r_o = 15$)

3.7 Centroid algorithms

The Center of Gravity (CoG) is the simplest and most direct way to calculate the position of a symmetric spot:

$$\widehat{x}_{CoG} = \frac{(\sum x.I_{(x,y)})}{(\sum I_{(x,y)})} \tag{8}$$

3.8 Wavefront reconstruction

After finding out the centroid of each spot, the slope of the wavefront at each sub aperture is calculated with the knowledge of focal length of the Shack-Hartmann sensor and the deviation of the centroid of each spot from the reference image. From this slope the wavefront is reconstructed using a modal approach. Reference and distorted centroid points are compared for slope determination. The wavefront is calculated with a modal approach using Zernike [30] basis functions using 21 modes. Figure 7 shows the wavefront reconstructed from the distorted wavefront at Turbulence $D/ro = 5$. One can see that distorted wavefront leading to speckles and centroid positions also shifted. In Figure 8, Zernike coefficients are plotted for

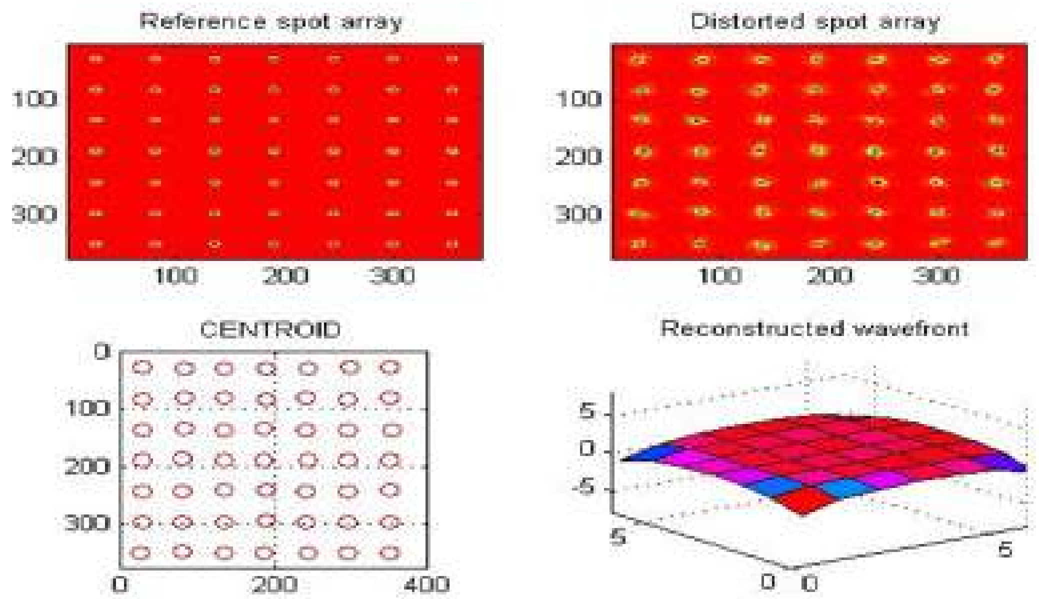

Figure 6: Shack Hartmann sensor Top left one is reference array pattern; Top right one is distorted array pattern, bottom left one is calculated centroid positions; bottom right one is wavefront constructed at D/ro =5.

Zernike index up to 21 modes. (Except piston). It is clearly seen that the values of the lower order terms are much higher than the higher orders ones.

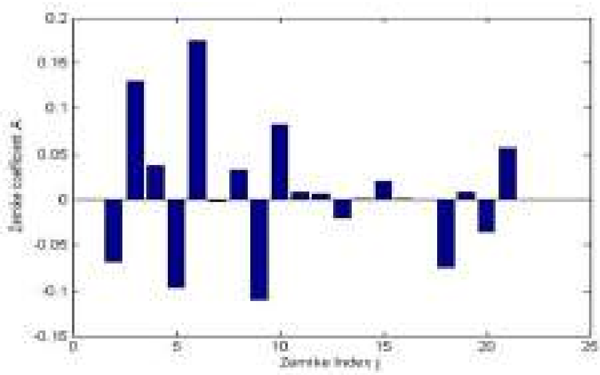

Figure 7: Zernike coefficients are plotted against Zernike index number

4 Conclusion

Estimation of the wavefront errors is a very important aspect in adaptive optics. Besides the telescope system errors, the atmospheric turbulence also accounts for the major contribution to the errors. The atmospheric turbulence is characterized by the Kolmogorov model. It is essential to accurately estimate these aberrations in dynamic situations and to apply real-time corrections. A phased screen based on the Fourier transform and sub-harmonics method has been tested and reconstructed the wavefront using Zernike polynomials.

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Relatively Prime Domination In Power Of Wheel Graph

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Abstract

A subset S of V is said to be dominating set in if every vertex in V-S is adjacent to at least one vertex in S. The domination number (G) is the minimum cardinality of a dominating set in G. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that (deg(u),deg(v))=1. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. If there is no such pair exist, then $\gamma_{rpd}(G) = 0$. This article focuses on exploring the relatively prime domination number within the context of power of wheel graph. The discussion reveals that for the power of wheel graph W_n the relatively prime domination number, denoted as (W_n), assumes values of 0 or 2. Additionally, the article describes the computation of the relatively prime domination number for the power of wheel graph using the Python programming language.

Key words::Dominating Set, Domination Number, Relatively Prime Dominating Set, Relatively Prime Dominating Number.

AMS classification:

1 Introduction

By a graph G=(V,E) we mean a finite undirected graph without loops and multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretical terms, we refer Harary [2] and for terms related to domination we refer to Haynes [5]. A subset S of V is said to be a dominating set in G if every vertex in V-S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in C. Berge [1] and Ore [4] formulated the concept of domination in graphs. It was further extended to define many other dominations related parameters in graphs. Let G be a non trivial graph. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices and in such that (deg(u), deg(v)) = 1. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. If there is no such pair exist, then $\gamma_{rpd}(G) = 0$ [3]. Switching in graphs was introduced by Lint and Seidel. For a finite undirected graph G(V, E) and a subset $\sigma \subseteq V$, the switching of G by is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and $V - \sigma$. For $\sigma = \{v\}$, we write G^v instead of $G^{\{v\}}$ and the corresponding switching is called as vertex switching. A Wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. It is denoted by W_n.

2 Relatively Prime Domination on Wheel graph

In this section, we discussed about the relatively prime domination number of a wheel graph.

Theorem 2.1 *For any Wheel graph* (W_n) *where* $n \geq 4$,

$$\gamma_{rpd}(W_n) = \begin{cases} 0 \ if \ \ n = k + 4 \ where \ \ \ k = 0, 3, 6, 9, \cdots \\ 2 \ \ otherwise \end{cases}$$

Proof: *Let* W_n *be a wheel graph and n denote number of vertices where* $n \geq 4$. *We Proceed by two cases.*
Case 1: $\gamma_{rpd}(W_n) = 0$ *if n=k+4 where k=0,3,6,9,*$\cdots$
Let v_1 *be the universal vertex of* (W_n) *of degree n-1 (a multiple of 3). Let* $u_1, u_2, \cdots, u_{n-1}$ *be the other vertices of degree 3. Since the degree of all the vertices have a common factor 3, relatively prime dominating set does not exist. Therefore,* $\gamma_{rpd}(W_n) = 0$
Case 2: Otherwise
Let v_1 *be the universal vertex of degree n-1. Let* $u_1, u_2, \cdots, u_{n-1}$ *be the other vertices of degree 3. Since the graph has n vertices and the vertex* v_1 *has degree n-1. The Vertex* v_1 *covers all the Vertices of* W_n. *Since relatively prime dominating set has at least two elements. We Choose two Vertices* v_1 *of degree n-1(not a multiple of 3) and* u_i *of degree 3, where* $1 \leq i \leq n-1$. *Then* $(d(v_1), d(u_i)) = 1$. *Thus, Relatively Prime Dominating set is* $\{v_1, u_i\}$ *and* $\gamma_{rpd}(W_n) = 2$.

3 Relatively Prime Domination on power of Wheel graph

Theorem 3.1 *Let* W_n *be a Wheel Graph. Then* $\gamma_{rpd}((W_n)^p) = 0$, *where* $p \geq 2, n \geq 4$.
Proof: *Since* $(W_n)^p$ *is a complete graph where* $n \geq 4$ *and* $p \geq 2$, *we have* $\gamma_{rpd}((W_n)^p) = 0 \ \forall p \geq 2$.
As relatively prime domination number of wheel graph is zero, we find the number for it switching. Its surprise that the relatively prime domination number for its switching graph is also zero.

Theorem 3.2 *Let* W_n *be a wheel graph. Then* $\gamma(((w_n)^p)^v) = 0$ *where* $p \geq 2, n \geq 4$.
Proof: *Let* $(w_n)^p$ *be a power of wheel graph where* $p \geq 2, n \geq 4$. *As power of wheel graph is a complete graph, degree of each vertex is same, ie) power of wheel graph is a complete graph of degree n-1. Therefore, degree of each vertex is n-1. Let v be any vertex in* $((w_n)^p)$. *Then in the resulting graph* $((W_n)^p)^v$, *deg(v) = 0 and deg(u) = n-2* $\forall \ u \neq v$ *Clearly the two vertices u and v cover all the vertices of* $((w_n)^p)^v)$. *But it is not relatively prime dominating set*
Therefore, relatively prime dominating set does not exist for $((w_n)^v)$ *and so* $\gamma(((w_n)^p)^v) = 0$.

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4 Python program to compute the

Relatively prime domination number of any wheel graph or power of wheel graph can be computed using the python program given below.

PROGRAM INPUT

```
#Relatively prime domination on wheel graph
def switching_graph(G,v):
import networkx as nx
import matplotlib.pyplot as plt
V = list(G.nodes())
E = list(G.edges())
for i in V:
    if (i,v) in E:
        E.remove((i,v))
    elif (v,i) in E:
        E.remove((v,i))
    else:
        if i!=v:
            E.append((i,v))
K=nx.Graph()
K.add_nodes_from(V)
K.add_edges_from(E)
nx.draw(K,with_labels=True)
plt.show()
return K

def rpd_ wheel_ graph(n,p=1):
#draw the wheel graph
import networkx as nx
import matplotlib.pyplot as plt
G = nx.wheel_ graph(n) K=nx.power(G,p)
nx.draw(K,with_ labels=True)
plt.show()
#condition for the relatively prime domination number
u=n-4
if u%3==0 or p >= 2:
print("Relatively prime domination number is 0")
```

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print("Since Relatively prime domination number is 0 we find the relatively prime domination number for switching graph ")

K=switching _ graph(K,0)

print("Relatively prime domination number of switching graph is 0")

else:

print("Relatively prime domination number is 2")

Example 4.1 ***INPUT***

rpd_wheel_graph(10)

OUTPUT

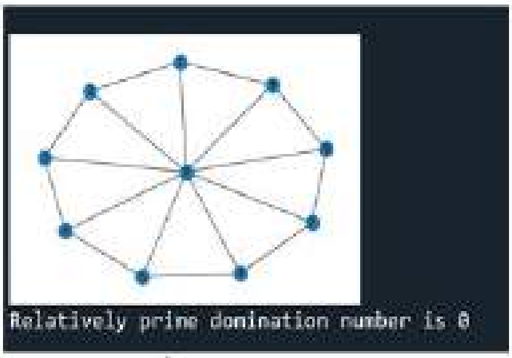

Figure 1:

Example 4.2 ***INPUT***

rpd_wheel_graph (12,4)

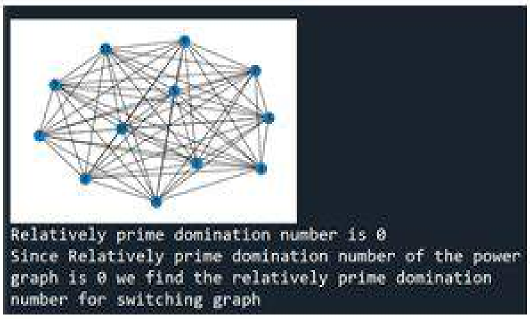

Figure 2:

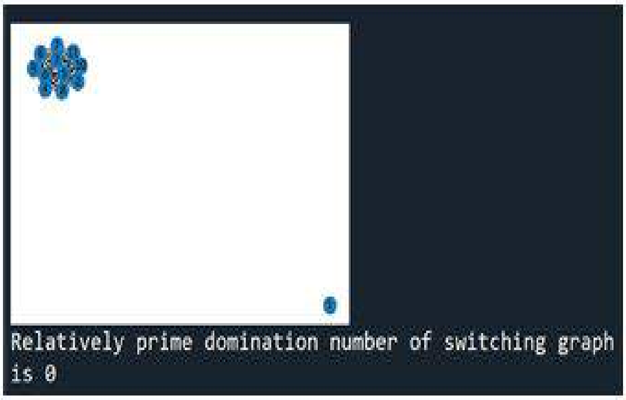

Figure 3:

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Cordial Labeling of Bistar Clunged with Square of Cycle Graph and Cartesian Product Graph P_2XC_n

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Abstract

Cordial labeling is defined as a function $g : V(\theta) \rightarrow \{0,1\}$ in which each edge ab is assigned the label $|g(a) - g(b)|$ with the conditions $|v_g(0) - v_g(1)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$ where $v_g(0)$ and $v_g(1)$ signify the number of vertices with 0s and 1s, similarly $e_g(0)$ and $e_g(1)$ signify the number of edges with 0s and 1s. In this paper, Bistar clunged with square of cycle graph and Cartesian product graph P_2XC_n are analyzed for cordial labeling.

Key words::Cordial labeling, Square of cycle graph, Bistar graph, Cartesian product graph.

AMS classification: 05C50, 05C78

1 Introduction

Graph labeling is an assignment of integers to vertices, or edges or both under certain conditions. Rosas[5] 1967 invention is credited for the majority of graph labeling methods. Cordial labeling was introduced by Cahit[1] in 1987. In[2] Devakirubanithi, et.al established graphs such as uniform sub-divided shell bow graph, uniform sub-divided shell flower graph, subdivided shell graph with star graphs coupled to the apex and path vertices and one point union of multiple sub-divided shell graph admits cordial labelling. In[4] Pariksha Gupta, et.al proved a Cordial labeling pattern for star of a bistar graph. Cordial labeling is useful in DNA code word design problem and noisy communication channels. In this paper, Bistar clunged with square of cycle graph and Cartesian product graph P_2XC_n are analyzed for cordial labeling.

2 Main Results

In this section, we provide all the fundamental notations and definitions which serve as prerequisites for the advancement of the topic.

Definition 2.1 [3] *Let f be a function from the vertices of G to $\{0,1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. f is called a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and the number of edges labeled 0 and the number of edges labeled 1 differ atmost by 1.*

Definition 2.2 [6] *Square of a graph G denoted by G^2 has the same vertex as of G and two vertices are adjacent in G^2 if they at a distance of 1 or 2 apart from G.*

Definition 2.3 *The Bistar clunged with square of cycle graph $B_{l,m}(C_n^2)$ is obtained by attaching the square of cycle graph C_n^2 to each pendant vertices of bistar graph $B_{l,m}$.*

Definition 2.4 *The Cartesian Product Graph G_1XG_2 of a graph G_1 and G_2 whose vertex set is $V(G_1)XV(G_2)$ can be defined as follows. Let u be a vertex in $V(G_1)$ and v be the vertex in $V(G_2)$. Then (u,v) is an element of G_1XG_2 and (u,v) is adjacent to (u',v') iff either $u = u'$ and edge vv' belongs to $E(G_2)$ or $v = v'$ and the edge uu' belongs to $E(G_1)$.*

Theorem 2.5 *The Bistar clunged with square of cycle graph $B_{l,m}(C_n^2)$ is cordial when n is even and $n \geq 4$.*

Proof: Let $G = B_{l,m}(C_n^2)$ be the bistar clunged with square of cycle graph where l and m are the number of vertices of the bistar graph and n is the number of vertices of the cycle. Fix the central vertices of the bistar graph as $a_0 = 0$ and $a_1 = 1$. Let the vertices of (C_n^2) be labeled as $a_i^j (1 \leq i \leq n, 1 \leq j \leq l+m)$. Here a_i^j denote the vertices of the j^{th} copy of C_n^2. We obtain $a_1^m, a_2^m, \cdots, a_n^{l+m}$ as the successive vertices of the n^{th} copy of the square of cycle graph

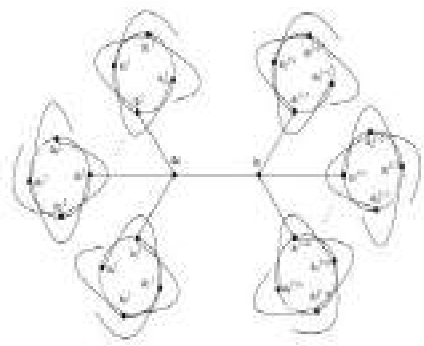

Figure 1: Generalized Bistar clunged with square of cycle graph

Case 1: *If l and m are same*
The number of vertices and edges in $B_{l,m}(C_n^2)$ is defined as 2ln+2 and 4ln+2l+1
Define the vertex labeling as follows

$$f(a_i) = \begin{cases} 0, if \quad i \equiv 0 \ (mod 2) \\ 1, if \quad i \equiv 1 \ (mod 2) \end{cases}$$

The number of vertices labeled with 0 and 1 is defined as follows:

$$V_f(0) = \frac{2ln+2}{2}; V_f(1) = \frac{2ln+2}{2}$$

The number of edges labeled with 0 and 1 is defined as follows

$$e_f(0) = \left\lfloor \frac{4ln + 2l + 1}{2} \right\rfloor ; e_f(1) = \left\lfloor \frac{(4ln + 2l + 1)}{2} \right\rfloor + 1$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ *and* $|e_f(0) - e_f(1)| \leq 1$ *Thus, the Bistar clunged with square of cycle graph* $B_{l,m}(C_n^2)$ *admits cordial labeling when* l *and* m *are same.*

Case 2: *If* l *and* m *are different*

Subcase (i): when l *is odd and* m *is even*

Define the vertex labeling as follows

If $j \equiv 1$*(mod 2)*

$$f(a_i^j) = \begin{cases} 0, if \ \ i \equiv 1(mod\ 2) \\ 1, if \ \ i \equiv 0(mod\ 2) \end{cases}$$

If $j \equiv 0(mod\ 2)$

$$f(a_i^j) = \begin{cases} 0, if \ \ i \equiv 0(mod\ 2) \\ 1, if \ \ i \equiv 1(mod\ 2) \end{cases}$$

Subcase (ii): when l *is even and* m *is odd*

Define the vertex labeling as follows

If $j \equiv 1(mod\ 2)$

$$f(a_i^j) = \begin{cases} 0, if \ \ i \equiv 0(mod\ 2) \\ 1, if \ \ i \equiv 1(mod\ 2) \end{cases}$$

$f\ j \equiv 0(mod\ 2)$

$$f(a_i^j) = \begin{cases} 0, if \ \ i \equiv 1(mod\ 2) \\ 1, if \ \ i \equiv 0(mod\ 2) \end{cases}$$

For both subcase (i) and subcase (ii), The number of vertices and edges in $B_{l,m}(C_n^2)$ *is defined as ln+mn+2 and 2ln+2mn+l+m+1*

The number of vertices labeled with 0 and 1 is defined as

$$V_f(0) = \frac{ln + mn + 2}{2}; V_f(1) = \frac{ln + mn + 2}{2}$$

The number of edges labeled with 0 and 1 is defined as follows

$$e_f(0) = \frac{2ln + 2mn + l + m + 1}{2}; e_f(1) = \frac{2ln + 2mn + l + m + 1}{2}$$

Subcase (iii): when l *and* m *are even or when* l *and* m *are odd*

Define the vertex labeling as follows

If $j \equiv 1(mod\ 2)$

$$f(a_i^j) = \begin{cases} 0, if \ \ i \equiv 1(mod\ 2) \\ 1, if \ \ i \equiv 0(mod\ 2) \end{cases}$$

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If $j \equiv 0(mod\ 2)$

$$f(a_i^j) = \begin{cases} 0, if\ \ i \equiv 0(mod\ 2) \\ 1, if\ \ i \equiv 1(mod\ 2) \end{cases}$$

The number of vertices and edges in $B_{l,m}(C_n^2)$ *is defined as ln+mn+2 and 2ln+2mn+l+m+1*
The number of vertices labeled with 0 and 1 is defined as

$$V_f(0) = \frac{ln + mn + 2}{2}; V_f(1) = \frac{ln + mn + 2}{2}$$

when l and m are even, the number of edges labeled with 0 and 1 is defined as follows

$$e_f(0) = \left\lfloor \frac{2ln + 2mn + l + m + 1}{2} \right\rfloor ; e_f(1) = \left\lfloor \frac{2ln + 2mn + l + m + 1}{2} \right\rfloor + 1$$

when l and m are odd, the number of edges labeled with 0 and 1 is defined as follows

$$e_f(0) = \left\lfloor \frac{2ln + 2mn + l + m + 1}{2} \right\rfloor + 1; e_f(1) = \left\lfloor \frac{2ln + 2mn + l + m + 1}{2} \right\rfloor$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ *and* $|e_f(0) - e_f(1)| \leq 1$
Thus, the Bistar clunged with square of cycle graph $B_{l,m}(C_n^2)$ *admits cordial labeling when l and m are different.*

Illustration 2.6 *Case 1*

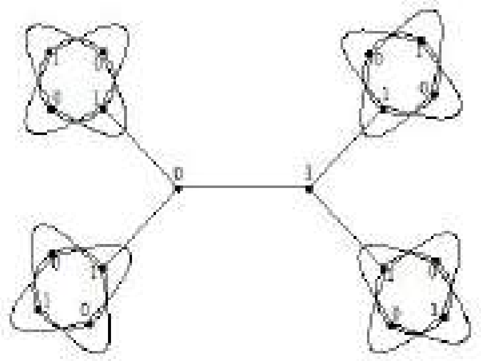

Figure 2: Cordial labeling of $B_{2,2}(C_4^2)$

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Illustration 2.7 *Case 2*

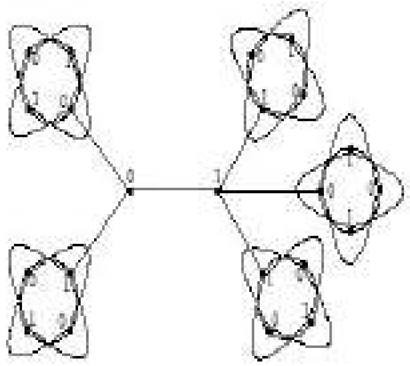

Figure 3: Cordial labeling of $B_{2,3}(C_4^2)$

Theorem 2.8 *The Cartesian product graph P_2XC_n is cordial when n is odd*

Proof: *Let $G = P_2XC_n$ be the Cartesian Product graph P_2XC_n where n is the number of vertices of the cycle. Let the vertices of P_2XC_n be labeled as a_i where i=1,2,3,$\cdots$,2n The number of*

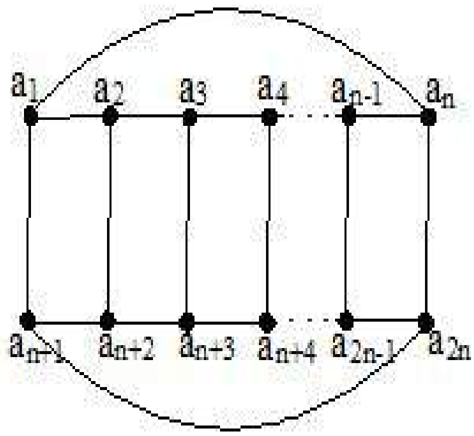

Figure 4: Generalized Cartesian product graph P_2XC_n

vertices in P_2XC_n are defined as 2n and 3n

Define the vertex labeling as follows

$$f(a_i) = \begin{cases} 0, if \ i \equiv 1, 0(mod \ 4) \\ 1, if \ i \equiv 2, 3(mod \ 4) \end{cases}$$

The number of vertices labeled with 0 and 1 is defined as

$$V_f(0) = n; V_f(1) = n$$

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Case 1: If $n \equiv 1(mod\ 4)$
The number of edges labeled with 0 and 1 is defined as follows

$$e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor + 1; e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$$

Case 2: If $n \equiv 3(mod\ 4)$
The number of edges labeled with 0 and 1 is defined as follows

$$e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor ; e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor + 1$$

In both the cases, $|V_f(0) - V_f(1)| \leq 1$ *and* $|e_f(0)e_f(1)| \leq 1$
Therefore, the Cartesian product graph P_2XC_n *admits cordial labelling when n is odd.*

Illustration 2.9 *Case 1*

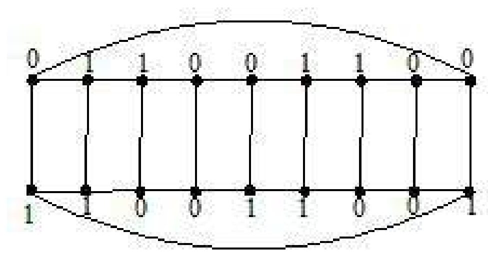

Figure 5: Cordial labeling of P_2XC_9

Illustration 2.10 *Case 2*

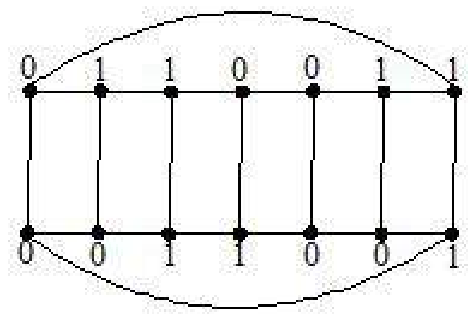

Figure 6: Cordial labeling of P_2XC_7

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3 Conclusion

Thus in this paper we have obtained Bistar clunged with square of cycle graph and Cartesian product graph P_2XC_n are analysed for cordial labeling.

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Subdivision of Stolarsky-3 Mean Graphs

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Abstract

Let $G = (V, E)$ be a graph with p vertices and q edges. G is said to be Stolarsky-3 Mean graph if each vertex $x \in V$ is assigned distinct labels $f(x)$ from $1, 2, ..., q+1$ and each edge $e = uv$ is assigned with labels $f(e) = \left\lceil \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rceil$ or $\left\lfloor \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rfloor$ then the resulting edge labels are distinct. In this case, f is called a Stolarsky-3 mean labeling of G and G is called a Stolarsky-3 Mean graph. In this paper we contribute some new results in Stolarsky-3 mean graphs. We prove that subdivision of Stolarsky-3 mean graphs are Stolarsky-3 mean graphs. We use some standard graphs to derive the results for subdivision of graphs.

Keywords : Graph Labeling, Stolarsky-3 mean labeling, Subdivision of graphs, Comb graph, Ladder graph, Triangular snake graph and Quadrilateral Snake graph.

AMS Subject Classification : 05C78

1 Introduction

Throughout this paper we consider finite, undirected and simple graphs. Let G be a graph with p vertices and q edges. There are several types of labeling and a detailed survey can be found in [2]. For all other standard terminology and notations, we follow [3]. Subdivision of Mean labeling was introduced in [8]. The concept of Stolarsky-3 mean labeling was introduced in [4]. In this paper we investigate the subdivision of Stolarsky-3 mean labeling of graphs. We will provide brief summary of definitions and other information which are necessary for our present investigation.

Definition 1.1 *A graph G with p vertices and q edges is said to be Stolarsky-3 Mean graph if each vertex $x \in V$ is assigned distinct labels $f(x)$ from $1, 2, ..., q+1$ and each edge $e = uv$ is assigned the distinct labels* $f(e) = \left\lceil \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rceil$ *or* $f(e) = \left\lfloor \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rfloor$ *then the resulting edge labels are distinct. In this case, f is called a Stolarsky-3 mean labeling of G and G is called a Stolarsky-3 Mean graph.*

Definition 1.2 *A walk in which all the vertices $u_1, u_2, ..., u_n$ are distinct is called a Path. It is denoted by P_n.*

Definition 1.3 *A closed path is called a cycle. A Cycle on n vertices is denoted by C_n.*

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Definition 1.4 *The Corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 vertices) and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.*

Definition 1.5 *The Cartesian product $G_1 \times G_2$ of two graphs is defined to be the graph with vertex set $V_1 \times V_2$ and two vertices $U = (U_1, U_2)$ and $V = (V_1, V_2)$ are adjacent in $G_1 \times G_2$ if either $U_1 = V_1$ and U_2 is adjacent to V_2 and U_1 is adjacent to V_1.*

Definition 1.6 *Comb $P_n \odot K_1$ is a graph obtained by joining a single pendant edge to each vertex of a path.*

Definition 1.7 *The Ladder graph L_n $(n \geq 2)$ is the product graph $P_2 \times P_n$ which contains $2n$ vertices and $3n - 2$ edges.*

Definition 1.8 *A Triangular Snake T_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n - 1$. That is, every edge of a path is replaced by a triangle C_3.*

Definition 1.9 *A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i. That is, every edge of a path is replaced by a cycle C_4.*

Definition 1.10 *If $e = uv$ is an edge of G and w is a vertex not in G, then e is said to be subdivided when it is replaced by the edges uw and vw. The graph obtained by subdividing each edge of a graph G is called the Subdivision graph G and is denoted by $S(G)$.*

Theorem 1.11. *Any Path P_n is Stolarsky-3 mean graph. [4]*
Theorem 1.12. *The Comb graph $P_n \odot K_1$ is a Stolarsky-3 mean graph.[4]*
Theorem 1.13. *Any Ladder L_n is a Stolarsky-3 mean graph. [4]*
Theorem 1.14. *Any Triangular snake T_n is a Stolarsky-3 mean graph [4]*
Theorem 1.15. *Any Quadrilateral Snake Q_n is a Stolarsky-3 mean graph. [4]*

2 Main Results

Theorem 2.1 *The subdivision of a Comb graph $(P_n \odot K_1)$ is a Stolarsky-3 mean graph.*
Proof: Here we subdivide the comb graph in three different cases. Let $u_1, u_2, ..., u_n$ be the vertices of the path P_n and let v_i be the pendant vertices attached to the path P_n. Let $G = S(P_n \odot K_1)$.
Case (i): *Subdivide the edge u_iu_{i+1}, $1 \leq i \leq n - 1$ of the path P_n. Let t_i, $1 \leq i \leq n - 1$ be the vertices which subdivide the edges of P_n.*

Define a function $f : V(G) \to 1, 2, ..., q+1$ by $f(u_i) = 3i-2$, $1 \leq i \leq n$, $f(v_i) = 3i-1$, $1 \leq i \leq n$, $f(t_i) = 3i$, $1 \leq i \leq n-1$, and the edges are labeled as $f(u_it_i) = 3i-1$, $1 \leq i \leq n-1$, $f(u_iv_i) = 3i-1$, $1 \leq i \leq n$, and $f(t_iu_{i+1}) = 3i$, $1 \leq i \leq n-1$. This gives a set of distinct edge labels which forms a Stolarsky-3 mean graph.

Example 2.2 *Subdivision of $P_5 \odot K_1$ is given below*

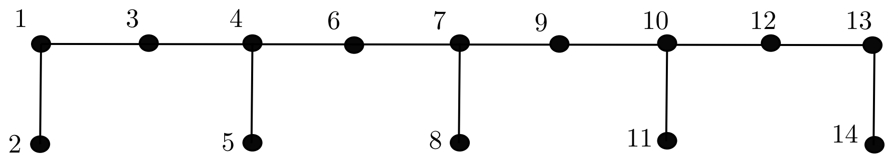

Figure 1

Case (ii): *Here we subdivide the edge u_iv_i, $1 \leq i \leq n$ of the comb $P_n \odot K_1$. Let t_i, $1 \leq i \leq n$ be the vertices which subdivide the edges u_iv_i, $1 \leq i \leq n$.*
Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_i) = 3i-2$, $1 \leq i \leq n$, $f(v_i) = 3i$, $1 \leq i \leq n$, $f(t_i) = 3i-1$, $1 \leq i \leq n-1$. Then the edges are labeled as $f(u_iu_{i+1}) = 3i$, $1 \leq i \leq n-1$ $f(u_it_i) = 3i-2$, $1 \leq i \leq n$ and $f(t_iv_i) = 3i-1$, $1 \leq i \leq n$. This forms a Stolarsky-3 mean graph.

Example 2.3 *Subdivision of $(P_5 \odot K_1)$ is given below*

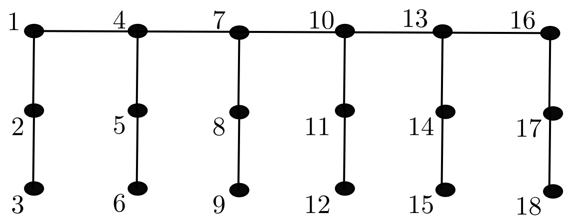

Figure 2

Case (iii): *We apply subdivision for all the edges of the comb $P_n \odot K_1$. Let t_i, $1 \leq i \leq n-1$ be the vertices which subdivide the edges u_iu_{i+1} and w_i, $1 \leq i \leq n$ be the vertices which subdivide the edge u_iv_i, $1 \leq i \leq n$.*
Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_i) = 4i-3$, $1 \leq i \leq n$, $f(v_i) = 4i-1$ $1 \leq i \leq n$, $f(t_i) = 4i$, $1 \leq i \leq n-1$, $f(w_i) = 4i-2$, $1 \leq i \leq n$, thus we get distinct edge labels $f(u_it_i) = 4i-1$, $1 \leq i \leq n-1$, $f(u_iw_i) = 4i-3$, $1 \leq i \leq n$, $f(w_iv_i) = 4i-2$, $1 \leq i \leq n$, $f(t_iu_{i+1}) = 4i$, $1 \leq i \leq n-1$. This forms a Stolarsky-3 mean graph.

Example 2.4 *$S(P_5 \odot K_1)$ is given below*

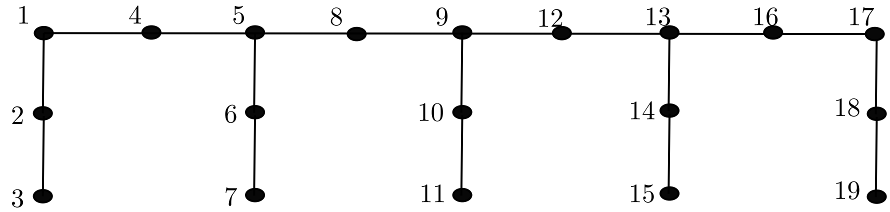

Figure 3

Theorem 2.5 *Subdivision of Ladder graph L_n is a Stolarsky-3 mean graph.*

Proof: Let L_n be a Ladder graph connecting two paths $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$. Let $G = S(L_n)$ be a graph obtained by subdividing all the edges of L_n.

Here we subdivide the Ladder in three different cases.

Case (i): *We subdivide each edge u_iu_{i+1}, and v_iv_{i+1} $1 \le i \le n-1$ of L_n. Let x_i, y_i, $1 \le i \le n-1$ be the vertices which subdivide the edges u_iu_{i+1}, and v_iv_{i+1}.*

Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_i) = 5i-4$, $1 \le i \le n$, $f(v_i) = 5i-3$, $1 \le i \le n$, $f(x_i) = 5i-2$, $1 \le i \le n-1$, and $f(y_i) = 5i-1$, $1 \le i \le n-1$. The edges are labeled as $f(u_ix_i) = 5i-3$, $1 \le i \le n-1$, $f(x_iu_{i+1}) = 5i-1$, $1 \le i \le n-1$, $f(v_iy_i) = 5i-2$, $1 \le i \le n-1$, $f(y_iv_{i+1}) = 5i$, $1 \le i \le n-1$, $f(u_iv_i) = 5i-4$, $1 \le i \le n$. Thus we get distinct edge labels which forms a Stolarsky-3 mean labeling of graphs.

Example 2.6 *The labeling pattern of subdivision of L_5 is given below.*

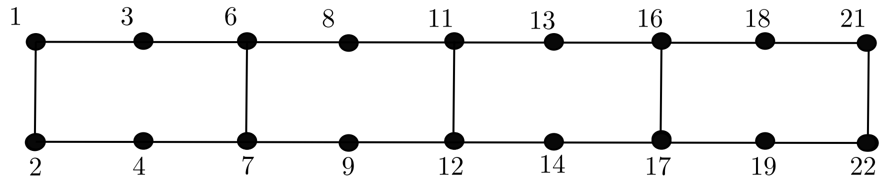

Figure 4

Case (ii): *Here we subdivide the edge u_iv_i, $1 \le i \le n$ of L_n. Let t_i, $1 \le i \le n-1$ be the vertices which subdivide u_i and v_i. Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_1) = 1$, $f(u_i) = 4(i-1)$, $2 \le i \le n$, $f(v_1) = 3$, $f(v_i) = 4(i-1)+1$, $2 \le i \le n$, $f(x_i) = 4i-2$, $1 \le i \le n$. Edges are labeled with $f(u_iu_{i+1}) = 4i-1$, $1 \le i \le n-1$, $f(v_iv_{i+1}) = 4i$, $1 \le i \le n-1$, $f(u_ix_i) = 4i-3$, $1 \le i \le n$, $f(x_iv_i) = 4i-2$, $1 \le i \le n$. Thus we get distinct edge labels. Hence f is Stolarsky-3 mean labeling.*

Example 2.7 *The Labeling pattern of $S(L_5)$ is given below*

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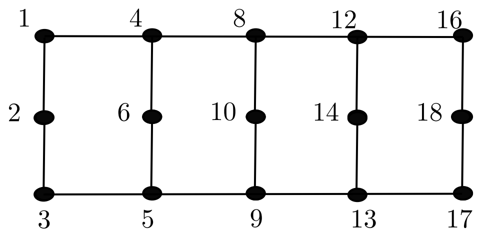

Figure 5

Case (iii): *We apply subdivision for all the edges of the ladder L_n. Let x_i, y_i, $1 \le i \le n-1$ be the vertices which subdivide the edges u_iu_{i+1}, and v_iv_{i+1} and let t_i, $1 \le i \le n-1$ be the vertices which subdivide the edges u_iv_i, $1 \le i \le n$. Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_1) = 6$, $f(u_i) = 6(i-1)$, $2 \le i \le n$, $f(v_1) = 3$, $f(v_i) = 6(i-1)+1$, $2 \le i \le n$, $f(x_i) = 6i-2$, $1 \le i \le n$, $f(y_i) = 6i-1$, $1 \le i \le n-1$, $f(t_i) = 6i-4$, $1 \le i \le n$. Edges are labeled as $f(u_ix_i) = 6i-3$, $1 \le i \le n-1$, $f(x_iu_{i+1}) = 6i-1$, $1 \le i \le n-1$, $f(u_iz_i) = 6i-5$, $1 \le i \le n$, $f(z_iv_i) = 6i-4$, $1 \le i \le n$, $f(v_iy_i) = 6i-2$, $1 \le i \le n-1$, $f(y_iv_(i+1)) = 6i$. Thus we get distinct edge labels. This forms a Stolarsky-3 mean labeling of graphs.*

Example 2.8 *The Labeling pattern of $S(L_5)$ is exhibited below*

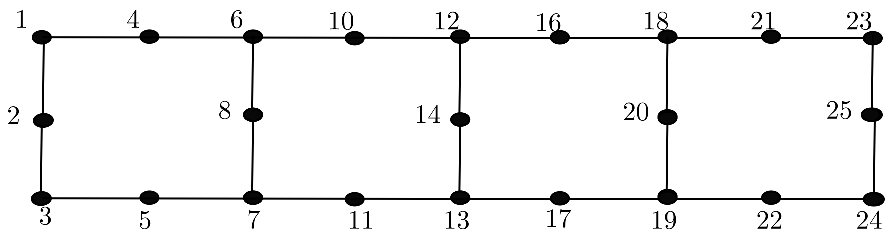

Figure 6

Theorem 2.9 *Subdivision of Triangular snake T_n is a Stolarsky-3 mean graph.*
Proof: In this theorem we make subdivision in three cases. In first case, we subdivide the edges of the triangle in its lower base and in second case we subdivide the remaining two sides. Then in the third case we divide three sides at the same time. Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the vertices of the triangular snake.
Case (i): *We subdivide the edge u_iu_{i+1} by x_i. Then the function defined here as $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_i) = 4i-3$, $1 \le i \le n$, $f(v_i) = 4i-2$, $2 \le i \le n-1$, $f(x_i) = 4i$, $1 \le i \le n-1$. Edges are labeled as $f(u_ix_i) = 4i-2$, $1 \le i \le n-1$, $f(x_iu_{i+1}) = 4i$, $1 \le i \le n-1$, $f(u_iv_i) = 4i-3$, $1 \le i \le n-1$, $f(v_iu_{i+1}) = 4i-1$, $1 \le i \le n-1$. Thus, we get distinct edge labels. This forms a Stolarsky-3 mean labeling of graphs.*

Example 2.10 *Stolarsky-3 mean labeling of subdivided Triangular snake is*

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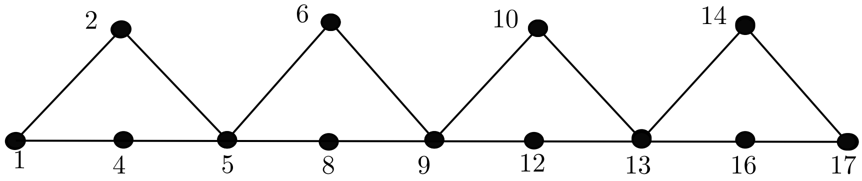

Figure 7

Case (ii): *In this case we subdivide the two sides of the triangle. Let* x_i, y_i, $1 \leq i \leq n-1$ *be the vertices which subdivide the edges* u_iv_i *and* $u_{i+1}v_i$, $1 \leq i \leq n-1$ *respectively. Define a function* $f : V(G) \rightarrow \{1, 2, ..., q+1\}$ *by* $f(u_i) = 5i-4$, $1 \leq i \leq n$, $f(v_i) = 5i-2$, $1 \leq i \leq n-1$, $f(x_i) = 5i-3$, $1 \leq i \leq n-1$, $f(y_i) = 5i-1$, $1 \leq i \leq n-1$. *Then the edges are labeled as* $f(u_ix_i) = 5i-4$, $1 \leq i \leq n-1$, $f(x_iv_i) = 5i-3$, $1 \leq i \leq n-1$, $f(v_iy_i) = 5i-2$, $1 \leq i \leq n-1$, $f(y_iu_{i+1}) = 5i$, $1 \leq i \leq n-1$, $f(y_iu_{i+1}) = 5i-1$, $1 \leq i \leq n-1$. *Thus we get distinct edge labels. This forms a Stolarsky-3 mean labeling of graphs.*

Example 2.11 *The labeling pattern of* $S(T_5)$ *is shown below*

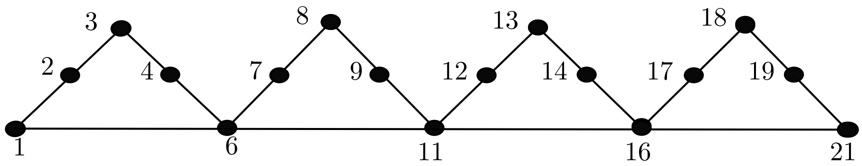

Figure 8

Case (iii): *In this case we subdivide each edge of a Triangular snake by a new vertex.*
Let x_i, y_i *and* t_i, $1 \leq i \leq n-1$ *be the vertices which subdivide the edges* u_iv_i, v_iu_{i+1}, *and* u_iu_{i+1} *respectively. Define a function* $f : V(G) \rightarrow \{1, 2, ..., q+1\}$ *by* $f(u_i) = 6i-5$, $1 \leq i \leq n$, $f(v_i) = 6i-3$, $1 \leq i \leq n-1$, $f(x_i) = 6i-4$, $1 \leq i \leq n-1$, $f(y_i) = 6i-1$, $1 \leq i \leq n-1$, $f(t_i) = 6i-2$, $1 \leq i \leq n-1$. *Then the edges are labeled as* $f(u_it_i) = 6i-3$, $1 \leq i \leq n-1$, $f(u_ix_i) = 6i-5$, $1 \leq i \leq n-1$, $f(x_iv_i) = 6i-4$, $1 \leq i \leq n-1$, $f(t_iu_{i+1}) = 6i-1$, $1 \leq i \leq n-1$, $f(v_iy_i) = 6i-2$, $1 \leq i \leq n-1$, $f(y_iu_{i+1}) = 6i$, $1 \leq i \leq n-1$. *Thus all edge labels are distinct. This forms a Stolarsky-3 mean labeling of graphs.*

Example 2.12 *Stolarsky-3 mean labeling of subdivided Triangular snake is*

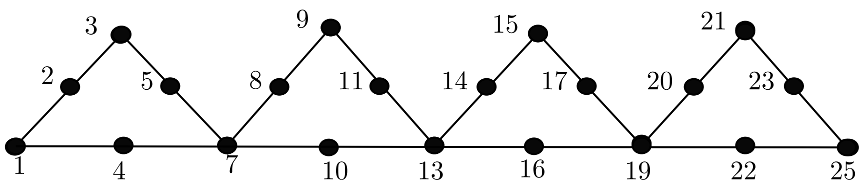

Figure 9

Theorem 2.13 *Subdivision of any Quadrilateral Snake Q_n is a Stolarsky-3 mean graph.*

Proof: Let Q_n be a Quadrilateral Snake obtained from a path $u_1, u_2, ..., u_n$. Join u_i and u_{i+1} to new vertices v_i and w_i, $1 \leq i \leq n-1$ respectively and then joining v_i and w_i. Let $G = S(Q_n)$ be the graph obtained by subdividing all the edges of Q_n.

Here we consider the following cases.

Case (i): *First we subdivide the edge u_iu_{i+1} of Q_n. Let x_i, $1 \leq i \leq n-1$ be the vertices which subdivide u_i and u_{i+1}. Define a function $f : V(G) \rightarrow \{1, 2, ..., q+1\}$ by $f(u_i) = 5i-4$, $1 \leq i \leq n$, $f(v_1) = 3$, $f(v_i) = 5(i-1)+2$, $2 \leq i \leq n-1$, $f(w_i) = 5i-1$, $1 \leq i \leq n-1$, $f(x_1) = 2$, $f(x_i) = 5(i-1)+3$, $2 \leq i \leq n-1$. Edges are labeled with $f(u_1x_1) = 1$, $f(u_ix_i) = 5(i-1)+2$, $2 \leq i \leq n-1$, $f(x_iu_{i+1}) = 5i-1$, $2 \leq i \leq n-1$, $f(u_1v_1) = 2$, $f(u_iv_i) = 5(i-1)+1$, $2 \leq i \leq n-1$, $f(v_iw_i) = 5i-2$, $1 \leq i \leq n-1$, $f(w_iu_{i+1}) = 5i$, $1 \leq i \leq n-1$. Hence f is Stolarsky-3 mean labeling.*

Example 2.14 *Stolarsky-3 mean labeling of subdivided Quadrilateral Snake is shown below*

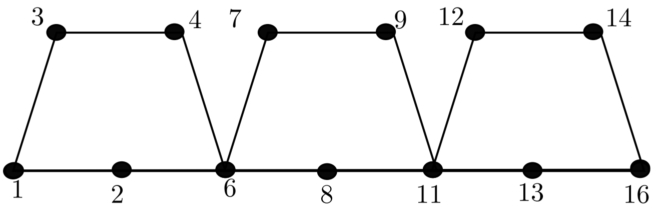

Figure 10

Case (ii): *In this case we subdivide the edges u_iv_i and $u_{i+1}w_i$ of Q_n. Let x_i and y_i, $1 \leq i \leq n-1$, be the vertices which subdivide the edges u_iv_i and $u_{i+1}w_i$.*

Define a function $f : V(G) \rightarrow \{1, 2, ..., q+1\}$ by $f(u_i) = 6i-5$, $1 \leq i \leq n$, $f(v_i) = 6i-3$, $1 \leq i \leq n-1$, $f(w_i) = 6i-2$, $1 \leq i \leq n-1$, $f(s_i) = 6i-4$, $1 \leq i \leq n-1$, $f(t_i) = 6i-1$, $1 \leq i \leq n-1$. Edges are labeled with $f(u_is_i) = 6i-5$, $1 \leq i \leq n-1$, $f(s_iv_i) = 6i-4$, $1 \leq i \leq n-1$, $f(u_{i+1}t_i) = 6i$, $1 \leq i \leq n-1$, $f(t_iw_i) = 6i-1$, $f(v_iw_i) = 6i-3$, $1 \leq i \leq n-1$. Clearly f is Stolarsky-3 mean labeling.

Example 2.15 *Stolarsky-3 mean labeling of $S(Q_4)$ is shown below*

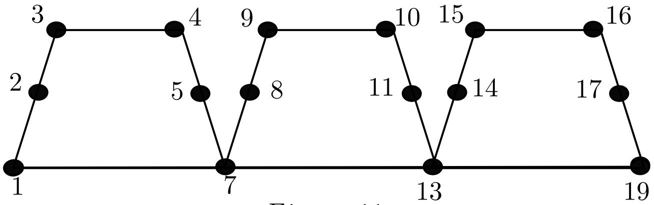

Figure 11

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Case (iii): *Here we subdivide the edge v_iw_i of Q_n. Let x_i, $1 \le i \le n-1$, be the vertices which subdivide v_i and w_i. Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_i) = 5i-4$, $1 \le i \le n$, $f(v_i) = 5i-3$, $1 \le i \le n$, $f(w_i) = 5i-1$, $1 \le i \le n-1$, $f(x_i) = 5i-2$, $1 \le i \le n-1$. Edges are labeled with $f(u_iv_i) = 5i-4$, $1 \le i \le n-1$, $f(v_ix_i) = 5i-3$, $1 \le i \le n-1$, $f(u_{i+1}w_i) = 5i$, $1 \le i \le n-1$, $f(x_iw_i) = 5i-1$, $1 \le i \le n-1$.*
Clearly f is Stolarsky-3 mean labeling.

Example 2.16 *Stolarsky-3 mean labeling of $S(Q_4)$ is shown below*

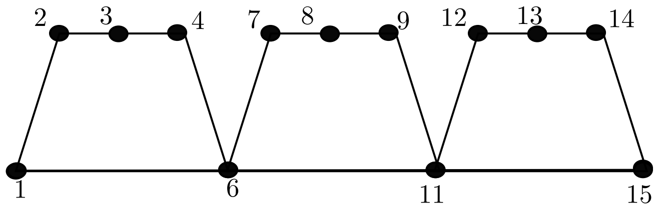

Figure 12

Case (iv): *In this case we subdivide all the edges of a Quadrilateral Snake Q_n. Let s_i, t_i, x_i and y_i, $1 \le i \le n-1$, be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, v_iw_i$ and w_iu_{i+1} respectively. Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by $f(u_i) = 8i-7$, $1 \le i \le n$, $f(v_i) = 8i-5$, $1 \le i \le n-1$, $f(w_i) = 8i-3$, $1 \le i \le n-1$, $f(s_i) = 8i$, $1 \le i \le n-1$, $f(t_i) = 8i-6$, $1 \le i \le n-1$, $f(x_i) = 8i-4$, $1 \le i \le n-1$, $f(y_i) = 8i-2$, $1 \le i \le n-1$. Then the edges are labeled as $f(u_it_i) = 8i-7$, $1 \le i \le n-1$, $f(t_iv_i) = 8i-6$, $1 \le i \le n-1$, $f(v_ix_i) = 8i-5$, $1 \le i \le n-1$, $f(x_iw_i) = 8i-4$, $1 \le i \le n-1$, $f(w_iy_i) = 8i-2$, $1 \le i \le n-1$, $f(y_iu_{i+1}) = 8i-1$, $1 \le i \le n-1$, $f(u_{i+1}s_i) = 8i$, $1 \le i \le n-1$, $f(s_iu_i) = 8i-3$, $1 \le i \le n-1$. Thus, all edge labels are distinct. This forms a Stolarsky-3 mean labeling of graph.*

Example 2.17 *Stolarsky-3 mean labeling of $S(Q_4)$ is shown below*

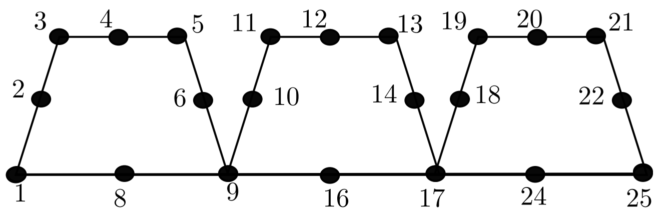

Figure 13

3 Conclusion

The study of labeled graph is important due to its different applications. It is very interesting to investigate subdivision of Stolarsky-3 mean graphs which admit Stolarsky-3 mean graphs.

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The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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Edge Product In Cluster Hypergraphs

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Abstract

The Cluster hypergraph $H = (V_x, E)$ is said to be an Edge Product Cluster Hypergraph if there exists an edge function $f : E \to I$ such that the edge function f and the corresponding edge product function F of f on V_x have the following two conditions:

(i) $F(v) \in I$ for every $v \in V_x$

(ii) if $f(e_1) \times f(e_2) \times \ldots \times f(e_i) \in I$ for some edges $e_1, e_2, \ldots e_i \in E(H)$ then the edges $e_1, e_2, \ldots e_i$ are all adjacent to some vertex $v \in V_x$.
An edge e whose edge function is labeled as one is the unit edge and the largest element in the set of positive integers I is the maximal edge of the cluster hypergraph. The edge product cluster hypergraph, unit edge product cluster hypergraph and some theorems based on this concept have been discussed in this paper.
Keywords : Edge function, Edge product function, Edge product cluster hypergraph, Unit edge product cluster hypergraph, Cluster hypergraph.
AMS Subject Classification : 05C65

1 Introduction

Harary F introduced the notation of sum graph [6]. A graph $G(V, E)$ is said to be a sum graph if there exists a bijection labeling f from the vertex set V to a set S of positive integers such that $xy \in E$ if and only if $f(x) + f(y) \in S$. The product analogue of sum graphs was first introduced by Thavamani in 2011 [8]. He introduced the edge product graphs and the edge product number of a graph [6], [13]. A graph G is said to be an edge product graph if the edges of G can be labelled with distinct positive integers such that the product of all label of the edges incident on a vertex is again an edge label of G [8] .

The hypergraph is a generalization of a graph in which edges can connect any number of vertices. Hypergraph was introduced by Berge in 1973 [1]-[2]. In 2020 S. Samantha introduced the concept of cluster hypergraphs [8]. An cluster hypergraph was introduced to generalize the concept of hypergraph in which cluster nodes are allowed. Jadhar and Pawar introduced the

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notation of an edge function and using this edge function an edge product hypergraph is defined [9]. In this paper the concept of edge product cluster hypergraph, unit edge product cluster hypergraph have been discussed here.

2 Preliminaries

Definition 2.1 *Let $H = (V_x, E)$ be a simple cluster hypergraph with vertex set $V_x(H)$ and edge set $E(H)$. Let I be the set of positive integers such that $|E| = |I|$. Then any bijection $f : E \to I$ is called an Edge Function of the cluster hyergraph H.*

Definition 2.2 *The function $F(v) = \prod\{f(e); edge\ e\ incident\ to\ the\ vertex\ v\}$ on $V_x(H)$ is called an Edge Product Function of the edge function f.*

Definition 2.3 *The cluster hypergraph $H = (V_x, E)$ is said to be an Edge Product Cluster Hypergraph if there exists an edge function $f : E \to I$ such that the edge function f and the corresponding edge product function F of f on $V_x(H)$ have the following two conditions:*
(i) $F(v) \in I$ for every $v \in V_x(H)$
(ii) if $f(e_1) \times f(e_2) \times \ldots \times f(e_i) \in I$ for some edges $e_1, e_2 \ldots e_i \in E(H)$, then the edges $e_1, e_2 \ldots e_i$ are all incident to some vertex $v \in V_x(H)$.

Example 2.3 Let $H = (V_x, E)$ be a cluster hypergraph, with vertex set $V_x(H) = \{v_1, v_2, v_3 = \{v_1, v_2\}, v_4, v_5, v_6 = \{v_4, v_5\}, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ and edge set $E(H) = \{e_1, e_2, \ldots e_8\}$. Here the edges of H are defined as follows $e_1 = \{v_1, v_3\}$, $e_2 = \{v_2, v_3\}$, $e_3 = \{v_3, v_6\}$, $e_4 = \{v_4, v_6\}$, $e_5 = \{v_5, v_6\}$, $e_6 = \{v_6, v_7, v_8\}$, $e_7 = \{v_6, v_7, v_9\}$, $e_8 = \{v_{10}, v_{11}, v_{12}\}$. Now we define the edge function $f : E \to I$ by $f(e_1) = 15$, $f(e_2) = 8$, $f(e_3) = 20$, $f(e_4) = 2$, $f(e_5) = 3$, $f(e_6) = 4$, $f(e_7) = 5$, $f(e_8) = 2400$. The edge product function F of f is defined by $F(v_1) = 15$, $F(v_2) = 8$, $F(v_3) = 2400$, $F(v_4) = 2$, $F(v_5) = 3$, $F(v_6) = 2400$, $F(v_7) = 20$, $F(v_8) = 4$, $F(v_9) = 5$, $F(v_{10}) = 2400$, $F(v_{11}) = 2400$, $F(v_{12}) = 2400$. Thus the given cluster hypergraph in figure 2 is an edge product cluster hypergraph.

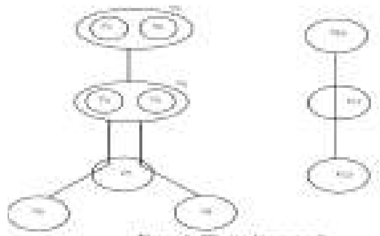

Figure 1: Cluster hypergraph

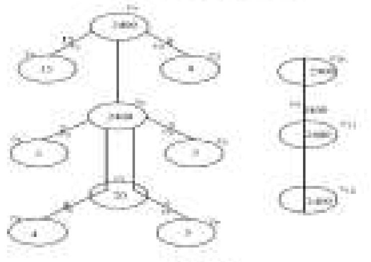

Figure 2.

Definition 2.4 *Let $H = (V_x, E)$ be an edge product cluster hypergraph. Then H is said to be a Unit edge Product cluster hypergraph if there exists an edge function $f : E \to I$ such that $1 \in I$.*

Definition 2.5 *An edge e in a unit edge product cluster hypergraph whose edge function label as the largest element in the set of positive integers I is called a $Maximal$ $Edge$ of that cluster hypergraph.*

Definition 2.6 *Let X be a nonempty set and let V_x be a subset of $P(X)$ such that $\phi \notin V_x$ and $X \subset V_x$. Now F be a multi-set whose element belong to $P(P(X))$ such that*
(i) $E \neq \phi$
(ii) for each element $e \in E$, there exist atleast one element $v \in V_x$ such that $v \in e$.
Then $H = (V_x, E)$ is said to be $Cluster$ $hypergraph$ where V_x is said to be a vertex set and E is said to be multi-hyper edge set[11].

3 Main Results

Theorem 3.1 *Let $H = (V_x, E)$ be a unit edge product cluster hypergraph with a unit edge e. Then every maximal edge in H is adjacent to only e.*
Proof: Let $H = (V_x, E)$ be a unit edge product cluster hypergraph with a unit edge e. Let $f : E \to I$ be an edge function and F be an edge product function of f. Now, consider the unit edge e. ie., $f(e) = 1$. Let m be a maximal edge in H such that $f(m) = i$. To prove m is adjacent only to e. By using theorem 3.3 in [1], e is adjacent to all the edges in H. Therefore it

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is enough to prove that m has e only as its neigbour. Suppose on the contrary m is adjacent to some other edge $h \in E(H)$. Choosing a vertex $v \in m \cap h$ or $v \in m \cap h \cap e$. So we obtained that $F(v) = f(m).f(h) > i$ or $F(v) = f(m).f(h).f(e) > i$. Both the ways, we get a contradiction to that i is the largest element in I. Hence m is the maximal edge in H which is adjacent to only the unit edge e. Hence the theorem.

Definition 3.2 *Let $H = (V_x, E)$ be a cluster hypergraph. Let $v \in V$. The edge degree of v is the number of edges containing the vertex v and is denoted by $d_E(v)$.*
The maximum edge degree of H is denoted by $\Delta_E(H)$
The minimum edge degree of H is denoted by $\delta_E(H)$
A vertex of edge degree one is called a pendant vertex or end vertex.

Theorem 3.3 *Let $H = (V_x, E)$ be a unit edge product cluster hypergraph with a unit edge e. If $v \in V$ be a vertex of maximum edge degree, then $v \in e$.*
Proof: Let $H = (V_x, E)$ be a unit edge product cluster hypergraph with a unit edge e. Let $f : E \to I$ be an edge function and F be an edge product function of f. Let v be the vertex in unit edge product cluster hypergraph with maximum edge degree d (say), where d is a positive integer. To prove $v \in e$. On the contrary, suppose $v \notin e$. Since $\Delta_E(H) = d$. Consider $e_1, e_2, \ldots, e_d$ be the edges incident to v. Then $F(v) = f(e_1) \times f(e_2) \times \ldots \times f(e_d) \in I$. But it is clear that $F(v) = F(v).1$. Therefore it follows that $F(v) = f(e_1) \times f(e_2) \times \ldots \times f(e_d) \times f(e) \in I$. This shows that the edges $e_1, e_2, \ldots, e_d, e$ is incident with v and so the edge degree of v is $d + 1 > \Delta_E(H)$, which is a contradiction. Hence $v \in e$.

Theorem 3.4 *Let $H = (V_x, E)$ be a cluster hypergraph. Let e be any edge of H with $f(e) = 1$. Let $e_i \neq e$ for any other edge in H, there is exactly one vertex $v_i \in e_i \cap e$ for $1 \leq i \leq m-1$ and $f \cap g = \phi$ for every distinct edges f and g connecting the maximal vertices (except edge e). Then H is a unit edge product cluster hypergraph.*
Proof: Let $H = (V_x, E)$ be a cluster hypergraph which satisfies the hypothesis of the theorem. Let $e, e_1, e_2, \ldots, e_{m-1}$ be the edges of H. Let $v_1, v_2, \ldots, v_{m-1}$ be the set of vertices incident with e and $e_i \cap e_j \neq \phi$ for all distinct edges e_i and e_j which connects the maximal vertices. Clearly, for every edge e_i $(1 \leq i \leq m-1)$ has only one distinct vertex $v_i \in e_i \cap e$. Let $l_1, l_2, \ldots, l_t$ be the other vertices in e for which $l_1, l_2, \ldots, l_t \notin e_i$, $1 \leq i \leq m-1$. This implies that $l_1, l_2, \ldots, l_t$ are end vertices in H.
*For $1 \leq i \leq m-1$, let e_i be the edges with vertices $u_1^i, u_2^i, \ldots, u_{q_i}^i$ in H. Here $q_1, q_2, \ldots, q_{m-1}$ are the non-negative integers which represents the number of members in $e_1, e_2, \ldots, e_{m-1}$ respectively. Therefore $V(H) = \{v_1, v_2, \ldots, v_{m-1},$
$u_1^1, u_2^1, \ldots, u_{q_1}^1, u_1^2, u_2^2, \ldots, u_{q_2}^2, \ldots, u_1^{m-1}, u_2^{m-1}, \ldots, u_{q_{m-1}}^{m-1}, l_1, l_2, \ldots, l_t\}$ and
$E(H) = \{e, e_1, e_2, \ldots, e_{m-1}\}$ where $e = \{v_1, v_2, \ldots, v_{m-1}, l_1, l_2, \ldots, l_t\}$ and $e_i = \{u_1^i, u_2^i, \ldots, u_{q_i}^i\}$ for $1 \leq i \leq m-1$ are the vertex set and edge set of the given cluster hypergraph respectively.*

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Now, define the edge product function for this cluster hypergraph H. Consider the set of positive integers I as $I = \{1, p_1, p_2, \ldots, p_{m-1}\}$ where p_i are the prime number which are taken in the increasing order.Define the edge function $f : E \to I$ by $f(e) = 1$, $f(e_i) = p_i$ for $1 \le i \le m-1$. So the edge product function of $F$of edge function f is defined by

$F(v_i) = p_i$ *for* $1 \le i \le m-1$

$F(l_j) = 1$ *for* $1 \le j \le t$

$F(u^1_1) = F(u^1_2) = F(u^1_3) = \ldots = F(u^1_{q_1}) = p_1$

$F(u^2_1) = F(u^2_2) = F(u^2_3) = \ldots = F(u^2_{q_2}) = p_2$

$\vdots$

$F(u^{m-1}_1) = F(u^{m-1}_2) = F(u^{m-1}_3) = \ldots = F(u^{m-1}_{q_{m-1}}) = p_{m-1}$

It is easily verified that for every vertex $x \in V_x(H)$, we have $F(x) \in I$. Also if the product of a collection more than one member of I is in I then the collection consists of exactly two members. That is 1 and p_i. This happens, the edges e and e_i is incident to a vertex $v_i \in V_x(H)$. Therefore it follows that H is a unit edge product cluster hypergraph.

Remark 3.5 *If the cluster hypergraph $H = (V_x, E)$ satisifies the hypothesis of the above theorem, then every cluster nodes contains exactly one simple node.*

Theorem 3.6 *Let $H = (V_x, E)$ be an edge product cluster hypergraph. Let x be a non-pendant vertex such that the edges $e_1, e_2, \ldots, e_i$ are incident to x and $e \in E$ with $F(x) = F(e)$. If e is adjacent to any edge $m \neq e_1, e_2, \ldots, e_i$ in H, then m must be adjacent to the edges $e_1, e_2, \ldots, e_i$ in H.*

Proof: Let $H = (V_x, E)$ be an edge product cluster hypergraph. Let $f : E \to I$ be an edge function and F be an edge product function of f. Let x be a non-pendant vertex and $e \in E$ with $F(x) = F(e)$. Let $e_1, e_2, \ldots, e_i$ be the edges incident with x. Since x is non-pendant implies that $i \ge 2$. Therefore, $F(x) = f(e_1) \times f(e_2) \times \ldots \times f(e_i) \in I$. Let m be any edge other than $e_1, e_2, \ldots, e_i$ which is adjacent to e. This shows that, there exist a vertex v such that $v \in e \cap m$ and so $F(v) = f(e) \times f(m) \in I$. It follows that $F(v) = f(e_1) \times f(e_2) \times \ldots \times f(e_i) \times f(m) \in I$. Hence m must be adjacent to th edges $e_1, e_2, \ldots, e_i$ in H.

Remark 3.7 *Let $H = (V_x, E)$ be any cluster hypergraph which satisfies the hypothesis of the above theorem. If u be a vertex of H incident with the edges $e_1, e_2, \ldots, e_i, m$ then $\delta_E(H) \ge 3$.*

4 Conclusion

The edge product of cluster hypergarph and some theorems related to this concept have been discussed here. In future studies, the edge product number can be determined for cluster hypergraphs.

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The Upper Edge-to-Edge Steiner Number of a Graph

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Abstract

For a nonempty subset $W \subseteq E(G)$, the edge-to-vertex Steiner distance d_{ev} (W) of W is the smallest size of a tree containing V(W). The smallest size of a tree containing V(W) is called a Steiner W_{ev} - tree of G. Every edge of a Steiner W_{ev}-tree of G must be present for a set $W \subseteq E$ to be referred to as an edge-to-edge Steiner set. The edge-to-edge Steiner number s_{ee} (G) of G is the least cardinality of its edge-to-edge Steiner sets. The Upper edge-to-edge Steiner set of cardinality $s_{ee}(G)$ is referred to as a minimum edge-to-edge Steiner number of G is the maximum cardinality of a minimal edge-edge Steiner set of G. It is denoted by $s^+_{ee}(G)$. Certain kinds of graphs have their upper edge-to-edge Steiner numbers determined. This concept's general qualities are investigated. It is shown that for any positive integers a and b with $2 \leq a \leq b$, there exists a connected graph G such that $s_{ee}(G) = a$ and $s^+_{ee}(G) = b$.

Key words:: Steiner distance, Steiner number, edge-to-vertex Steiner distance, edge-to-vertex Steiner set, edge-to-edge Steiner set.
AMS classification: 05C12

1 Introduction

Let $G = (V, E)$ be a graph having a vertex set V(G) and an edge set E(G). In addition, we state that a graph G has size $m = \|E(G)\|$ and order $n = \|V(G)\|$. We refer to [2] for the fundamental terms used in graph theory. If and only if an edge $e = uv \in E(G)$ exists, a vertex v is next to a vertex u. If $uv \in E(G)$, then u is a neighbor of v, and the set of neighbors of v is denoted by $N_G(v)$.The formula for a vertex's degree is $deg_G(v) = |N_G(v)|$. If $deg_G(v) = n - 1$, a vertex is said to be a universal vertex. If the subgraph induced by vertex v is complete, then vertex v is said to be an extreme vertex. The length of the shortest $u - v$ path in a connected graph G is equal to the distance $d(u, v)$ between two vertices, denoted as u and v. For a nonempty set W of vertices in a connected graph G, the Steiner distance d(W) of W is the minimum size of a connected subgraph of G containing W. The Steiner distance for [17] was examined. Let S(W) be the collection of all Steiner W-tree vertices. A set $W \subseteq V(G)$ is referred to as a Steiner

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set of G if $S(W) = V(G)$. The lowest cardinality for a Steiner set, commonly referred to as a minimum Steiner set or simply an s-set, is the Steiner number s(G) of G. The Steiner number s(G) of G determines the minimal cardinality of a Steiner set, which is also known as an s-set. The Steiner number was introduce in [5] and further studied in [5,6,9-14,19-22].

Definition 1.1 [15] *A connected graph with at least three vertices should be $G = (V, E)$. If every edge of G is lies on a Steiner $W_{ev} - tree$ of G, a set $W \subseteq E$ is referred to as an edge-to-edge Steiner set. The edge-to-edge Steiner number $s_{ee}(G)$ of G is the minimum cardinality of its edge-to-edge Steiner sets. Any edge-to-edge Steiner set with cardinality $s_{ee}(G)$ is a minimum edge-to-edge Steiner set of G or s_{ee}-set of G.*

Example 1.2 *Let $W_1 = v_1v_6, v_2v_5, v_3v_4$ for the graph G shown in Figure 1.1. W is an edge-to-edge Steiner set of G since each edge of G is contained in one of the two Steiner W_{ev}-trees, and as a result, $s_{ee}(G) \leq 3$. No edge-to-edge Steiner set of G is a two elements subset of E, hence $s_{ee}(G) = 3$.*

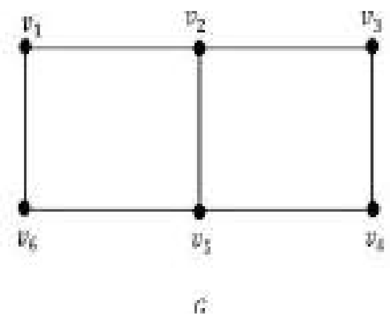

Figure 1: 1

Remark 1.3 [11] *If G is adjacent vertices $e = uv$ and $f = vw$ are, then the Steiner $W_{ev} - tree$ is a path between u,v and w. It should be emphasised that the Steiner $W_{ev} - tree$ only contains the elements of V(W) if $W \subseteq E$ such that W is connected.*

Corollary 1.4 [15] *Each end vertex of G belongs to every edge-to-edge Steiner set of G.*

Theorem 1.5 [15] *Every edge-to-edge Steiner set of a connected graph of G is an edge-to-vertex Steiner set of G.*

2 The Upper Edge-to-Edge Steiner Number of a Graph

Definition 2.1 *A minimum edge-to-edge Steiner set of G is an edge-to-edge Steiner set W in a connected graph G where no appropriate subset of W is an edge-to-edge Steiner set of G. The*

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highest cardinality of a minimal edge-to-edge Steiner set of G is known as the upper edge-to-edge Steiner number of G, it is denoted by $s^+_{ee}(G)$.

Example 2.2 *Consider the graph G shown in Figure 1.1. The only two minimal edge-to-edge Steiner sets of G are* $W_1 = \{v_1v_6, v_2v_5, v_3v_4\}$*and* $W_2 = \{v_1v_2, v_2v_3, v_4v_5, v_5v_6\}$ *and so that* $s^+_{ee}(G) = 4$.

Remark 2.3 *Every minimal edge-to-edge Steiner set of G is a minimal edge-to-edge Steiner set of G and the converse need not true.* $W_2 = v_1v_2, v_2v_3, v_4v_5, v_5v_6$ *is a minimal edge-to-edge Steiner set for the graph G shown in Figure 1.1, but it is not a minimal edge-to-edge Steiner set for G.*

observation 2.4 *1. In a connected graph G, each end-edge is a member of every minimum edge-to-edge G Steiner set.*

1. Let G be a connected graph and let W be the minimal edge-to-edge Steiner set of G. Therefore, W is not the owner of any cut edge of G that are not G end edges.

Theorem 2.5 ***For a connected graph G size*** $n \geq 2$,$2 \leq s_{ee}$ ***(G)*** $\leq s^+_{ee}(G) \leq n$.
Proof: There must be at least two edges in any edge-to-edge Steiner set, so $s_{ee}(G) \geq 2. s_{ee}(G) \leq^+_{ee} (G)$ *because every minimal edge-to-edge Steiner set is a minimal edge-to-edge Steiner set. The fact that E(G) is an edge-to-edge Steiner set of G also makes it obvious that* $s^+_{ee}(G) \leq n$. *Consequently,* $2 \leq s_{ee}(G) \leq s^+_{ee}(G) \leq n$.

Remark 2.6 *In Theorem 2.5, there may be a sharp bound. For the path of length* $n \geq 3, s_{ee}(G) = 2$. *For the star* $G = K_{(1,n)}, s^+_{ee}(G) = n$, *and for the double star G,*$s_{ee}(G) = s^+_{ee}(G)$. *Additionally, Theorem 2.5 inequalities are all strict. For the graph G given in Figure 1.1,* $s_{ee}(G) = 3, s^+_{ee}(G) = 4$*and n=7. Thus* $2 < s_{ee}(G) < s^+_{ee}(G) < n$.

Theorem 2.7 *Consider the connected graph G,* $s_{ee}(G) = n$ *if and only if* $s^+_{ee}(G) = n$.
Proof: *Suppose* $s_{ee}(G) = n$. *Then the unique minimal edge-to-edge Steiner set of G is* $W = E(G)$. *It is obvious that W is the one and only minimum edge-to-edge Steiner set of G because no proper subset of W is an edge-to-edge Steiner set of G, and as a result,* $s^+_{ee}(G) = n$. *Theorem 2.5 leads to the Converse.*

Theorem 2.8 *G be a connected graph of size* $n \geq 5$, *which is not a star. Then* $s^+_{ee}(G) \leq n - 2$.
proof:*Assume that* $s^+_{ee}(G) \geq n - 1$. *So, according to Theorem 2.7,* $s^+_{ee}(G) = n - 1$. *Let* $W = E(G) - \{e\}$ *be the smallest edge-to-edge Steiner set of G. Let e be an edge of G that is not an*

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end edge of G. e is connected to $[E(G)e]$ since it is not a cut edge of G. Let f be an edge of $[E(G)e]$ that is not an end edge of G and is independent of e (this is possible since $n \geq 5$). An edge-to-edge Steiner set of G is then $W_1 = W\{f\}$. W is not a minimal edge-to-edge Steiner set of G since $W_1 \subseteq W$, which is a contradiction. Consequently, $s^+_{ee}(G) \leq n-2$.

Remark 2.9 *There can be sharp bounds in Theorem 2.8. For the cycle $G = C_4, s_{ee}(G) = 2 = n-2$.*

Theorem 2.10 *For a connected graph of G with $n \geq 4, s_{ee}(G) = n-1$ if and only if $s^+_{ee}(G) = n-1$.*
proof: *Let $s_{ee}(G) = n-1$. Then it follows from Theorem 2.5, $s^+_{ee}(G) = n$ or $n-1$. According to Theorem 2.6, $s^+_{ee}(G) = n$, which is a contradiction, if $s^+_{ee}(G) = n$. Therefore, $s^+_{ee}(G) = n-1$. Conversely, if we assume that $s^+_{ee}(G) = n-1$, it follows from Theorem 2.8 that G is not a star. Thus, according to Theorem 2.8, G has a cut-edge, say e. It follows that $W = E(G) - e$ is the only minimal edge-to-edge Steiner set of G since $s^+_{ee}(G) = n-1$. In our assertion, $s_{ee}(G) = n-1$. Assume that $s_{ee}(G) = n-1$. Then there exists a Steiner set W_1that is edge-to-edge minimal and such that $|W_1| < n-1$. If $e \notin W_1$, then it follows that $W_1 \subset W$, which is a contradiction. Consequently, $s^+_{ee}(G) \leq n-1$.*

Theorem 2.11 *For a complete graph of K_m with $m \geq 4$,$s^+_{ee}(K_m) = m-1$.*
Proof: *Let W be any collection of K_m adjacent edges that are incident at a vertex, say x. It is obvious that W is an edge-to-edge Steiner set of G because every vertex of K_m is on the Steiner W_{ev}-tree of G. There exists a proper subset of W' such that W' is an upper edge-to-edge Steiner set of G if W is not a minimal edge-to-edge Steiner set of G. At least one vertex, say y of K_m, exists where y is not incident with any of the edges of W'. This results in a contradiction because y does not occur with any Steiner $W^{'}_{ev}$ -tree of G. Therefore, W is an edge-to-edge Steiner minimum set of G. Consequently, $S^+_{ee}(K_m) \geq m-1$. Consider the case where $|S| \geq m$and there exists a minimal edge-to-edge set of S. There is at least one cycle in S because it has at least m edges. Let $S^{'} = S - \{e\}$, be the edge of a cycle which lies [S] . It is obvious that S' is an edge-to-edge Steiner set with $S^{'} \in S$, which is a contradiction . Consequently,$s^+_{ee}(K_m) = m-1$.*

Theorem 2.12 *For the complete bipartite graph $G = K_{r,s}(2 \leq r \leq s), s^+_{ee}(G) = r+s-2$.*
Proof. *The bipartite sets of G are $U = \{u_1, u_2, u_3, u_4, \cdots, u_r\}$ and $V = \{v_1, v_2, v_3, u_4, \cdots, v_s\}$. Let $W_i = \{u_iv_1, u_iv_2, \cdots, u_iv_(s-1), u_2v_s, \cdots, u_(i-1)v_s, u_(i+1)v_s, \cdots, u_rv_s\}, (1 \leq i \leq r)$,$S_j = \{u_1v_j, u_2v_j, \cdots, u_(r-1)v_j, u_rv_1, u_rv_2, \cdots, u_rv_(i-1), u_rv_(i+1), \cdots, u_rv_s\}, (1 \leq j \leq s)$ and $W_k = \{u_1v_1, u_2v_(2), \cdots, u_{(r-1)}v_{(r-1)}, u_rv_r, u_rv_{(mr+1)}, \cdots, u_rv_s\}$with $|W_i| = |S_j| = r+s-2$ and $|W_k| = s$. Any smallest edge-to-edge Steiner set of G is easily verifiable to be of the type W_i or S_j or W_k. It follows that $s^+_{ee}(G) = r+s-2$ since no proper subset of $W_i(1 \leq i \leq r), S_j(1 \leq j \leq s)$, and W_k is an edge-to-edge Steiner set of G.*

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Theorem 2.13 *For any non-trivial tree T with k end vertices, $s^+_{ee}(G) = k$.*
Proof: *This follows from Observation 2.4. In viva, Theorem 2.5. We have the realisation result listed below.*

Theorem 2.14 *There is a connected graph G there exists for every pair of positive integers a and b with $2 \leq a \leq b$ the property that $s_{ee}(G) = a$ and $s^+_{ee}(G) = b$.*
Proof: *Let $G = T$ if $a = b$. Consequently, $s_{ee}(G) = s^+_{ee}(G) = a$. Let $a < b$. Let T : $u_0, u_1, u_2, u_3, u_4, u_5$ be the six-vertex path. Let $P_i : x_i, y_i(1 \leq i \leq b - a + 1)$ be represent a path on two vertices. Let H be the graph created by adding each $x_i(1 \leq i \leq b - a + 1)$ to u_0 and each $y_i(1 \leq i \leq b - a + 1)$ to u_5 from P and P_i. Figure 2.2 displays the graph G. The collection of end edges in G is given by $Z = \{zz_1, zz_2, , zz_{(a-2)}\}$. Z is a subset of each edge-to-edge Steiner set of G. It is clear that, Z is not an edge-to-edge Steiner set of G. Additionally, it is established that $Z \cup e, e \notin Z$ is not an edge-to-edge Steiner set of G and that, thus, $s_{ee}(G) \geq a$. Let $Z' = Z \cup \{u_0, u_1, u_4, u_5\}$. So that $s_{ee}(G) = a$, Z' is an edge-to-edge Steiner set of G.*

We then demonstrate that $s^+_{ee}(G) = b$. Now $W = Z \cup \{u_2u_3, x_1y_1, x_2y_2, \cdots, x_{(b-a+1)}y_{(b-a+1)}\}$ is an edge-to-edge Steiner set of G. We demonstrate that W is an minimum edge-to-edge Steiner set of G. Let W' represent any proper subset of W. Then at least one edge, say $f \in W$, such that $f \notin W'$. If $f \notin zz_i(1 \leq i \leq a - 2)$ is determine. There is a contradiction if $f = x_iy_i(1 \leq i \leq b - a + 1)$, since $x_iy_i(1 \leq i \leq b - a + 1)$ does not lie on a Steiner W_{ev}-tree of G. Therefore W is a minimum edge-to-edge Steiner set of G, and as a result, $s^+_{ee}(G) \leq b$. We demonstrate that $s^+_{ee}(G) = b$. Suppose that $s^+_{ee}(G) \geq b + 1$. Then, there exists a minimum edge-to-edge Steiner set W' and such that $|W'| \geq b + 1$. Then W' being a minimum edge-to-edge Steiner set of G is then clearly demonstrated. Hence, $s^+_{ee}(G) = b$.

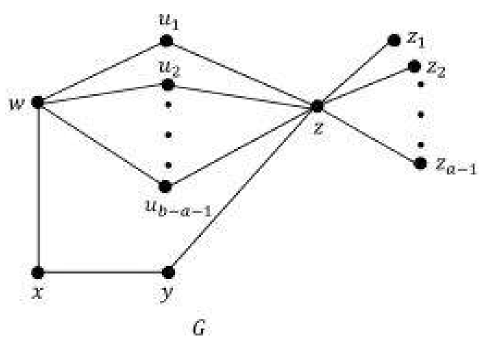

Figure 2: G

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3 Conclusion

In this article we studied the upper edge-to-edge Steiner number of a Graph and generalized same properties of some standard graphs we will develop this concept with other distance related parameters of graphs in the further studies.

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Enhancing Online Food Recommendations Through Singular Value Decomposition

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Abstract

With the exponential growth of online food platforms, the need for effective food recommendation systems has become imperative to enhance user experience and satisfaction. This paper explores the application of Singular Value Decomposition (SVD) as a powerful technique for personalized food recommendations. SVD, a matrix factorization method, is employed to analyse user-item interaction matrices, extracting latent features that capture user preferences and item characteristics. The motivation behind this research stems from the evolving landscape of online user behaviour, where individuals seek tailored and relevant suggestions in the vast array of available food options. Traditional recommendation systems often face challenges in providing accurate and personalized suggestions due to the complexity of user preferences. By utilising SVD, this study aims to address these challenges and improve recommendation accuracy by uncovering latent patterns within user-item interactions. Through a comprehensive analysis of SVD-based food recommendation systems, this research emphasizes the significance of personalized recommendations in enhancing user engagement and satisfaction. The proposed approach aims to contribute to the evolution of online food platforms, providing users with a more enjoyable and personalized culinary experience while fostering increased loyalty and user retention.

Key words:: SVD (Singular Value Decomposition)
AMS classification: 05C15, 05C69

1 Introduction

In the dynamic landscape of modern living, online food delivery applications have become an integral part of our daily routines, significantly impacting the way we experience and indulge in culinary delights. As the availability of diverse food options continues to burgeon, users face the challenge of navigating through extensive menus to discover dishes that resonate with their preferences. This paper explores the application of Singular Value Decomposition (SVD) to revolutionize the food recommendation system within online delivery platforms. By leveraging the power of SVD, we aim to provide users with tailored and enticing food suggestions, navigating the vast array of options a vailable. This study delves into the intersection of technology and gastronomy, addressing the growing significance of food delivery apps in shaping contemporary dining experiences.

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2 Singular Value Decomposition

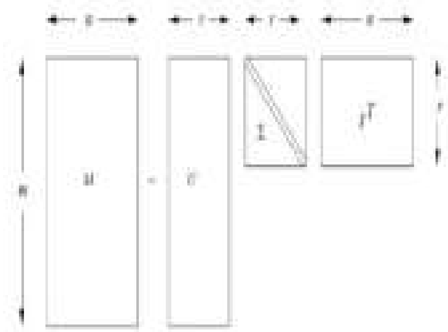
Let M be an $m \times n$ matrix, and let r be the rank of M. Then there exists a matrix factorization called Singular value Decomposition (SVD)1 of M with the form $U \sum V^T$, where

1. U is an column-orthonormal matrix; That is, each of its columns is a unit vector and the dot product of any two columns is 0.

2. $\sum$ is a diagonal matrix where the diagonal entries are called the singular values of M.

3. V^T is an $r \times n$ row-orthonormal matrix; That is, each of its rows is a unit vector and the dot product of any two columns is 0.

   The SVD of a matrix M has strong connections to the eigenvectors of the matrix $M^T M$



and MM^T

3 Enhancing Online Food Recommendations

After completing their orders on the online food delivery app, users have the opportunity to provide ratings based on their dining experiences. These user ratings play a pivotal role in assessing the quality and impact of the digital culinary offerings. To enhance the feedback process and gain more specific insights, users are categorized into distinct groups: North Indian and South Indian dishes. This categorization ensures that the assessments are tailored to the unique preferences and expectations of users from these two regional cuisines. By collecting ratings from both categories, we can use Singular Value Decomposition (SVD) to find suggestions for new users or others with similar taste preferences. Consider the data collected from users who mainly fall under

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these two categories: preference for North Indian and South Indian dishes. Five users from each category are chosen, and five dishes, where dishes 1, 2, and 3 belong to the North Indian category, and dishes 4 and 5 belong to the South Indian category. The ratings given by users under two main categories as North Indian and South Indian food category as follow:

User	Food 1	Food 2	Food 3	Food 4	Food 5
A	4	4	4	0	0
B	5	5	5	0	0
C	0	0	0	1	1
D	0	0	0	2	2
E	0	0	0	3	3

Where the first two A and B users are representing North Indian category and next three users C, D, E represent South Indian category. Now we construct a matrix using the ratings given by the user as follow:

$$A = \begin{bmatrix} 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}$$

We consider the above rating matrix as A. the first step is to decompose the matrix A using singular value decomposition. the SVD can be done accurately and quickly using the python software. Thus, a matrix A is decomposed into $U \sum V^T$.

$$\begin{bmatrix} 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix} =$$

$$\begin{pmatrix} 0.62 & 0 & -0.78 & 0 & 0 \\ 0.78 & 0 & 0.62 & 0 & 0 \\ 0 & 0.27 & 0 & -0.89 & -0.36 \\ 0 & 0.53 & 0 & 0.45 & -0.72 \\ 0 & 0.8 & 0 & 0 & 0.59 \end{pmatrix} \begin{pmatrix} 11.09 & 0 & 0 & 0 & 0 \\ 0 & 5.29 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.7 \\ -0.7 & 0.7 & 0 & 0 & 0 \\ -0.4 & -0.4 & 0.81 & 0 & 0 \\ 0 & 0.8 & 0 & -0.7 & 0.7 \end{pmatrix}$$

Neglecting the smaller eigen values by dimensionality reduction we get,

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$$\approx \begin{bmatrix} 0.62 & 0 \\ 0.78 & 0 \\ 0 & 0.27 \\ 0-0.53 & \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 11.09 & 0 \\ 0 & 5.29 \end{bmatrix} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.7 \end{bmatrix}$$

Where, U= $\begin{bmatrix} 0.62 & 0 \\ 0.78 & 0 \\ 0 & 0.27 \\ 0-0.53 & \\ 0 & 0.8 \end{bmatrix}$, $\sum = \begin{bmatrix} 11.09 & 0 \\ 0 & 5.29 \end{bmatrix}$ $V^T = \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.7 \end{bmatrix}$

U-Connects user to foods.

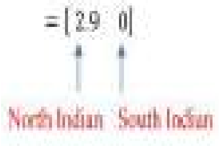
$\sum$-Each diagonal entry represents the strength of each category.

V^T-Connects category to foods.

Here the strength of North Indian Category is higher than South Indian since the data provides more information about the South Indian Category. By using the constructed SVD we can find a suggestion list for new user having similar interest of foods from the datas which are already collected and calculated. Suppose there is a new user H who only read Food 3. We could recommend foods to her in following ways: If H be choice of new users ratings to find the suggestion list for new user, we have to find the inner product of the new user ratings and the transpose of the matrix V that is H.V.This inner product of gives the category of foods which should be suggested for the new user.

We can represent the rating given by user H as $H = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \end{bmatrix}$

Now $H.V = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.7 \\ 0 & 0.7 \end{bmatrix}$ From this it is clear that user H is interested in



North Indian related foods, but not in South Indian foods. we now have a representation of user H in North Indian Category. We can also map user C back to foods by computing $[2.90]V^T$ the matrix V^T represent the category each food comes under. if we compute

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$[2.90]V^T$ we get the list of foods that is to be suggested for user C.
Let the rating of new user I be represented as I=[0 4 5 0 0]Then

$$I.V=\begin{bmatrix}0 & 4 & 5 & 0 & 0\end{bmatrix}\begin{bmatrix}0.58 & 0\\0.58 & 0\\0.58 & 0\\0 & 0.7\\0 & 0.7\end{bmatrix}$$

User I is also interested in North Indian related foods. we now have a representation of the user I in North Indian category. we can also map user I back as follow:

$$\begin{bmatrix}5.22 & 0\end{bmatrix} V^T = \begin{bmatrix}5.22 & 0\end{bmatrix}\begin{bmatrix}0.58 & 0.58 & 0.58 & 0 & \\0 & 0 & 0 & 0.7 & 0.7\end{bmatrix}$$

$$= \begin{bmatrix}3.03 & 3.03 & 3.03 & 0 & 0\end{bmatrix}$$

This indicates User I would like dish1, dish 2 and dish 3 but not dish 4 and dish 5. In similar way the SVD can computed for m number of dishes and n number of users (for larger value of m and n) and the user can be categorised accordingly and required suggestion for the food can be made. Further this recommendation system for foods to different category of users in online food delivery applications can be completely automated using programming by Python software.

4 Conclusion

In conclusion, the implementation of a recommendation system for food using Singular Value Decomposition holds great promise in elevating the user experience on online food delivery applications. Harnessing the capabilities of SVD enables us to offer diners personalized and relevant food recommendations, ensuring they serve a delightful culinary experience in todays diverse gastronomic landscape. With the exponential growth in the availability of North Indian and South Indian dishes on these platforms, navigating the extensive menu options has become a challenge for users. A food recommendation system proves to be a valuable tool, guiding users to explore and relish new dishes aligned with their taste preferences and culinary interests.

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Utilizing Partial Differential Equations For Enhanced Brain Tumour Detection In Human And Animal Subjects

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Abstract

Recent advancements in human-computer interaction have emphasized the development of logical communication channels between humans and computers. Vision-based interface technologies have emerged as a promising avenue, enabling the extraction of nuanced information from input images without the need for expensive hardware. This approach holds significant potential for enhancing interaction systems. This paper describes image processing using anisotropic diffusion and by applying partial differential equations the similarity between human and animal brain tumour and the type of tumour in the image detected. Utilizing the capabilities of the MATLAB toolbox and its associated functions, we can efficiently and accurately process and analyze images in real-time.
Key words:: Anisotropic diffusion, Matrix Laboratory, Medical imaging techniques.
AMS classification: 35Q68

1 Introduction

Software for high-performance numerical computing and visualization is available under the name MATLAB. With hundreds of built-in functions for technical computation, graphics, and animation, it offers an interactive environment. The best part is that it also offers simple extension using a high-level programming language of its own. MATrix LABoratory is the meaning behind the term MATLAB. Anisotropic diffusion is often described using a partial differential equation that models the behavior of the image over time. The most commonly used PDE in anisotropic diffusion is the Perona-Malik equation

$$\partial I \partial t = \nabla \cdot (c\,(||\nabla I||)\,\nabla I) \tag{1}$$

Where
I - is the image intensity,
t - is time,
∇ - is the gradient operator,
$C\,(||\nabla I||)$ - is the diffusion coefficient that varies based on the gradient magnitude $||\nabla I||$,
. - represents the dot products and
∇I . - Represents the divergence

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2 Brain Tumour

A brain tumour is a mass or growth of abnormal cells in the brain, which are what make up a human body. For medical and scientific research, the interior of the human body is visualized using medical imaging techniques, and this technology can also be used to identify non-invasive conditions. The numerous medical imaging technologies, such as MRI, Ultrasound, CT scan, SPECT, PET, and X-ray, are based on non-invasive methods. MRI pictures can be used to detect brain tumours. The detection of brain cancers using image processing methods is the main topic of this research.

2.1 INPUT FOR MATLAB

ANISOTROPIC DIFFUSION

```
function diff_im = anisodiff(im, num_iter, delta_t, kappa, option)
fprintf('Removing noise\n');
fprintf('Filtering Completed !!');
% Convert input image to double.
im = double(im);
% PDE (partial differential equation) initial condition.
diff_im = im;
% Center pixel distances.
dx = 1;
dy = 1;
dd = sqrt(2);
% 2D convolution masks - finite difference

hN = [0 1 0; 0 -1 0; 0 0 0];
hS = [0 0 0; 0 -1 0; 0 1 0];
hE = [0 0 0; 0 -1 1; 0 0 0];
hW = [0 0 0; 1 -1 0; 0 0 0];
hNE = [0 0 1; 0 -1 0; 0 0 0];
hSE = [0 0 0; 0 -1 0; 0 0 1];
hSW = [0 0 0; 0 -1 0; 1 0 0];
hNW = [1 0 0; 0 -1 0; 0 0 0];
% Anisotropic diffusion.
for t = 1:num_iter

  % Finite differences. [imfilter(.,.,'conv') can be replaced by conv2(.,.,'same')]
```

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```
  nablaN = imfilter(diff_im,hN,'conv');
  nablaS = imfilter(diff_im,hS,'conv');
  nablaW = imfilter(diff_im,hW,'conv');
  nablaE = imfilter(diff_im,hE,'conv');
  nablaNE = imfilter(diff_im,hNE,'conv');
  nablaSE = imfilter(diff_im,hSE,'conv');
  nablaSW = imfilter(diff_im,hSW,'conv');
  nablaNW = imfilter(diff_im,hNW,'conv');

  % Diffusion function.
  if option == 1
    cN = exp(-(nablaN/kappa).^2);
    cS = exp(-(nablaS/kappa).^2);
    cW = exp(-(nablaW/kappa).^2);
    cE = exp(-(nablaE/kappa).^2);
    cNE = exp(-(nablaNE/kappa).^2);
    cSE = exp(-(nablaSE/kappa).^2);
    cSW = exp(-(nablaSW/kappa).^2);
    cNW = exp(-(nablaNW/kappa).^2);

  elseif option == 2
    cN = 1./(1 + (nablaN/kappa).^2);
    cS = 1./(1 + (nablaS/kappa).^2);
    cW = 1./(1 + (nablaW/kappa).^2);
    cE = 1./(1 + (nablaE/kappa).^2);
    cNE = 1./(1 + (nablaNE/kappa).^2);
    cSE = 1./(1 + (nablaSE/kappa).^2);
    cSW = 1./(1 + (nablaSW/kappa).^2);
    cNW = 1./(1 + (nablaNW/kappa).^2);
 end


% Discrete PDE solution.
 diff_im = diff_im + ...
     delta_t*(...
     (1/(dy^2))*cN.*nablaN + (1/(dy^2))*cS.*nablaS + ... (1/(dx^2))*cW.*nablaW + (
  End
```

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3 For Brain Tumour

```
clc
close all
clear all
%% Input
[I,path]=uigetfile('*.jpg','select a input image');
str=strcat(path,I);
s=imread(str);
figure;
imshow(s);
title('Input image','FontSize',20);
%% Filter num_iter = 10;
   delta_t = 1/7;
   kappa = 15;
   option = 2;

   disp('Preprocessing image please wait . . .');
   inp = anisodiff(s,num_iter,delta_t,kappa,option);
   inp = uint8(inp);
   inp=imresize(inp,[256,256]);
if size(inp,3)>1
   inp=rgb2gray(inp);
end
figure;
imshow(inp);
title('Filtered image','FontSize',20);
%% thresholding
sout=imresize(inp,[256,256]);
t0=mean(s(:));
th=t0+((max(inp(:))+min(inp(:)))./2);
for i=1:1:size(inp,1)
  for j=1:1:size(inp,2)
    if inp(i,j)>th
      sout(i,j)=1;
    else
      sout(i,j)=0;
```

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```
    end
  end
end
%% Morphological Operation
label=bwlabel(sout);

stats=regionprops(logical(sout),'Solidity','Area','BoundingBox');

density=[stats.Solidity];

area=[stats.Area]; high_dense_area=density>0.7;
max_area=max(area(high_dense_area));

tumor_label=find(area==max_area); tumor=ismember(label,tumor_label);
if max_area>200
  figure;
  imshow(tumor)
  title('tumor alone','FontSize',20);
else
  h = msgbox('No Tumor!!','status');
  %disp('no tumor'); return;
end

%% Bounding box
box = stats(tumor_label);
wantedBox = box.BoundingBox;
figure
imshow(inp);
title('Bounding Box','FontSize',20);
hold on;
rectangle('Position',wantedBox,'EdgeColor','y');
hold off;
%% Getting Tumor Outline - image filling, eroding, subtracting
% erosion the walls by a few pixels
dilationAmount = 5;
rad = floor(dilationAmount);
[r,c] = size(tumor);
```

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```
filledImage = imfill(tumor, 'holes');
for i=1:r
 for j=1:c
   x1=i-rad;
   x2=i+rad;

   y1=j-rad;
   y2=j+rad;
   if x1<1
     x1=1;
   end



   if x2>r
     x2=r;
   end
   if y1<1
     y1=1;
   end
   if y2>c
     y2=c;
   end
   erodedImage(i,j) = min(min(filledImage(x1:x2,y1:y2)));
  end
end

figure
imshow(erodedImage);
title('eroded image','FontSize',20);
%% subtracting eroded image from original BW image tumorOutline=tumor;
tumorOutline(erodedImage)=0;
figure;
imshow(tumorOutline);
title('Tumor Outline','FontSize',20);
%% Inserting the outline in filtered image in red color
rgb = inp(:,:,[1 1 1]);
red = rgb(:,:,1);
```

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```
red(tumorOutline)=255;
green = rgb(:,:,2);
green(tumorOutline)=0;
blue = rgb(:,:,3);
blue(tumorOutline)=0;

tumorOutlineInserted(:,:,1) = red;
tumorOutlineInserted(:,:,2) = green;
tumorOutlineInserted(:,:,3) = blue;
figure
imshow(tumorOutlineInserted);
title('Detected Tumer','FontSize',20);
%% Display Together

 figure
subplot(231);imshow(s);title('Input image','FontSize',20);
subplot(232);imshow(inp);title('Filtered image','FontSize',20);
subplot(233);imshow(inp);title('Bounding Box','FontSize',20);

hold on;rectangle('Position',wantedBox,'EdgeColor','y');
hold off;
subplot(234);imshow(tumor);title('tumor alone','FontSize',20);
subplot(235);imshow(tumorOutline);title('TumorOutline','FontSize',20);
subplot(236);imshow(tumorOutlineInserted);title('Detected Tumor','FontSize',20);
```

3.1 Input Images

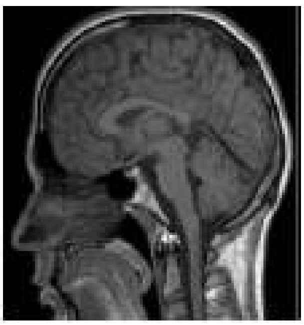

Figure. 1.

Figure. 2.

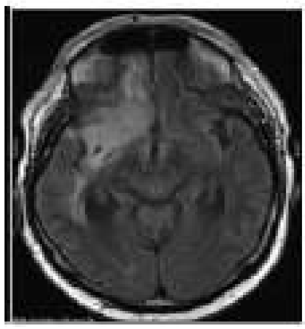

Figure. 3.

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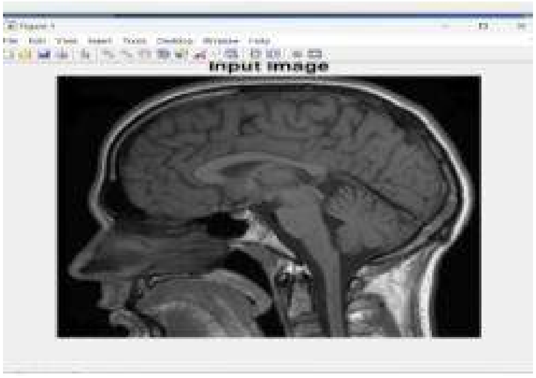
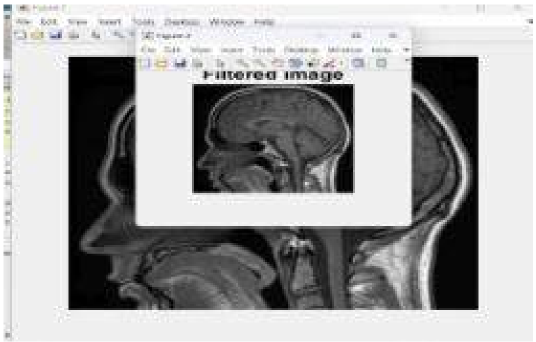
4 Results and Discussion

4.1 Output



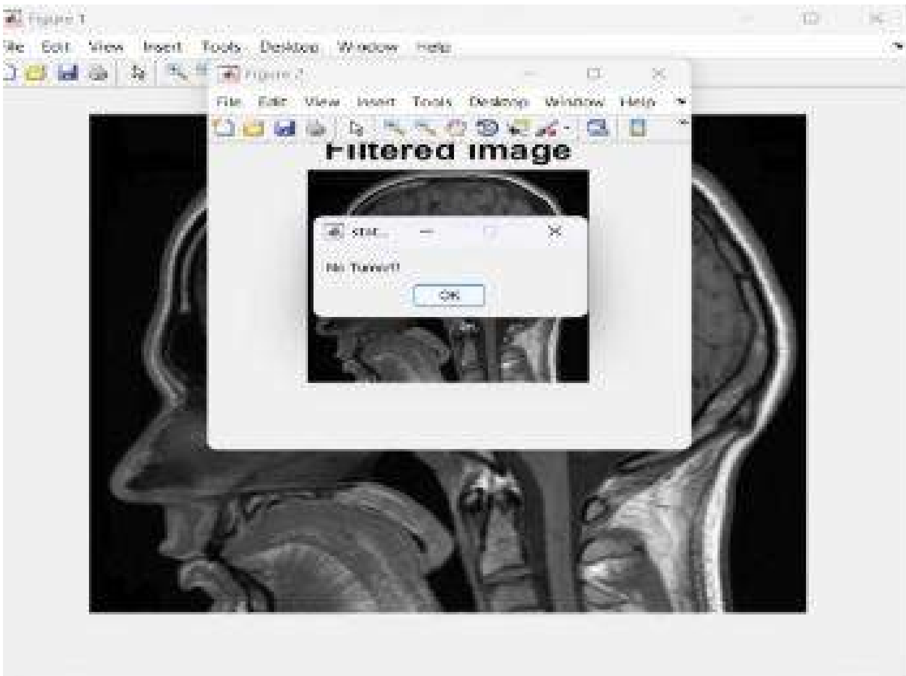
Input Image



Filtered Image

Output

Detected Tumour



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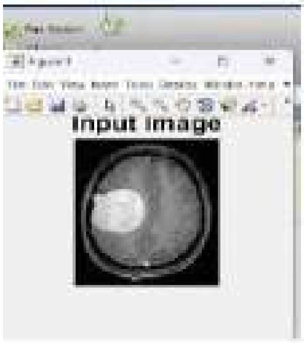
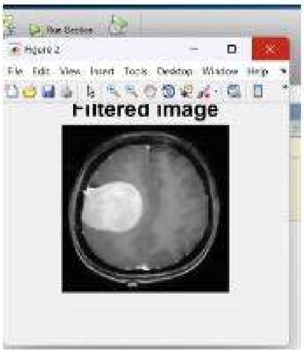
Output for Image 2

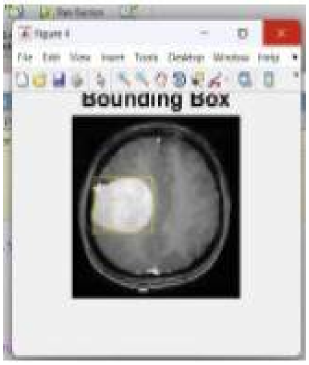


Input image

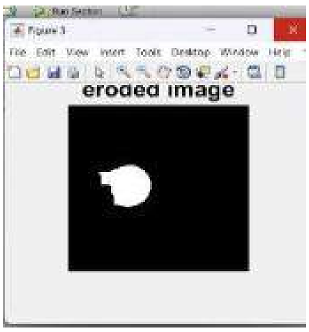


Filtered image

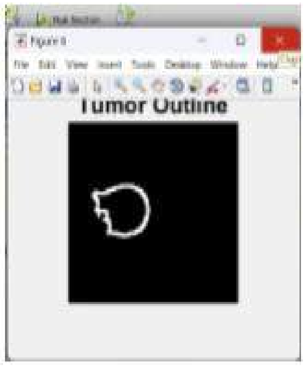
Tumour Alone



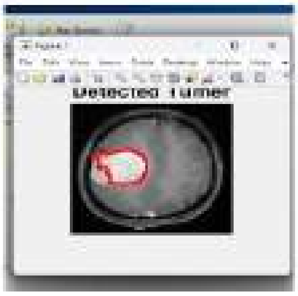
Bounding Box



Eroded image



Tumour Outline



Detected Tumour

Output image

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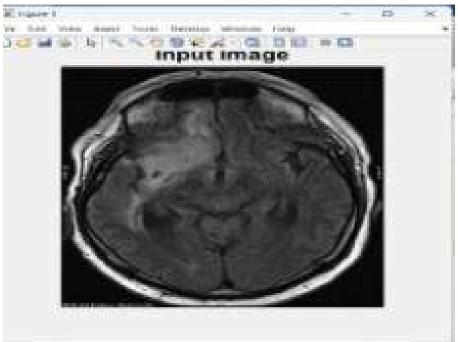
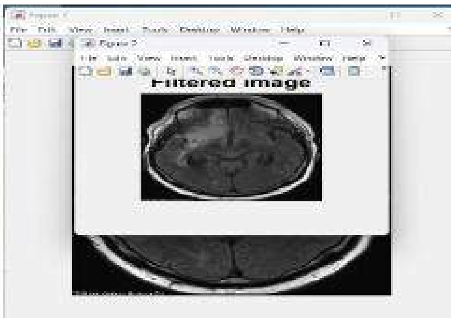
Output for Image 3

Input image



Filtered image



Tumour Detection

No Tumour

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5 Characteristics of Chimpanzees

Among all animals, the brains of great apes, particularly chimpanzees are the most similar to the human brain in terms of structure and function. This similarity is due to the close evolutionary relationship between humans and these primates. Chimpanzees, for example, share about 98% of their DNA with humans

⇒ similarities between the brains of humans and chimpanzees

1. Brain Structure
2. Complexity
3. Social Cognition
4. Tool Use
5. Emotional Expression

5.1 Input for Matlab

Anisotropic Diffusion

```
function diff_im = anisodiff_chimp(im, num_iter, delta_t, kappa, option)
fprintf('Removing noise\n');
fprintf('Filtering Completed !!');
% Convert input image to double.
im = double(im);
% PDE (partial differential equation) initial condition.
diff_im = im;
% Center pixel distances.
dx = 1;
dy = 1;
dd = sqrt(2);
% 2D convolution masks - finite difference
hN = [0 1 0; 0 -1 0; 0 0 0];
hS = [0 0 0; 0 -1 0; 0 1 0];
hE = [0 0 0; 0 -1 1; 0 0 0];
hW = [0 0 0; 1 -1 0; 0 0 0];
hNE = [0 0 1; 0 -1 0; 0 0 0];
hSE = [0 0 0; 0 -1 0; 0 0 1];
```

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```
hSW = [0 0 0; 0 −1 0; 1 0 0];
hNW = [1 0 0; 0 −1 0; 0 0 0];
% Anisotropic diffusion.
for t = 1:num_iter
    % Finite differences. [imfilter(.,.,'conv') can be replaced by conv2(.,.,'same')]
    nablaN = imfilter(diff_im,hN,'conv');
    nablaS = imfilter(diff_im,hS,'conv');
    nablaW = imfilter(diff_im,hW,'conv');
    nablaE = imfilter(diff_im,hE,'conv');
    nablaNE = imfilter(diff_im,hNE,'conv');
    nablaSE = imfilter(diff_im,hSE,'conv');
    nablaSW = imfilter(diff_im,hSW,'conv');
    nablaNW = imfilter(diff_im,hNW,'conv');
    % Diffusion function.
    if option == 1
        cN = exp(−(nablaN/kappa).^2);
        cS = exp(−(nablaS/kappa).^2);
        cW = exp(−(nablaW/kappa).^2);
        cE = exp(−(nablaE/kappa).^2);
        cNE = exp(−(nablaNE/kappa).^2);
        cSE = exp(−(nablaSE/kappa).^2);
        cSW = exp(−(nablaSW/kappa).^2);
        cNW = exp(−(nablaNW/kappa).^2);
    elseif option == 2
        cN = 1./(1 + (nablaN/kappa).^2);
        cS = 1./(1 + (nablaS/kappa).^2);
        cW = 1./(1 + (nablaW/kappa).^2);
        cE = 1./(1 + (nablaE/kappa).^2);
        cNE = 1./(1 + (nablaNE/kappa).^2);
        cSE = 1./(1 + (nablaSE/kappa).^2);
        cSW = 1./(1 + (nablaSW/kappa).^2);
        cNW = 1./(1 + (nablaNW/kappa).^2);
    end
 % Discrete PDE solution.
    diff_im = diff_im + ...
        delta_t*(...
        (1/(dy^2))*cN.*nablaN + (1/(dy^2))*cS.*nablaS + ...
```

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```
        (1/(dx^2))*cW.*nablaW + (1/(dx^2))*cE.*nablaE + ...
        (1/(dd^2))*cNE.*nablaNE + (1/(dd^2))*cSE.*nablaSE + ...
        (1/(dd^2))*cSW.*nablaSW + (1/(dd^2))*cNW.*nablaNW
        );
End
End
```

For Brain Tumour

```
clc;
close all;
clear all;

%% Input
[I, path] = uigetfile('*.jpg', 'Select an input image');
str = strcat(path, I);
s = imread(str);
figure;
imshow(s);
title('Input image', 'FontSize', 20);

%% Preprocessing
num_iter = 10;
delta_t = 1/7;
kappa = 15;
option = 2;
disp('Preprocessing image please wait . . .');
inp = anisodiff(s, num_iter, delta_t, kappa, option);
inp = uint8(inp);
inp = imresize(inp, [256,256]);
if size(inp, 3) > 1
    inp = rgb2gray(inp);
end

figure;
imshow(inp);
title('Filtered image', 'FontSize', 20);
```

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```
%% Thresholding
sout = imresize(inp, [256, 256]);
t0 = mean(s(:));
th = t0 + ((max(inp(:)) + min(inp(:))) / 2);
for i = 1:size(inp, 1)
    for j = 1:size(inp, 2)
        if inp(i, j) > th
            sout(i, j) = 1;
        else
            sout(i, j) = 0;
        end
    end
end

%% Morphological Operations
label = bwlabel(sout);
stats = regionprops(logical(sout), 'Solidity', 'Area', 'BoundingBox');
density = [stats.Solidity];
area = [stats.Area];
high_dense_area = density > 0.7;
max_area = max(area(high_dense_area));
tumor_label = find(area == max_area);
tumor = ismember(label, tumor_label);


if max_area > 200
    figure;
    imshow(tumor)
    title('Tumor alone', 'FontSize', 20);
else
    h = msgbox('No Tumor!!', 'status');
end


%% Bounding Box
box = stats(tumor_label);
wantedBox = box.BoundingBox;
```

```
figure
imshow(inp);
title('Bounding Box', 'FontSize', 20);
hold on;
rectangle('Position', wantedBox, 'EdgeColor', 'y');
hold off;


%% Getting Tumor Outline
dilationAmount = 5;
rad = floor(dilationAmount);
[r, c] = size(tumor);
filledImage = imfill(tumor, 'holes');
for i = 1:r
    for j = 1:c
        x1 = i - rad;
        x2 = i + rad;
        y1 = j - rad;
        y2 = j + rad;
        if x1 < 1
            x1 = 1;
        end
        if x2 > r
            x2 = r;
        end
        if y1 < 1
            y1 = 1;
        end
        if y2 > c
            y2 = c;
end
        erodedImage(i, j) = min(min(filledImage(x1:x2, y1:y2)));
    end
end

%% Subtracting eroded image from original BW image
tumorOutline = tumor;
```

```
tumorOutline(erodedImage) = 0;
figure;
imshow(tumorOutline);
title('Tumor Outline', 'FontSize', 20);

%% Inserting the outline in filtered image in red color
rgb = inp(:,:,[1 1 1]);
red = rgb(:, :, 1);
red(tumorOutline) = 255;
green = rgb(:, :, 2);
green(tumorOutline) = 0;
blue = rgb(:, :, 3);
blue(tumorOutline) = 0;


tumorOutlineInserted(:, :, 1) = red;
tumorOutlineInserted(:, :, 2) = green;
tumorOutlineInserted(:, :, 3) = blue;
figure
imshow(tumorOutlineInserted);
title('Detected Tumor', 'FontSize', 20);


%% Display Together
figure
subplot(231); imshow(s); title('Input image', 'FontSize', 20);
subplot(232); imshow(inp); title('Filtered image', 'FontSize', 20);
subplot(233); imshow(inp); title('Bounding Box', 'FontSize', 20);
hold on;
rectangle('Position', wantedBox, 'EdgeColor', 'y');
hold off;
subplot(234); imshow(tumor); title('Tumor alone', 'FontSize', 20);
subplot(235); imshow(tumorOutline); title('Tumor Outline', 'FontSize', 20);
subplot(236); imshow(tumorOutlineInserted); title('Detected Tumor', 'FontSize', 20);
```

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Input images

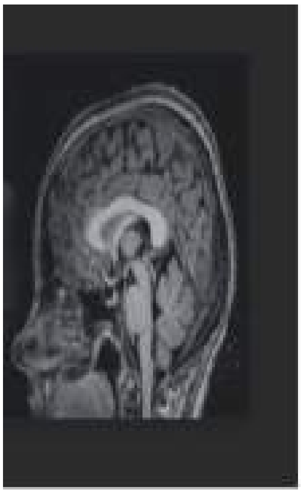


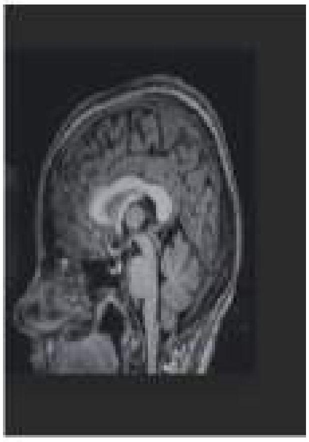
Figure. 1.

Figure. 2.

Figure. 3.

Results and Discussion

Output



Input image

Filtered image

Output

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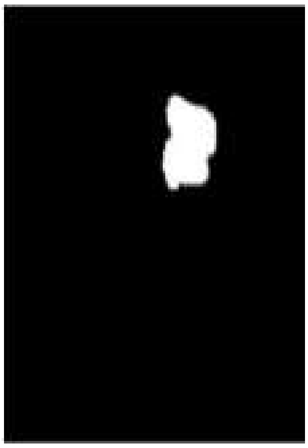
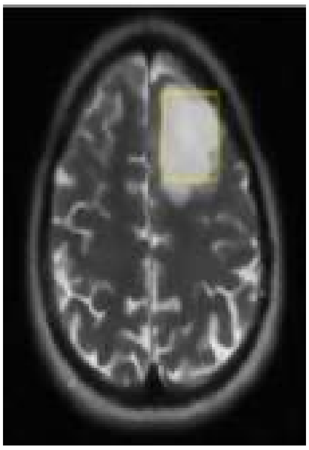
Output for Image 2

Input image

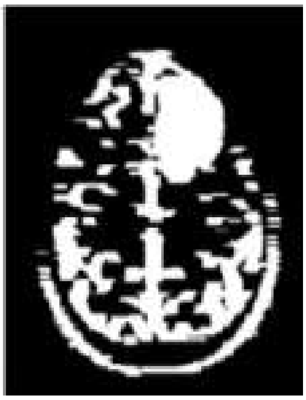
Filtered image



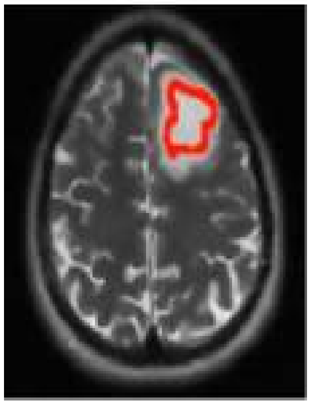
Tumour Alone



Bounding Box

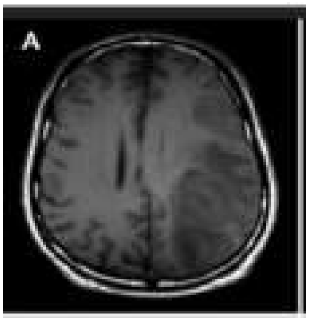


Eroded image

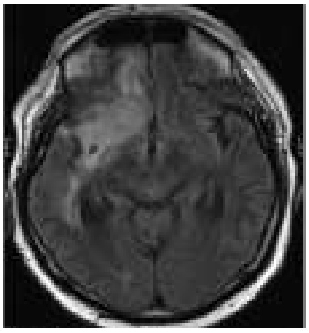


Detected Tumour

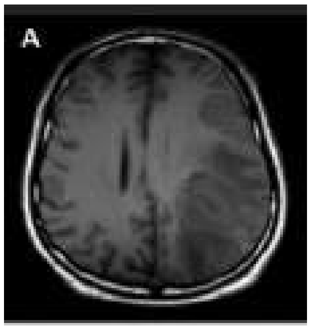
Output for Image 3



Input image



Filtered image



Tumour Detection

6 Conclusion

In conclusion, By applying mathematical models like PDEs to analyze brain imaging data, we can improve our ability to accurately identify and characterize tumors. Since brain tumours are a lethal form of cancer, early and precise identification is essential for effective therapy. Tumour detection by hand can be time-consuming and error-prone. This experiment suggests a technique for locating a tumour, if one is there, in a brain MRI scan. The image noise is removed using an isotropic filtering technique. The main advantage of this kind of detection is less time consuming for long time. Moreover, the use of PDEs allows for a more comprehensive understanding of tumor growth and behavior, leading to advancements in diagnostic techniques and therapeutic strategies for brain tumors in both humans and animals.

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[3] Sudharson M, Rajapandiyan SRT, Ilavarasi PU, Brain tumour detection by image processing using MATLAB, Middle-East journal of scientific research, 2016;24(SI):143-148.

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